International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 10 Issue 02, February-2021

Intuitionistic L-Fuzzy Structures in Z-Algebras

S. Sowmiya
Assistant Professor,
Department of Mathematics,
Sri Ramakrishna Engineering College
Vattamalaipalayam,Coimbatore-22,
Tamilnadu,India

P. Jeyalakshmi
Professor and Head,
Department of Mathematics,
Avinashilingam Institute for Home Science and Higher
Education for Women, Coimbatore-43,
Tamilnadu,India.

***Abstract*—In this article, we initiate and explore the idea of intuitionistic L-Fuzzy Z-Subalgebras and intuitionistic L-Fuzzy Z-ideals in Z-algebras. We further explore some of their properties of intuitionistic L-Fuzzy Z-Subalgebras and intuitionistic L-Fuzzy Z-ideals under Z-homomorphism and cartesian product in Z-algebras.**

2010 Mathematics Subject Classification. 03B20, 03B52

***Keywords*—Z-algebra, Z-ideal, Z-homomorphism, ,intuitionistic L-fuzzy Z-Subalgebra, intuitionistic L-fuzzy Z-ideal .**

I. INTRODUCTION

Atanassov and Stoeva [1] introduced the notion of intuitionistic L-fuzzy sets in 1984 as an extension of Goguen's [3] notion of L-fuzzy set. Here they have defined both membership and non-membership functions from the Universe of discourse X to the set L, where (L, ≤, ∧, ∨) is a complete lattice. Motivated by this, many mathematicians started to review various concepts and theorems in intuitionistic L-fuzzy structures. Currently, in the year 2017, Chandramouleeswaran et al.[2] introduced a new class of algebra called Z-algebra that arise from the notion of propositional calculi. In our previous articles [4, 5, 6, 7, 8, 9, 10, 11] we have introduced fuzzy Z-Subalgebras, fuzzy Z-ideals, fuzzy H-ideals, fuzzy p-ideals, fuzzy implicative ideals, intuitionistic fuzzy Z-Subalgebras and intuitionistic fuzzy Z-ideals in Z-algebras. In this article, we have initiated intuitionistic L-fuzzy Z-Subalgebras and intuitionistic L-fuzzy Z-Ideals in Z-algebras.

II. PRELIMINARIES

In this section, we recall some basic definitions that are required for our work

Definition 2.1[2] A Z-algebra $(X,*,0)$ is a nonempty set X with a constant 0 and a binary operation $*$ satisfying the following conditions:

(Z1) $x*0=0$

(Z2) $0*x=x$

(Z3) $x*x=x$

(Z4) $x*y=y*x$ when $x\neq 0$ and $y\neq 0$ $\forall$ x, y $\in$ X.

Definition 2.2[2] Let $(X,*,0)$ and $(Y,*',0')$ be two Z-algebras. A mapping $h:(X,*,0)\to(Y,*',0')$ is said to be a **Z-homomorphism** of Z-algebras if $h(x*y)=h(x)*'h(y)$ for all $x,y\in X$.

Definition 2.3:**[12]** Let X be a nonempty set. A fuzzy set A in X is characterized by a membership function μA which associates with each point x in X, a real number in the interval [0,1] with the value $\mu_A(x)$ representing the "grade of membership" of x in A. That is, a fuzzy set A in X is characterized by a membership function $\mu_A(x)$: X → [0, 1].

Definition 2.4:[1] Let (L, ≤, ∧, ∨) be a complete lattice with least element 0 and greatest element 1 and an involutive order reversing operation $N:L\to L$. Then Intuitionistic L-Fuzzy Set $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle \mid x\in X\}$ in a nonempty set X is an object having the form where $\mu_A:X\to L$ is the degree of membership function and $\nu_A:X\to L$ is the degree of non-membership function of the element $x\in X$ satisfying $\mu_A(x)\le N(\nu_A(x))$.

