

Intelligent Controlling of an Inverted Pendulum Using PSO-PID Controller

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Abstract –Stabilizing the inverted pendulum is a standard problem in the field of control system. When force is applied to cart its position and pendulum angle deviate from its position. Many researchers have been applying different control algorithm and design techniques such as Neural Network, Genetic Algorithm (GA), Fuzzy logic, Particle swarm optimization on to a PID controller for stabilization of cart position and pendulum angle. The particle swarm optimization is a new evolutionary computation technique and has been introduced to solve several industrial problems [1-6]. Particle swarm optimization has better computational efficiency and it requires less number of parameter to adjust [13]. In this paper Particle Swarm Optimization (PSO) technique has been discussed to control the inverted pendulum problem and result is compared with conventional PID controller.

Keywords: *Inverted pendulum; PID controller; PSO; System stability*

I. Introduction

For control engineers inverted pendulum is a very good platform to verify different problems in the field of control theory. It is an excellent test benchmark for testing various complicated control problems. Normally inverted pendulum is stable when put on a cart, if a force is applied to the cart pendulum becomes unbalance until a suitable control strategy is applied. It is a Single input multiple output problem because the system has one input the force applied to the cart, and two outputs position of the cart and the angle of the pendulum.

The standard linear techniques cannot solve this type of non-linear dynamics of the system. This system is challenging for analysis. Due to the good features of particle swarm optimization (PSO) algorithm, presently it has been used as a new optimizer and applied to various types of research problems. PSO was developed through simulation of simplified social systems, and is robust in solving nonlinear optimization problems [8]. Objective of this paper is to design simulink model of inverted pendulum system, stabilize it with PID controller using particle swarm optimization technique (PSO) and compare the results with conventional PID

technique.

II. Modeling an Inverted Pendulum

The inverted pendulum model is shown in Fig.1. Its modeling is done for analysis the pendulum position when force is applied to cart and pendulum is stabilize using PSO-PID controller.

2.1 Inverted Pendulum on a Cart

The cart with an inverted pendulum, is shown in Fig. 1. An impulse force F Newton is applied to the cart. Some assumptions are made for modeling of an inverted pendulum which is given below in table 1.

Table I. Assumption for Inverted Pendulum

Symbol	Parameter	Value
M	Mass of the cart	0.5 Kg
m	Mass of Pendulum	0.2 Kg
b	Friction of the cart	0.1/N/m/sec
l	Length to pendulum centre of mass	0.3 m
I	Inertia of pendulum	0.006Kg*m ²
F	Force applied to the cart	1 Newton
θ	Pendulum angle from vertical	radian

Below are the two Free Body Diagrams of the system. Summing the forces in the Free Body Diagram of the cart in the horizontal direction, we get the following equation of motion:

$$M\ddot{x} + b\dot{x} + N = F \tag{1}$$

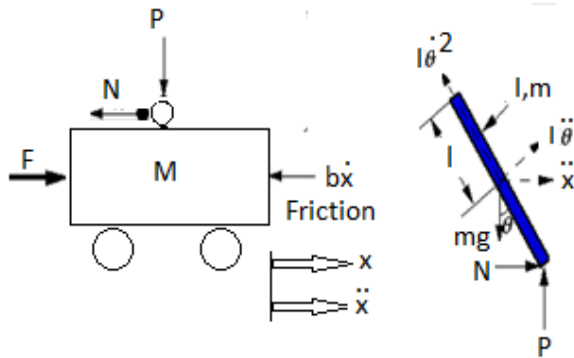


Fig.1.Free Body Diagram of Inverted Pendulum

Note that the forces can be sum in the vertical direction, but no useful information would be gained. Summing the forces in the Free Body Diagram of the pendulum in the horizontal direction, we can get an equation for N

$$N = m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta \tag{2}$$

By substituting (2) equation into the (1) equation, we get the equation of motion for this system

$$(M + m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F \tag{3}$$

To get the second equation of motion, sum the forces perpendicular to the pendulum

$$P\sin\theta + N\cos\theta - mg\sin\theta = ml\ddot{\theta} + m\ddot{x}\cos\theta \tag{4}$$

