Intuitionistic Fuzzy Almost Generalized $\beta$ Closed Mappings

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ABSTRACT:
In this paper we introduce intuitionistic fuzzy almost generalized $\beta$ closed mappings and intuitionistic fuzzy almost generalized $\beta$ open mappings. We investigate some of their properties. Also we provide the relation between intuitionistic fuzzy almost generalized $\beta$ closed mappings and other intuitionistic fuzzy closed mappings.

Key words and phrases : Intuitionistic fuzzy topology, intuitionistic fuzzy generalized $\beta$ $T_{1/2}$ space, intuitionistic fuzzy almost generalized $\beta$ closed mappings.

1. Introduction

The notion of intuitionistic fuzzy sets is introduced by Atanassov [1]. Using this notion, Coker [2] introduced the notion of intuitionistic fuzzy topological spaces. D. Jayanthi [4] introduced the notion of intuitionistic fuzzy generalized $\beta$ closed mappings and intuitionistic fuzzy generalized $\beta$ open mappings. In this paper we introduce intuitionistic fuzzy almost generalized $\beta$ closed mappings. We investigate some of its properties. Also we provide the relation between an intuitionistic fuzzy almost generalized $\beta$ closed mapping and other intuitionistic fuzzy closed mappings.

2. Preliminaries

Definition 2.1: [1] An intuitionistic fuzzy set (IFS) $A$ in $X$ is an object having the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$$

where the functions $\mu_A: X \rightarrow [0,1]$ and $\nu_A: X \rightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set $A$, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2: [1] Let $A$ and $B$ be IFSs of the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \} \quad \text{and} \quad B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}.$$ 

Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$

(d) $A \cap B = \{ \langle x, \mu_A(x) \land \mu_B(x), \nu_A(x) \lor \nu_B(x) \rangle / x \in X \}$

(e) $A \cup B = \{ \langle x, \mu_A(x) \lor \mu_B(x), \nu_A(x) \land \nu_B(x) \rangle / x \in X \}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$. 

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The intuitionistic fuzzy sets $0_\ldots = \{\langle x, 0, 1\rangle / x \in X\}$ and $1_\ldots = \{\langle x, 1, 0\rangle / x \in X\}$ are respectively the empty set and the whole set of $X$.

**Definition 2.3:** [2] Let $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (IFP), written as $p_{(\alpha, \beta)}$, is defined to be an IFS of $X$ given by

$$p_{(\alpha, \beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0, 1) & \text{otherwise.} \end{cases}$$

**Definition 2.4:** [2] An intuitionistic fuzzy topology (IFT) on $X$ is a family $\tau$ of IFSs in $X$ satisfying the following axioms.

(i) $0_\ldots, 1_\ldots \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair $(X, \tau)$ is called an intuitionistic fuzzy topological space (IFTS) and any IFS in $\tau$ is known as an intuitionistic fuzzy open set (IFOS) in $X$. The complement $A^c$ of an IFOS $A$ in IFTS $(X, \tau)$ is called an intuitionistic fuzzy closed set (IFCS) in $X$.

**Definition 2.5:** [2] Let $(X, \tau)$ be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in $X$. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$$\text{int} (A) = \cup \{G / G \text{ is an IFOS in } X \text{ and } G \subseteq A\}$$

$$\text{cl} (A) = \cap \{K / K \text{ is an IFCS in } X \text{ and } A \subseteq K\}.$$

**Definition 2.6:** [3] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS $(X, \tau)$ is said to be an

(i) intuitionistic fuzzy regular closed set (IFRCS) if $\text{cl} (\text{int} (A)) = A$

(ii) intuitionistic fuzzy semi closed set (IFSCS) if $\text{int} (\text{cl} (A)) \subseteq A$

(iii) intuitionistic fuzzy pre closed set (IFPCS) if $\text{cl} (\text{int} (A)) \subseteq A$

(iv) intuitionistic fuzzy a closed set (IFaCS) if $\text{cl} (\text{int} (\text{cl} (A))) \subseteq A$

(v) intuitionistic fuzzy $\beta$ closed set (IF$\beta$CS) if $\text{int} (\text{cl} (\text{int} (A))) \subseteq A$

The respective complements of the above IFCSs are called their respective IFOSs.