Definition 2.5:[1] Let $A=(\mu_A,\nu_A)$ and $B=(\mu_B,\nu_B)$ be any two intuitionistic L-fuzzy set of a set X. Then we have

1. $A^c=\{\langle x,\nu_A(x),\mu_A(x)\rangle \mid x\in X\}$
2. $A\cap B=\{\langle x,\mu_A(x)\wedge\mu_B(x),\nu_A(x)\vee\nu_B(x)\rangle \mid x\in X\}$
3. $\oplus A=(\mu_A,(\mu_A)^c)=\{\langle x,\mu_A(x),1-\mu_A(x)\rangle \mid x\in X\}$
4. $\otimes A=((\nu_A)^c,\nu_A)=\{\langle x,1-\nu_A(x),\nu_A(x)\rangle \mid x\in X\}$
5. For any subset T of X ∃ $x_0\in T$ such that $\mu_A(x_0)=\sup_{t\in T}\mu_A(t)$ and $\nu_A(x_0)=\inf_{t\in T}\nu_A(t)$.
is called sup-inf property of A.
6. The Cartesian product $A\times B=(\mu_{A\times B},\nu_{A\times B})$ whose membership function $\mu_{A\times B}:X\times X\to L$ and non-membership function $\nu_{A\times B}:X\times X\to L$ are defined by $\mu_{A\times B}(x,y)=\mu_A(x)\wedge\mu_B(y)$ and $\nu_{A\times B}(x,y)=\nu_A(x)\vee\nu_B(y)$ for all $x,y\in X$.

Definition 2.6:[1] Let h be a mapping from a set X into a set Y.

(i) Let $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle \mid x\in X\}$ be an intuitionistic L-fuzzy set in X. Then the image of A under h, denoted by $h(A)=\{\langle y,\mu_{h(A)}(y),\nu_{h(A)}(y)\rangle \mid y\in Y\}$ is an intuitionistic L-fuzzy set in Y, defined by:

$$\mu_{h(A)}(y)=\begin{cases}\sup\limits_{z\in h^{-1}(y)} \mu_A(z) & \text{if } \ h^{-1}(y)=\{x\mid h(x)=y\}\neq\phi\\ 0 & \text{otherwise}\end{cases}$$

and

$$\nu_{h(A)}(y)=\begin{cases}\inf\limits_{z\in h^{-1}(y)} \nu_A(z) & \text{if } \ h^{-1}(y)=\{x\mid h(x)=y\}\neq\phi\\ 1 & \text{otherwise}\end{cases}$$

is

(ii) Let $B=\{\langle y,\mu_A(y),\nu_A(y)\rangle \mid y\in Y\}$ be an intuitionistic fuzzy set in Y. The pre-image of B under h, symbolized by $h^{-1}(B)=\{\langle x,\mu_{h^{-1}(B)}(x),\nu_{h^{-1}(B)}(x)\rangle \mid x\in X\}$ defined by: $\mu_{h^{-1}(B)}(x)=\mu_B(h(x))$ and $\nu_{h^{-1}(B)}(x)=\nu_B(h(x))$ for all $x\in X$ is an intuitionistic L-fuzzy set of X.

III. INTUITIONISTIC L-FUZZY Z-SUBALGEBRAS IN Z-ALGEBRAS

In this section we introduce the notion of Intuitionistic L-Fuzzy Z-Subalgebra of a Z-algebra. Also we prove some interesting results.

Definition 3.1 : An **Intuitionistic L-fuzzy Set** $A=(\mu_A,\nu_A)$ in a Z-algebra $(X,*,0)$ is called an **Intuitionistic L-fuzzy Z-Subalgebra** of X if it satisfies the following conditions:

(i) $\mu_A(x*y)\geq\mu_A(x)\wedge\mu_A(y)$

(ii) $\nu_A(x*y)\leq\nu_A(x)\vee\nu_A(y)$ for all $x,y\in X$.

Example 3.2: Consider a Z-algebra X= {0,1,2,3} with the following Cayley table as in [8]:

*	0	1	2	3
0	0	1	2	3
1	0	1	3	2
2	0	3	2	1
3	0	2	1	3

An intuitionistic L-fuzzy set $A=(\mu_A,\nu_A)$ in X defined by

$$\mu_A(x)=\begin{cases}0.6 \text{ if } x=0\\ 0.4 \text{ if } x=1\\ 0.3 \text{ if } x=2,3\end{cases}$$ and

$$\nu_A(x)=\begin{cases}0.4 \text{ if } x=0\\ 0.5 \text{ if } x=1\\ 0.6 \text{ if } x=2,3\end{cases}$$

is an intuitionistic L-fuzzy Z-Subalgebra of X.