To get rid of the P and N terms in the equation above, sum the moments around the centroid of the pendulum to get the following equation

$$-Pl\sin\theta - Nl\cos\theta = I\ddot{\theta} \tag{5}$$

Combining equation (4) & (5), we get the second dynamic equation:

$$(I + ml^2)\ddot{\theta} + mgl\sin\theta = -ml\ddot{x}\cos\theta \tag{6}$$

These set of equations (3) & (6) should be linearized about $\theta = 0$. Assume that theta = θ (represents a small angle from the vertical upward direction). Therefore, $\cos\theta = 1$, $\sin\theta = -\theta$ and $(d\theta/dt)^2 = 0$. After linearization the two equations of motion become (where u represents the input):

$$(I + ml^2)\ddot{\theta} - mgl\theta = ml\ddot{x} \tag{7}$$

$$(M + m)\ddot{x} + b\dot{x} - ml\ddot{\theta} = u \tag{8}$$

2.2. Transfer Function of Pendulum Model

To obtain the transfer function of the linearized system equations analytically, we must first take the Laplace transform of the system equations (7) & (8). The Laplace transforms are:

$$(I + ml^2)\phi(s)s^2 - mgl\phi(s) = mlX(s)s^2 \tag{9}$$

$$(M + m)X(s)s^2 + bX(s)s - ml\phi(s)s^2 = U(s) \tag{10}$$

Since we will be looking at the angle, θ as the output of interest, solve the equation (9) for $X(s)$,

$$X(s) = \left[\frac{(I + ml^2)}{ml} - \frac{g}{s^2} \right] \phi(s) \tag{11}$$

Substitute value of $X(s)$ from equation (11) to (10) and re-arrange. The transfer function is:

$$\frac{\phi(s)}{U(s)} = \frac{\frac{mls^2}{q}}{s^4 + \frac{b(I + ml^2)}{q}s^3 - \frac{(M + m)mgl}{q}s^2 - \frac{bmgI}{q}s} \tag{12}$$

where,

$$q = [(M + m)(I + ml^2) - (ml)^2] \tag{13}$$

From the transfer function above it can be seen that there is both a pole and a zero at the origins. These can be canceled and the transfer function becomes:

$$\frac{\phi(s)}{U(s)} = \frac{\frac{mls}{q}}{s^3 + \frac{b(I + ml^2)}{q}s^2 - \frac{(M + m)mgl}{q}s - \frac{bmgI}{q}} \tag{14}$$

From table I by putting the values of M, m, b, l, I in equation (14) and get the transfer function of Fig. I

$$\frac{\phi(s)}{U(s)} = \frac{4.545s}{s^3 + 0.1818s^2 - 31.18s - 4.455} \tag{15}$$

III. Particle Swarm Optimization

The PSO was originally designed by Kennedy and Eberhart and is one of the modern heuristic algorithm[10]. This technique involves simulating social behavior among individuals (particles) “flying” through a multidimensional search space, each particle representing a single intersection of all search dimensions. The particles evaluate their positions relative to a goal (fitness) at every iteration, and particles in local neighborhood share memories to adjust their own velocities and thus subsequent positions.

The various steps involved in Particle Swarm Optimization Algorithm are as follows:

Step 1: The velocity and position of all particles are randomly set to within pre-defined ranges.

Step 2: Velocity updating – At each iteration, the velocities of all particles are updated according to, $v_i = v_i + c_1R_1(p_i, best - p_i) + c_2R_2(g_i, best - p_i)$ where,

p_i and v_i - are the position and velocity of particle i, respectively;

$p_i, best$ and $g_i, best$ - is the position with the ‘best’ objective value found so far by Particle i and the entire population respectively;

w - is a parameter controlling the dynamics of flying;

R_1 and R_2 - are random variables in the range [0, 1];

c1 and **c2** - are factors controlling the related weighting of corresponding terms. The random variables help the PSO with the ability of stochastic searching.

Step 3: Position updating – The positions of all particles are updated according to,
 $p_i = p_i + v_i$

After updating, p_i should be checked and limited to the allowed range.

Step 4: Memory updating – Update p_i , $best_{p_i}$ and g_i , $best_{g_i}$ when condition is met,

$$p_i, best_{p_i} = p_i \quad \text{if } f(p_i) > f(p_i, best_{p_i})$$

$$g_i, best_{g_i} = g_i \quad \text{if } f(g_i) > f(g_i, best_{g_i})$$

where $f(x)$ is the objective function to be optimized.

Step 5: Stopping Condition – The algorithm repeats steps 2 to 4 until certain stopping conditions are met, such as a pre-defined number iterations. Once stopped, the algorithm reports the values of g_{best} and $f(g_{best})$ as its solution