**Definition 2.7:** [2] Let $A$ be an IFS in an IFTS $(X, \tau)$. Then the $\beta$ interior and the $\beta$ closure of $A$ are defined by

$$\beta \text{int} (A) = \cup \{G / G \text{ is an IF$\beta$OS in } X \text{ and } G \subseteq A\}$$

$$\beta \text{cl} (A) = \cap \{K / K \text{ is an IF$\beta$CS in } X \text{ and } A \subseteq K\}.$$

**Definition 2.8:** [5] An IFS $A$ in an IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy generalized $\beta$ closed set (IFG$\beta$CS) if $\beta \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IFOS in $(X, \tau)$. The complement $A^c$, is called an intuitionistic fuzzy generalized $\beta$ open set (IFG$\beta$OS) in $X$.

**Definition 2.9:** [3] Let $p_{(\alpha, \beta)}$ be an IFP of $(X, \tau)$. An IFS $A$ of $X$ is called an intuitionistic fuzzy neighborhood (IFN) of $p_{(\alpha, \beta)}$ if there exists an IFOS $B$ in $X$ such that $p_{(\alpha, \beta)} \in B \subseteq A$. 

**Definition 2.10:** [5] If every IFG\(\beta\)CS in \((X, \tau)\) is an IF\(\beta\)CS in \((X, \tau)\), then the space can be called as an intuitionistic fuzzy \(\beta\) \(T_{1/2}\) space (IF\(\beta\)\(T_{1/2}\) space).

**Definition 2.11:** [3] A map \(f: X \rightarrow Y\) is called an intuitionistic fuzzy closed mapping (IFCM) if \(f(A)\) is an IFCS in \(Y\) for each IFCS \(A\) in \(X\).

**Definition 2.12:** [3]
(i) intuitionistic fuzzy semi open mapping (IFSOM) if \(f(A)\) is an IFOS in \(Y\) for each IFOS \(A\) in \(X\).
(ii) intuitionistic fuzzy \(\alpha\) open mapping (IF\(\alpha\)OM) if \(f(A)\) is an IF\(\alpha\)OS in \(Y\) for each IFOS \(A\) in \(X\).
(iii) intuitionistic fuzzy preopen mapping (IFPOM) if \(f(A)\) is an IFPOS in \(Y\) for each IFOS \(A\) in \(X\).
(iv) intuitionistic fuzzy \(\beta\) open mapping (IF\(\beta\)OM) if \(f(A)\) is an IF\(\beta\)OS in \(Y\) for each IFOS \(A\) in \(X\).

**Definition 2.13:** [3] The intuitionistic fuzzy semi closure and the intuitionistic fuzzy \(\alpha\) closure of an IFS \(A\) in an IFTS \((X, \tau)\) are defined by
\[
\text{scl}(A) = \cap \{ K / K \text{ is an IFSCS in } X \text{ and } A \subseteq K \}.
\]
\[
\alpha\text{cl}(A) = \cap \{ K / K \text{ is an IF}\alpha\text{CS in } X \text{ and } A \subseteq K \}.
\]

**Definition 2.14:** [5] Let \(p(\alpha, \beta)\) be an IFP in \((X, \tau)\). An IFS \(A\) of \(X\) is called an intuitionistic fuzzy \(\beta\) neighborhood (IF\(\beta\)N) of \(p(\alpha, \beta)\) if there is an IF\(\beta\)OS \(B\) in \(X\) such that \(p(\alpha, \beta) \in B \subseteq A\).

**Definition 2.15:** [4] A mapping \(f : X \rightarrow Y\) is said to be an intuitionistic fuzzy generalized \(\beta\) closed mapping (IFG\(\beta\)CM) if \(f(A)\) is an IFG\(\beta\)CS in \(Y\) for every IFCS \(A\) in \(X\).