By applying the definition of intuitionistic L-fuzzy set, we can prove easily the following result.

Theorem 3.3: Let A_1 and A_2 be two intuitionistic L-fuzzy Z- Subalgebras of a Z-algebra X. Then $A_1\cap A_2$ is an intuitionistic L-fuzzy Z-Subalgebra of X.

We can generalize the above theorem as follows.

Corollary 3.4: Let $\{A_i \mid i\in\Omega\}$ be a family of intuitionistic L-fuzzy Z-Subalgebras of a Z-algebra X. Then $\bigcap\limits_{i\in\Omega} A_i$ is an intuitionistic L-fuzzy Z-Subalgebra of X.

By using the definition of A^c, we can prove the following result.

Theorem 3.5: An intuitionistic L-fuzzy set $A=(\mu_A,\nu_A)$ is an intuitionistic L-fuzzy Z-Subalgebra of a Z-algebra X if and only if the L-fuzzy sets μ_A and $(\nu_A)^c$ are L-fuzzy Z-Subalgebras of X.

Theorem 3.6: $A=(\mu_A,\nu_A)$ is an intuitionistic L-fuzzy Z-Subalgebra of a Z-algebra X if and only if (i) $\oplus A=(\mu_A,(\mu_A)^c)$ and (ii) $\otimes A=((\nu_A)^c,\nu_A)$, both are intuitionistic L-fuzzy Z-Subalgebras of X.

Proof: Let $A=(\mu_A,\nu_A)$ be an intuitionistic L-fuzzy Z-Subalgebra of a Z-algebra X.

Let $x,y\in X$. Then,

(i) $\mu_A(x*y)\geq\mu_A(x)\wedge\mu_A(y)$

(ii) $\nu_A(x*y)\leq\nu_A(x)\vee\nu_A(y)$

(iii) $(\mu_A)^c(x*y)=1-\mu_A(x*y)=(\mu_A)^c(x)\vee(\mu_A)^c(y)$

(iv)

$$(\nu_A)^c(x*y)=1-\nu_A(x*y)$$
$$=(\nu_A)^c(x)\wedge(\nu_A)^c(y)$$

From (i) and (iii), we get $\oplus A$ is an intuitionistic L-fuzzy Z-Subalgebra of X.

And, from (ii) and (iv), we get $\otimes A$ is an intuitionistic L-fuzzy Z-Subalgebra of X.

Conversely, assume that $\oplus A=(\mu_A,(\mu_A)^c)$ and $\otimes A=((\nu_A)^c,\nu_A)$ are intuitionistic L-fuzzy Z-Subalgebras of a Z-algebra X. For any $x,y\in A$,

$\mu_A(x*y)\geq\mu_A(x)\wedge\mu_A(y)$ and $\nu_A(x*y)\leq\nu_A(x)\vee\nu_A(y)$

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 10 Issue 02, February-2021

Hence $A=(\mu_A,\nu_A)$ is an intuitionistic L-fuzzy Z-Subalgebra of X.

Analogously, we can prove the following result.

Theorem 3.7: An intuitionistic L-fuzzy set $A=(\mu_A,\nu_A)$ in a Z-algebra X is an intuitionistic L-fuzzy Z-Subalgebra of X if and only if $U(\mu_A;s)$ and $L(\nu_A;t)$ are Z-Subalgebras of X for all $s,t\in[0,1]$.

As a consequence, we have the following corollary.

Corollary 3.8: Any Z-Subalgebra of a Z-algebra X can be realized as both the upper s-level and lower t-level Z-Subalgebras of some intuitionistic L-fuzzy Z-Subalgebras of X.

Analogously, the following theorems can be proved.

Theorem 3.9: Let X be a Z-algebra. Then any given chain of Z-Subalgebras $Q_0\subset Q_1\subset\ldots\subset Q_r=X$, there exists an intuitionistic L-fuzzy Z-Subalgebra A of X whose upper s-level and lower t-level Z-Subalgebras are exactly the Z-Subalgebras of this chain.

