Published by :
https://www.ijert.org/
An International Peer-Reviewed Journal

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 04 , April - 2026

Interval -Valued Fuzzy Menger Space

Durgi Kol, Dr. Ruchi Singh, Manish Tiwari
(Department of Mathematics)
Pandit shambhunath Shukla University Shahdol Madhya Pradesh

Abstract : - The Interval -Valued Fuzzy Menger Space is a Mathematics concept that, combines fuzzy set theory and Menger Space. It's used to handle understand imprecies data. We define Interval -Valued Fuzzy Menger Space, discussing it's characteristics and relation to Fuzzy Menger Space

Key words - Fuzzy Menger Space , Fuzzy metric space , interval -valued fuzzy set, interval -Valued Fuzzy Menger Space,

1.INTRODUCTION

The concept of fuzzy set was introduced of Lotfi A. Zadeh in 1965 to model uncertainty using membership values between 0 and 1 Later, interval -valued fuzzy set were developed to represent uncertainty more effectively by assigning an interval (instead of a single number)as the membership degree. This allows a better representation of vagueness in real word problem. The development of Interval -Valued Fuzzy Menger Space involved contribution from several mathematics.An interval – valued fuzzy Menger space set is characterized by an interval – valued membership function, and it is taken as another generalization of the fuzzy set. In 2009, Li [20] introduced three kinds of distance between two interval valued fuzzy set (or number s) defined on real line R moreover, he noted that each kind of distance is a metric on the corresponding set and in interval -valued fuzzy numbers metric space is complete. A.George and P. Veeramani – Modified fuzzy metric spaces in 1994 and studies their properties [1][3][4]. S. Nadaban introduced fuzzy b- metric spaces and studies their properties. Khedmati yaghoobi and Morteza Saheli researched interval Valued Fuzzy Menger Space, other mathematics also contributed in this field, their work established interval -Valued Fuzzy Menger Space as a significant mathematical structure for studying uncertainty.

2.PRELIMINARIES :

Diffinition 2.1 A triplet (X, F t) where X is a set, t is a continuous t- norm (like min or product) , and F: X × X(0,∞) →[0,1] satisfies condition for all x, y, z € X ,t ,s > 0

1. F(x,y, t) > 0
2. F(x, y, t) = 1 if and only if x= y ;
3. F(x,y, t) = F(y,x,t);
4. F(x,z,t+s) ≥ F(x,y, t) • F(y,z, s) (tringle inequality/ Menger property).

Diffinition 2.2 The triple (X,F, t) is said to be an Interval -Valued Fuzzy Menger Space if X is an arbitrary set, t is a continuous interval – valued t- norm on [1] and F is an interval – valued fuzzy set on X × (0,∞) satisfying the following condition: -

1. F(x,y, t) > 0 ;
   -
2. F(x, y, t) = 1 if and only if x= y

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 04 , April - 2026

3. F(x, y, t) = F(y,x, t) ;
4. F(x,y,t) • F(y, z, s) ≤ F (x, z, t+s) ;
5. F(x, y,•): (0,∞) →[1] is continuous;

-

6. limn→∞ F(x,y, t)= [1] where x, y, z € X and t, s > 0 ;

Diffinition 2.3 An Interval -Valued Fuzzy Menger Space (X, F, t) in which for every sequence is Convergent is called a complete Interval -Valued Fuzzy Menger Space.

Diffinition 2.4 Let X be an ordinary non- empty set. The mapping A : X → [I] is called an interval – valued fuzzy set on X. All interval -valued fuzzy sets on X denoted by IVF (X).

Diffinition 2.5 Commutative associative and non – decreasing mapping t : [0,1] × [0,1] → [0,1] is a t- norm if and only if t(a,1) = a for all a € [0,1] , t(0,0) = 0 and t(c,d) for c ≥ a , d ≥ b.