**Definition 2.16:** [4] A mapping \(f : X \rightarrow Y\) is said to be an intuitionistic fuzzy M-generalized \(\beta\) closed mapping (IFMG\(\beta\)CM) if \(f(A)\) is an IFG\(\beta\)CS in \(Y\) for every IFG\(\beta\)CS \(A\) in \(X\).

**Definition 2.17:** [5] An IFS \(A\) is said to be an intuitionistic fuzzy dense (IFD) in another IFS \(B\) in an IFTS \((X, \tau)\), if \(\text{cl}(A) = B\).

**Definition 2.18:** [6] A mapping \(f : X \rightarrow Y\) is said to be an intuitionistic fuzzy generalized \(\beta\) continuous mapping (IFG\(\beta\)cts.M) if \(f^{-1}(A)\) is an IFG\(\beta\)CS in \(X\) for every IFCS \(A\) in \(Y\).

3. Intuitionistic fuzzy almost generalized \(\beta\) closed mappings and intuitionistic fuzzy almost generalized \(\beta\) open mappings.

In this section we introduce intuitionistic fuzzy almost generalized \(\beta\) closed mappings and intuitionistic fuzzy almost generalized \(\beta\) open mappings. We study some of their properties.
Definition 3.1: A map $f: X \to Y$ is called an intuitionistic fuzzy almost generalized $\beta$ closed mapping (IF$\alpha$g$\beta$CM) if $f(A)$ is an IFG$\beta$CS in $Y$ for each IFRCS $A$ in $X$.

Example 3.2: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.5, 0.4), (0.5, 0.6) \rangle$, $G_2 = \langle y, (0.2, 0.3), (0.8, 0.7) \rangle$. Then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are IFTs on $X$ and $Y$ respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then $f$ is an IF$\alpha$g$\beta$CM.

The relation between various types of intuitionistic fuzzy closedness is given in the following diagram.

The reverse implications are not true in general in the above diagram. This can be seen from the following examples.

Example 3.3: In Example 3.2, $f$ is an IF$\alpha$g$\beta$CM but not an IFCM, since $G_1^c = \langle x, (0.5, 0.6), (0.5, 0.4) \rangle$ is an IFCS in $X$ but $f(G_1^c) = \langle y, (0.5, 0.6), (0.5, 0.4) \rangle$ is not an IFCS in $Y$.

Example 3.4: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.5, 0.4), (0.5, 0.6) \rangle$, $G_2 = \langle y, (0.5, 0.4), (0.2, 0.3) \rangle$. Then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are IFTs on $X$ and $Y$ respectively. Define a mapping $f: X \to Y$ by $f(a) = u$ and $f(b) = v$. Then $f$ is an IF$\alpha$g$\beta$CM but not an IFSCM, since $G_1^c = \langle x, (0.5, 0.6), (0.5, 0.4) \rangle$ is an IFCS in $X$ but $f(G_1^c) = \langle y, (0.5, 0.6), (0.5, 0.4) \rangle$ is not an IFCS in $Y$, since $\text{int}(\text{cl}(f(G_1^c))) = 1 \not\subset f(G_1^c)$.

Example 3.5: In Example 3.4 $f$ is an IF$\alpha$g$\beta$CM but not an IFCM, since $G_1^c = \langle x, (0.5, 0.6), (0.5, 0.4) \rangle$ is an IFCS in $X$ but $f(G_1^c) = \langle y, (0.5, 0.6), (0.5, 0.4) \rangle$ is not an IFCS in $Y$, since $\text{cl}(\text{int}(f(G_1^c))) = G_2^c \not\subset f(G_1^c)$.

Example 3.6: In Example 3.2 $f$ is an IF$\alpha$g$\beta$CM but not an IFCM, since $G_1^c = \langle x, (0.5, 0.6), (0.5, 0.4) \rangle$ is an IFCS in $X$ but $f(G_1^c) = \langle y, (0.5, 0.6), (0.5, 0.4) \rangle$ is not an IFCS in $Y$, since $\text{cl}(\text{int}(f(G_1^c))) = G_2^c \not\subset f(G_1^c)$.