Theorem 3.10: Let A be an intuitionistic L-fuzzy Z-Subalgebra of a Z-algebra X. Then

(i) two upper s-level Z- Subalgebras $U(\mu_A;s_1)$ and $U(\mu_A;s_2)$ (with $s_1<s_2$) of A are equal if and only if there is no $x\in X$ such that $s_1\le\mu_A(x)<s_2$.

(ii) two lower t-level Z- Subalgebras $L(\nu_A;t_1)$ and $L(\nu_A;t_2)$ (with $t_1>t_2$) of A are equal if and only if there is no $x\in X$ such that $t_1\ge\nu_A(x)>t_2$

Theorem 3.11: Let X be a finite Z-algebra and A be an intuitionistic L-fuzzy Z-Subalgebra of X.

(i) If $Im(\mu_A)=\{s_1,\ldots,s_n\}$, then the family of Z-Subalgebras $U(\mu_A;s_i)$, $i=1,2,\ldots,n$ constitutes all the upper s-level Z-Subalgebras of A.

(ii) If $Im(\nu_A)=\{t_1,\ldots,t_r\}$, then the family of Z-Subalgebras $L(\nu_A;t_i)$, $i=1,2,\ldots,n$ constitutes all the lower t-level Z-Subalgebras of A.

Theorem 3.12: Let A be an intuitionistic L-fuzzy Z-Subalgebra of a Z-algebra X. Then

(i) If $Im(\mu_A)$ is finite, say $\{s_1,\ldots,s_n\}$, then for any $s_i,s_j\in Im(\mu_A)$, $U(\mu_A;s_i)=U(\mu_A;s_j)$ implies $s_i=s_j$.

(ii) If $Im(\nu_A)$ is finite, say $\{t_1,\ldots,t_n\}$, then for any $t_i,t_j\in Im(\nu_A)$, $L(\nu_A;t_i)=L(\nu_A;t_j)$ implies $t_i=t_j$.

Theorems 3.13: Let A and B be any two intuitionistic L-fuzzy Z-Subalgebras of a Z-algebra X. Then $A\times B$ is an intuitionistic L-fuzzy Z-Subalgebra of $X\times X$.

Theorem 3.14: Let h be a Z-homomorphism from a Z-algebra $(X,*,0)$ onto a Z-algebra $(Y,*',0')$ and $A=(\mu_A,\nu_A)$ be an intuitionistic L-fuzzy Z-Subalgebra of X with sup-inf property. Then the image $h(A)=\{\langle y,\mu_{h(A)}(y),\nu_{h(A)}(y)\rangle | y\in Y\}$ of A under h is an intuitionistic L-fuzzy Z-Subalgebra of Y.

Theorem 3.15 : Let $h:(X,*,0)\to(Y,*',0')$ be a Z-homomorphism of Z-algebras and B be an intuitionistic L-fuzzy Z-Subalgebra of Y. Then the inverse image of B, $h^{-1}(B)=\{\langle x,\mu_{h^{-1}(B)}(x),\nu_{h^{-1}(B)}(x)\rangle | x\in X\}$ is an intuitionistic L-fuzzy Z-Subalgebra of X. Converse is true if h is an Z-epimorphism.

Proof: If h is an Z-epimorphism and $h^{-1}(B)$ is an intuitionistic L-fuzzy Z-Subalgebra of a Z-algebra X and for y_1, y_2 ∈ Y there exists x_1, x_2 ∈ X such that $h(x_1)=y_1$ and $h(x_2)=y_2$.

This implies $x_1=h^{-1}(y_1)$ and $x_2=h^{-1}(y_2)$.

Now, $\mu_B(y_1*'y_2)=\mu_B(h(x_1)*'h(x_2))=\mu_B(h(x_1*x_2))$

$$=\mu_{h^{-1}(B)}(x_1*x_2)$$
$$\ge\mu_{h^{-1}(B)}(x_1)\wedge\mu_{h^{-1}(B)}(x_2)$$
$$=\mu_B(h(x_1))\wedge\mu_B(h(x_2))$$
$$=\mu_B(y_1)\wedge\mu_B(y_2)$$

Analogously, we can prove that $\nu_B(y_1*'y_2)\le\nu_B(y_1)\vee\nu_B(y_2)$

Thus B is an intuitionistic L-fuzzy Z-Subalgebra of a Z-algebra Y.