Diffinition 2.6 A mapping Δ : [0,1]×[0,1]→[0,1] is called a t – norm if for all a,b,c € [0,1]

1. Δ (a,1) = a, Δ(0,0) = 0
2. Δ(a,b) = Δ(b,a) ;
3. Δ (c,d) ≥ Δ(a,b) for c ≥a ,d ≥ b;
4. Δ(Δ (a,b,c)) = Δ (a,Δ(b,c));

Diffinition 2.7 A fuzzy Menger Space (X,F,t) with the continuous t- norm is said to be complete if every couchy sequence in X converges point in X.

3.RESULT:

Theorem : Let (X,F,t) be an Interval -Valued Fuzzy Menger Space, for all x, y € X ,iff s,t > 0 , then F(x, y, t) = F(x,y, t) .

Proof – According to diffinition 2.2 if s > t > 0,

then we know that F(x,y, t) • F(y,y, s-t) ≤ F(x,y,s)

Since

–

F(y,y, s-t) = 1 we have

–

F(x,y, t) • F (y,y, s-t) = F (x, y, t) • 1 = F (x,y,t)

Therefore,

F(x,y, t) ≤ F (x,y,s)

Remark : In an Interval -Valued Fuzzy Menger Space (X, F, t), whenever

- - - -

F ((x, y, t) ≥ 1- r for all x,y € X , t> 0, r € (I) there exists a t0 with 0< t0<t such

- -

that F (x, y, t) > 1 - r .

International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 15 Issue 04 , April - 2026

CONCLUSION

An Interval -Valued Fuzzy Menger Space as a genrealization of Fuzzy Menger Space . Various properties and characterizetion of Interval -Valued Fuzzy Menger Space introduced by a continuous t- norm have been discussed. We have established several important results related to Convergence, Cauchy sequence and completeness in Interval- Valued Fuzzy Menger Space.The obtaind results extend and improve many known results of fuzzy metric spaces and Menger Space. This study shows that Interval -Valued Fuzzy Menger Space provide a more flexible and realistic framework for dealing uncertainty in mathematical modeling .

Interval -Valued Fuzzy Menger Space is an advanced tooling fuzzy mathematics, offering greater flexibility and accuracy then traditional fuzzy space.

REFERENCE:

[1] D. Mihet, " On the existence and the uniqueness of fixed points of sehgal contraction", Fuzzy ses syst. 156,pp. 135-141(2005).

[2] L. A. Zadeh, " Fuzzy set ", Information and control, 8 (3), 338-353(1965).

[3] B. Schweizer and A. Sklar, " Statistical metric space ", Pracific Journal of math.,10(1) 313-334(1960).

[4] Conell, " Properties fixed point space", Proc. Amer. Math. Science 10, 974979(1959).

[5] S. N. Mishra, " Common fixed point of Compatible mappings in PM- Space" , Math. Japonica , 36(2) , 283-289(1991).

[6] S. L. Singh and B. D. Pant, " Common fixed point theorems probabilistic metric space and extension to inform space", Hanam math. J.,6, 1-12 (1984).

[7] A . George and P. Veeramani, " On some results in fuzzy metric spaces", Fuzzy sets syst. 64, 395-399(1994)

[8] I. Kramosil and Michalek, " Fuzzy metric space and statistical metric space", Kybernetika,vol. 11,pp. 336-344(1975).

[9] K. Menger, " Statistical metrics" , Proceedings of the National Academy of Science the United States of America 28,535-537(1942).

[10] M. A. Kutbi, " Common fixed point and invariant approximation results" , Fixed point theory application 135(2013).

[11] D. Pant, " R. C. Dimri and S.L. Singh, " Fixed point theorems for expansion mapping on probabilistic metric space" ,Honam Math. J. 9, 77-81(1987).

[12] Z. K. Deng, " Fuzzy pseudo metric space" , Journal of mathematics analysis and application vol. 86, pp. 74-95(1982).

[13] O Hadzic, " Some theorems on the fixed points in probabilistic metric and random normed spaces" , Boll. Math. Ital; 13(5) 18 (1981) , 1-11(1981).