Example 3.7: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.2, 0.2), (0.4, 0.4) \rangle$, $G_2 = \langle x, (0.2, 0), (0.5, 0.4) \rangle$, $G_3 = \langle y, (0.5, 0.6), (0.2, 0) \rangle$ and $G_4 = \langle x, (0.4, 0.1), (0.2, 0.1) \rangle$. Then $\tau = \{0, G_1, G_2, 1\}$ and $\sigma = \{0, G_3, G_4, 1\}$ are IFTs on $X$ and $Y$ respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then $f$ is an IF$\alpha$g$\beta$M but
not an IFGβCM, since $G_2^c = \langle x, (0.5, 0.4), (0.2, 0) \rangle$ is an IFCS in X but $f(G_2^c) = \langle y, (0.5, 0.4), (0.2, 0) \rangle$ is not an IFGβCS in Y, since $f(G_2^c) \subseteq G_3$ but $\beta\text{cl}(f(G_2^c)) = 1. \not\subseteq G_3$.

**Example 3.8:** In Example 3.7 $f$ is an IFaGβCM but not an IFMGβCM, since $A = \langle x, (0.4, 0.2), (0.2, 0) \rangle$ is an IFGβCS in X but $f(A) = \langle y, (0.4, 0.2), (0.2, 0) \rangle$ is not an IFGβCS in Y, since $f(A) \subseteq G_3$ but $\beta\text{cl}(f(A)) = 1. \not\subseteq G_3$.

**Definition 3.9:** A map $f$: $X \rightarrow Y$ is called an intuitionistic fuzzy almost generalized $\beta$ open mapping (IFaGβOM) if $f(A)$ is an IFGβOS in Y for each IFROS A in X.

An IFaGβOM is an IFaGβCM if it is a bijective mapping.

**Theorem 3.10:** Let $p(\alpha, \beta)$ be an IFP in X. A bijective mapping $f$: $X \rightarrow Y$ is an IFaGβCM if for every IFOS A in X with $f^{-1}(p(\alpha, \beta)) \in A$, there exists an IFOS B in Y with $p(\alpha, \beta) \in B$ such that $f(A)$ is IFD in B.

**Proof:** Let A be an IFROS in X. Then A is an IFOS in X. Let $f^{-1}(p(\alpha, \beta)) \in A$, then there exists an IFOS B in Y such that $p(\alpha, \beta) \in B$ and $cl(f(A)) = B$. Since B is an IFOS, $cl(f(A)) = B$ is also an IFOS in Y. Therefore $int(cl(f(A))) = cl(f(A))$. Now $f(A) \subseteq cl(f(A)) \subseteq cl(int(cl(f(A))))$. This implies $f(A)$ is an IFβOS in Y and hence an IFGβOS in Y. Thus f is an IFaGβCM.

**Theorem 3.11:** Let f: $X \rightarrow Y$ be a bijective mapping where Y is an IFβT_{1/2} space. Then the following are equivalent

(i) $f$ is an IFaGβCM.
(ii) $\beta\text{cl}(f(A)) \subseteq f(cl(A))$ for every IFβOS A in X
(iii) $\beta\text{cl}(f(A)) \subseteq f(cl(A))$ for every IF βOS A in X
(iv) $f(A) \subseteq \beta\text{int}(f(int(cl(A))))$ for every IFPOS A in X.

**Proof:** (i) ⇒ (ii) Let A be an IFβOS in X. Then $cl(A)$ is an IFRCS in X. By hypothesis $f(A)$ is an IFGβCS in Y and hence is an IFβCS in Y, since Y is an IFβT_{1/2} space. This implies $\beta\text{cl}(f(cl(A))) = f(cl(A))$. Now $\beta\text{cl}(f(A)) \subseteq \beta\text{cl}(f(cl(A))) = f(cl(A))$. Thus $\beta\text{cl}(f(A)) \subseteq f(cl(A))$.