IV INTUITIONISTIC L-FUZZY Z-IDEALS IN Z-ALGEBRAS

In this section we introduce the notion of Intuitionistic L-fuzzy Z-ideal of a Z-algebra and some interesting results are obtained.

Definition 4.1: An intuitionistic L-fuzzy set $A=(\mu_A,\nu_A)$ in a Z-algebra $(X,*,0)$ is called an **intuitionistic L-fuzzy Z-ideal** of X if it satisfies the following conditions:

(i) $\mu_A(0)\ge\mu_A(x)$ and $\nu_A(0)\le\nu_A(x)$

(ii) $\mu_A(x)\ge\mu_A(x*y)\wedge\mu_A(y)$

(iii) $\nu_A(x)\le\nu_A(x*y)\vee\nu_A(y)$, for all $x,y\in X$.

Example 4.2: Let X={0,1,2,3} be a set with the following Cayley table as in [8]:

*	0	1	2	3
0	0	1	2	3
1	0	1	1	3
2	0	1	2	2
3	0	3	2	3

Then $(X,*,0)$ is a Z-algebra. Define an intuitionistic L-fuzzy set $A=(\mu_A,\nu_A)$ in X as follows: μ_A (x)=0.8 for all x = 0,1,2,3 and ν_A (x)=0.1 for all x = 0,1,2,3 .

Then, $A=(\mu_A,\nu_A)$ is an intuitionistic L-fuzzy Z-ideal of a Z-algebra X.

By applying the definition of an intuitionistic L-fuzzy set, we can easily prove the following result.

Theorem 4.3: Intersection of any two intuitionistic L-fuzzy Z-ideals of a Z-algebra X is again an intuitionistic L-fuzzy Z-ideal of X.

We generalize the above theorem as follows.

Theorem 4.4: Let $\{A_i \mid i\in\Omega\}$ be a family of intuitionistic L-fuzzy Z-ideals of a Z-algebra X. Then $\bigcap_{i\in\Omega} A_i$ is an intuitionistic L-fuzzy Z-Ideal of X.

Lemma 4.5: An intuitionistic L-fuzzy set $A=(\mu_A,\nu_A)$ is an intuitionistic L-fuzzy Z-ideal of a Z-algebra X if and only if the L-fuzzy sets μ_A and $(\nu_A)^c$ are L-fuzzy Z-ideals of X.

Theorem 4.6: Let $A=(\mu_A,\nu_A)$ be an intuitionistic L-fuzzy set in a Z-algebra X. Then $A=(\mu_A,\nu_A)$ is an intuitionistic L-fuzzy Z-ideal of X if and only if $\oplus A=(\mu_A,(\mu_A)^c)$ and $\otimes A=((\nu_A)^c,\nu_A)$ are intuitionistic L-fuzzy Z-ideals of X.

Theorem 4.7: An intuitionistic L-fuzzy set $A=(\mu_A,\nu_A)$ is an intuitionistic L-fuzzy Z-ideal of a Z-algebra X if and only if for all $s,t\in L$, the sets $U(\mu_A;s)$ and $L(\nu_A;t)$ are either empty or Z-ideals of X.

Theorem 4.8: Let h be a homomorphism from a Z-algebra $(X,*,0)$ onto a Z-algebra $(Y,*',0')$ and A be an intuitionistic L-fuzzy Z-ideal of X with sup-inf property. Then image of A, $h(A)=\{\langle y,\mu_{h(A)}(y),\nu_{h(A)}(y)\rangle \mid y\in Y\}$ is an intuitionistic L-fuzzy Z-ideal of Y.

Theorem 4.9: Let $h:(X,*,0)\to(Y,*',0')$ be a Z-homomorphism of Z-algebras and B be an intuitionistic L-fuzzy Z-ideal of Y. Then the inverse image of B, $h^{-1}(B)=\{\langle x,\mu_{h^{-1}(B)}(x),\nu_{h^{-1}(B)}(x)\rangle \mid x\in X\}$ is an intuitionistic L-fuzzy Z-ideal of X.