(ii) ⇒ (iii) Since every IFPOS is an IFβOS, the proof directly follows.

(iii) ⇒ (i) Let A be an IFRCS in X. Then $A = cl(int(A))$. Therefore A is an IFPOS in X. By hypothesis, $\beta\text{cl}(f(A)) \subseteq f(cl(A)) = f(A) \subseteq \beta\text{cl}(f(A))$. Hence $f(A)$ is an IFβCS and hence is an IFGβCS in Y. Thus f is an IFaGβCM.

(i) ⇒ (iv) Let A be an IFPOS in X. Then $A \subseteq int(cl(A))$. Since $int(cl(A))$ is an IFROS in X, by hypothesis, $f(int(cl(A)))$ is an IFGβOS in Y. Since Y is an IFβT_{1/2} space, $f(int(cl(A)))$ is an IFβOS in Y. Therefore $f(A) \subseteq f(int(cl(A))) \subseteq \beta\text{int}(f(int(cl(A))))$.

(iv) ⇒ (i) Let A be an IFROS in X. Then A is an IFPOS in X. By hypothesis, $f(A) \subseteq \beta\text{int}(f(int(cl(A)))) = \beta\text{int}(f(A)) \subseteq f(A)$. This implies $f(A)$ is an IFβOS in Y and hence is an IFGβOS in Y. Therefore f is an IFaGβCM.

**Theorem 3.12:** Let f: $X \rightarrow Y$ be a map. Then f is an IFaGβCM if for each IFP $p(\alpha, \beta) \in Y$ and for each IFβOS B in X such that $f^{-1}(p(\alpha, \beta)) \in B$, $\beta\text{cl}(f(B))$ is an IFβN of $p(\alpha, \beta) \in Y$.

**Proof:** Let $p(\alpha, \beta) \in Y$ and let A be an IFROS in X. Then A is an IFβOS in X. By hypothesis $f^{-1}(p(\alpha, \beta)) \in A$, that is $p(\alpha, \beta) \in f(A)$ in Y and $\beta\text{cl}(f(A))$ is an IFβN of $p(\alpha, \beta)$.
in Y. Therefore there exists an IFβOS B in Y such that p(α, β) ∈ B ⊆ βcl(f(A)). We have p(α, β) ∈ f(A) ⊆ βcl(f(A)). Now B = ∪ { p(α, β) / p(α, β) ∈ B } = f(A). Therefore f(A) is an IFβOS in Y and hence an IFGβOS in Y. Thus f is an IFaGβOM. By Theorem f is an IFaGβCM.

**Theorem 3.13:** Let f: X → Y be a mapping where Y is an IFβT_{1/2} space. Then the following are equivalent.

(i) f is an IFaGβOM
(ii) for each IFP p(α, β) in Y and each IFROS B in X such that f^{-1}(p(α, β)) ∈ B, cl(f(cl(B))) is an IFβN of p(α, β) in Y.

**Proof:** (i) ⇒ (ii) Let p(α, β) ∈ Y and let B be an IFROS in X such that f^{-1}(p(α, β)) ∈ B. That is p(α, β) ∈ f(B). By hypothesis, f(B) is an IFGβOS in Y. Since Y is an IFβT_{1/2} space, f(B) is an IFβOS in Y. Now p(α, β) ∈ f(B) ⊆ f(cl(B)) ⊆ cl(f(cl(B))). Hence cl(f(cl(B))) is an IFβN of p(α, β) in Y.

(ii) ⇒ (i) Let B be an IFOS in X. Then f^{-1}(p(α, β)) ∈ B. This implies p(α, β) ∈ f(B). By hypothesis, cl(f(cl(B))) is an IFβN of p(α, β). Therefore there exists an IFβOS A in Y such that p(α, β) ∈ A ⊆ cl(f(cl(B))). Now A = ∪ { p(α, β) / p(α, β) ∈ A } = f(B). Therefore f(B) is an IFβOS and hence an IFGβOS in Y. Thus f is an IFaGβOM.

**Theorem 3.14:** The following are equivalent for a mapping f: X → Y where y is an IFβT_{1/2} space.

(i) f is an IFaGβCM
(ii) βcl(f(A)) ⊆ f(αcl(A)) for every IFβOS A in X
(iii) βcl(f(A)) ⊆ f(αcl(A)) for every IFSOS A in X
(iv) f(A) ⊆ βint(f(scl(A))) for every IFPOS A in X.

**Proof:** (i) ⇒ (ii) Let A be an IFβOS in X. Then cl(A) is an IFRCS in X. By hypothesis f(A) is an IFβCS in Y and hence is an IFβCS in Y, since Y is an IFβT_{1/2} space. This implies f(cl(A)) = f(cl(A)). Now βcl(f(A)) ⊆ βcl(f(cl(A))) = f(cl(A)). Since cl(A) is an IFRCS, cl(int(cl(A))) = cl(A). Therefore βcl(f(A)) ⊆ f(cl(A)) = f(cl(int(cl(A)))) ⊆ f(A ∪ cl(int(cl(A)))) ⊆ f(αcl(A)). Hence βcl(f(A)) ⊆ f(αcl(A)).

(ii) ⇒ (iii) Let A be an IFSOS in X. Since every IFSOS is an IFβOS, the proof is obvious.

(iii) ⇒ (i) Let A be an IFRCS in X. Then A = cl(int(A)). Therefore A is an IFSOS in X. By hypothesis, βcl(f(A)) ⊆ f(αcl(A)) ⊆ f(cl(A)) = f(A) ⊆ βcl(f(A)). That is βcl(f(A)) = f(A). Hence f(A) is an IFβCS and hence is an IFGβCS in Y. Thus f is an IFaGβCM.

(i) ⇒ (iv) Let A be an IFPOS in X. Then A ⊆ int(cl(A)). Since int(cl(A)) is an IFROS in X, by hypothesis, f(int(cl(A))) is an IFβOS in Y. Since Y is an IFβT_{1/2} space, f(int(cl(A))) is an IFβOS in Y. Therefore f(A) ⊆ f(int(cl(A))) ⊆ βint(f(int(cl(A)))) = βint(f(A ∪ int(cl(A)))) = βint(f(scl(A))). That is f(A) ⊆ βint(f(scl(A))).

(iv) ⇒ (i) Let A be an IFROS in X. Then A is an IFPOS in X. By hypothesis, f(A) ⊆ βint(f(scl(A))). This implies f(A) ⊆ βint(f(A ∪ int(cl(A)))) ⊆ βint(f(A ∪ A)) = βint(f(A)) ⊆ f(A). Therefore f(A) is an IFβOS in Y and hence an IFGβOS in Y. Thus f is an IFaGβCM.

Next we provide the characterization theorem for an IFaGβOM.
Theorem 3.15: Let $f: X \rightarrow Y$ be a bijective mapping. Then the following are equivalent.

(i) $f$ is an IFaGβOM
(ii) $f$ is an IFaGβCM
(iii) $f^{-1}$ is an IFaGβCts.M.

Proof: (i) $\Leftrightarrow$ (ii) is obvious.

(ii) $\Rightarrow$ (iii) Let $A \subseteq X$ be an IFRCS. Then by hypothesis, $f(A)$ is an IFGβCS in $Y$. That is $(f^{-1})^{-1}(A)$ is an IFGβCS in $Y$. This implies $f^{-1}$ is an IFaGβCts.M.

(iii) $\Rightarrow$ (ii) Let $A \subseteq X$ be an IFRCS. Then by hypothesis $(f^{-1})^{-1}(A)$ is an IFGβCS in $Y$. That is $f(A)$ is an IFGβCS in $Y$. Hence $f$ is an IFaGβCM.

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