Theorem 4.10: Let $h:(X,*,0)\to(Y,*',0')$ be an Z-epimorphism of Z-algebras. Let B be an intuitionistic L-fuzzy set of Y. If $h^{-1}(B)$ is an intuitionistic L-fuzzy Z-ideal of X then B is an intuitionistic L-fuzzy Z-ideal of Y.

Proof: Assume that If $h^{-1}(B)$ is an intuitionistic L-fuzzy Z-ideal of X.

Let y $\in$ Y, there exists x $\in$ X such that h (x) = y. Then (i)

$$\mu_B(y)=\mu_B(h(x))=\mu_{h^{-1}(B)}(x)\le\mu_{h^{-1}(B)}(0)=\mu_B(h(0))=\mu_B(0')$$

(ii) $$\nu_B(y)=\nu_B(h(x))=\nu_{h^{-1}(B)}(x)\ge\nu_{h^{-1}(B)}(0)=\nu_B(h(0))=\nu_B(0')$$

Let x, y $\in$ Y. Then there exists a, b $\in$ X such that h(a) = x and h (b) = y. It follows that

(iii) $$\begin{aligned}\mu_B(x)&=\mu_B(h(a))=\mu_{h^{-1}(B)}(a)\\&\ge\mu_{h^{-1}(B)}(a*b)\wedge\mu_{h^{-1}(B)}(b)\\&=\mu_B(h(a*b))\wedge\mu_B(h(b))\\&=\mu_B(h(a)*'h(b))\wedge\mu_B(h(b))\\&=\mu_B(x*'y)\wedge\mu_B(y')\end{aligned}$$

(iv) $$\begin{aligned}\nu_B(x)&=\nu_B(h(a))=\nu_{h^{-1}(B)}(a)\\&\le\nu_{h^{-1}(B)}(a*b)\vee\nu_{h^{-1}(B)}(b)\\&=\nu_B(h(a*b))\vee\nu_B(h(b))\\&=\nu_B(h(a)*'h(b))\vee\nu_B(h(b))\\&=\nu_B(x*'y')\vee\nu_B(y')\end{aligned}$$

Hence B is an intuitionistic L-fuzzy Z-ideal of Y.

Theorem 4.11: Let A and B be two intuitionistic L-fuzzy Z-ideals in a Z-algebra X. Then $A\times B$ is an intuitionistic L-fuzzy Z-ideal of $X\times X$.

Proof: Take $(x_1,x_2)\in X\times X$.

Then $$\mu_{A\times B}(0,0)=\mu_A(0)\wedge\mu_B(0)\ge\mu_A(x_1)\wedge\mu_B(x_2)=\mu_{A\times B}(x_1,x_2)$$

and $$\nu_{A\times B}(0,0)=\nu_A(0)\vee\nu_B(0)\le\nu_A(x_1)\vee\nu_B(x_2)=\nu_{A\times B}(x_1,x_2)$$

Now take $(x_1,x_2),(y_1,y_2)\in X\times X$. Then

$$\mu_{A\times B}(x_1,x_2)=\mu_A(x_1)\wedge\mu_B(x_2)$$

$$\ge(\mu_A(x_1*y_1)\wedge\mu_A(y_1))\wedge(\mu_B(x_2*y_2)\wedge\mu_B(y_2))$$

$$\begin{aligned}&=(\mu_A(x_1*y_1)\wedge\mu_B(x_2*y_2))\wedge(\mu_A(y_1)\wedge\mu_B(y_2))\\&=\mu_{A\times B}((x_1*y_1),(x_2*y_2))\wedge\mu_{A\times B}(y_1,y_2)\\&=\mu_{A\times B}((x_1,x_2)*(y_1,y_2))\wedge\mu_{A\times B}(y_1,y_2)\end{aligned}$$

$$\beta_{A\times B}(x_1,x_2)=\beta_A(x_1)\vee\beta_B(x_2)$$

$$\begin{aligned}&\le(\beta_A(x_1*y_1)\vee\beta_A(y_1))\vee(\beta_B(x_2*y_2)\vee\beta_B(y_2))\\&=(\beta_A(x_1*y_1)\vee\beta_B(x_2*y_2))\vee(\beta_A(y_1)\vee\beta_B(y_2))\\&=\beta_{A\times B}((x_1*y_1),(x_2*y_2))\vee\beta_{A\times B}(y_1,y_2)\end{aligned}$$

$$= \beta_{A\times B}((x_1,x_2)*(y_1,y_2)) \vee \beta_{A\times B}(y_1,y_2)$$

Hence $A\times B$ is an intuitionistic L-fuzzy Z-ideal of $X\times X$.

Theorem 4.12: Let A and B be two intuitionistic L-fuzzy sets in a Z-algebra X. If $A\times B$ is an intuitionistic L-fuzzy Z-ideal of $X\times X$, the following are true.

(i) $\mu_A(0)\geq\mu_B(y)$ and $\mu_B(0)\geq\mu_A(x)$ for all $x,y\in X$.

(ii) $\nu_A(0)\leq\nu_B(y)$ and $\nu_B(0)\leq\nu_A(x)$ for all $x,y\in X$.

Theorem 4.13: Let A and B be two intuitionistic L-fuzzy sets in a Z-algebra X such that $A\times B$ is an intuitionistic L-fuzzy Z-ideal of $X\times X$. Then either A or B is an intuitionistic L-fuzzy Z-Ideal of X.

V CONCLUSION

In this article, we have introduced intuitionistic L-fuzzy Z-Subalgebras and intuitionistic L-fuzzy Z-ideals in Z-algebras and discussed their properties. We extend this concept in our research work.

VI ACKNOWLEDGMENT

Authors wish to thank Dr.M.Chandramouleeswaran, Professor and Head, PG Department of Mathematics, Sri Ramanas College of Arts and Science for Women, Aruppukottai, for his valuable suggestions to improve this paper a successful one.

REFERENCES

[1] K.T. Atanassov and S.Stoeva, “Intuitionistic L-Fuzzy Sets”, Cybernetics and Systems Research,2(1984) , pp.539–540.

[2] M.Chandramouleeswaran,P.Muralikrishna,K.Sujatha and S.Sabarinathan, “A note on Z-algebra”, Italian Journal of Pure and Applied Mathematics-N. 38 (2017) , pp.707–714.

[3] J.A.Goguen, “L-Fuzzy Sets”,J.Math.Anal.Appl. 18 (1967), pp. 145–174.

[4] S.Sowmiya and P.Jeyalakshmi, “Fuzzy Algebraic Structure in Z-Algebras”, World Journal of Engineering Research and Technology, 5(4) (2019) ,pp.74–88.

[5] S.Sowmiya and P.Jeyalakshmi.P., “On Fuzzy Z-ideals in Z-algebras”, Global Journal of Pure and Applied Mathematics, 15(4),(2019), pp. 505–516.

[6] S.Sowmiya and P.Jeyalakshmi, “Fuzzy α-Translations and Fuzzy β-Multiplications of Z-algebras”, Advances in Mathematics : Scientific Journal, 9(3) (2020) ,pp.1287–1292.

[7] S.Sowmiya and P.Jeyalakshmi, “Z-Homomorphism and Cartesian product on Fuzzy α-Translations and Fuzzy β-Multiplications of Z-algebras”,AIP Conference Proceedings 2261 (2020) , pp.030098-1–030098-5.

[8] S.Sowmiya and P.Jeyalakshmi, “Intuitionistic Fuzzy Sets in Z-Algebras”, Journal of Advanced Mathematical Studies, 13(3) (2020), pp.302–310.

[9] S.Sowmiya and P.Jeyalakshmi, “On Fuzzy H-ideals in Z-Algebras”(Submitted).

[10] S.Sowmiya and P.Jeyalakshmi, “On Fuzzy p-ideals in Z-Algebras”(Submitted).

[11] S.Sowmiya and P.Jeyalakshmi, “On Fuzzy implicative ideals in Z-Algebras”(Submitted).

[12] L.A.Zadeh, “Fuzzy sets”, Information and Control,8 (1965), pp.338–353