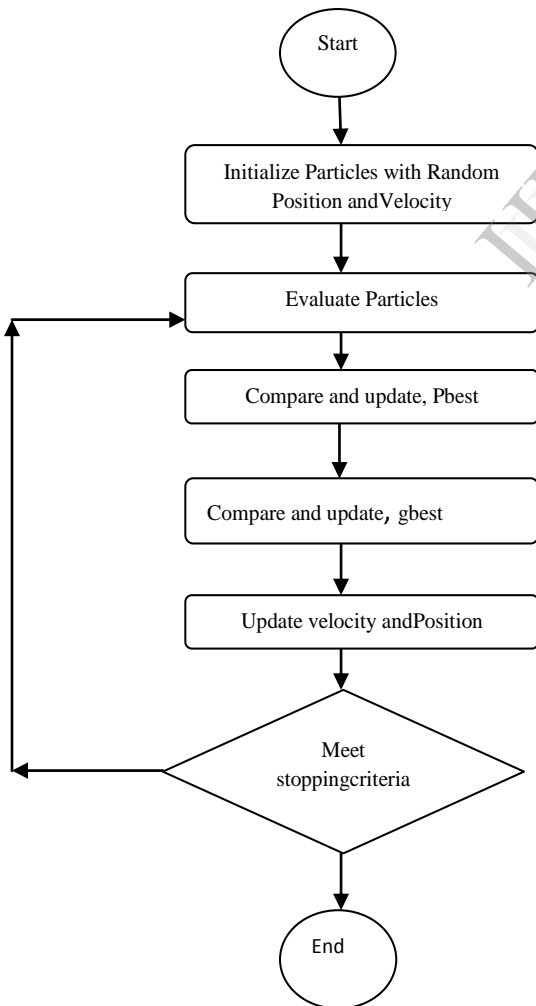


Fig 2: Flowchart of PSO Algorithm

IV. Simulation and Results

Simulink model has been developed in MATLAB. It is tried to tune inverted pendulum system using particle swarm optimization by nonlinear equations. Simulink model with impulse disturbance has been shown in Fig.3.

Fig.4 depicted the response of inverted pendulum with conventional (Zeigler Nichols) PID controller and with impulse disturbance. It is observed from Fig 4. that response is not stable before 20 sec.

The PID values for this PID controller are

$$K_p = 2, K_i = 5 \text{ and } K_d = 2$$

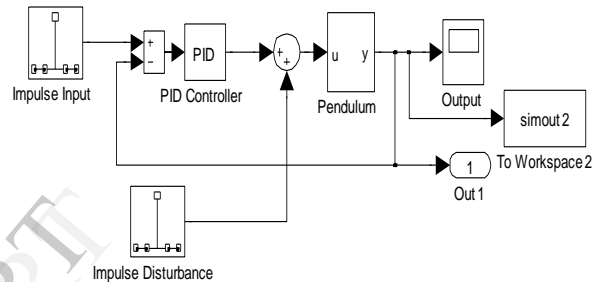


Fig.3. PSO-PID Model in Simulink

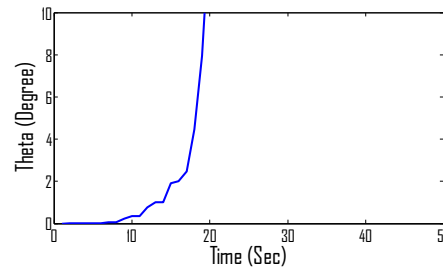


Fig.4 Impulse response of Pendulum with Conventional PID Controller

When PSO – PID controller is applied with following settings

- Initial population size (N) = 200
- Step size = 40
- $c_1 = c_2 = 2.0$
- $w = 0.8$

The response is shown in Fig.5

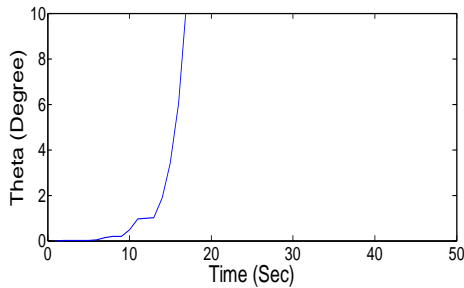


Fig.5 Impulse response of pendulum with PSO- PID controller

It is observed from Fig.5 that by using PSO technique to tune PID controller system is stable and response is less oscillatory.

The value of integral error performance indices is also obtained. The integral of the absolute value of the error (IAE) is an appropriate measure of control performance when the effect on control performance is linear with the deviation. The integral of square of the error (ISE) is appropriate when large deviations cause greater performance degradation than small deviation.

Table 1. Comparison of PID Techniques

S.No.	Type of Controller	IAE	ISE
1	Conventional (Zeigler-Nichols)	4.58	7.14
2	PSO PID	1.61	2.37

It is observed from table 1 that on IAE & ISE error performance indices the result is much better with PSO-PID Controller.

V. Conclusion

The results of the simulation have been shown. The proposed PSO improves the performance of inverted pendulum and can be easily introduced to any other nonlinear control problem. It is observed that the inverted pendulum position is stable very soon using PSO-PID controller compare to conventional PID controller. Future work can be done on tuning PID controller with Genetic algorithm and type- two fuzzy cascade controller.

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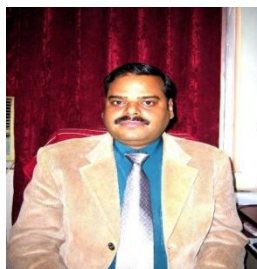
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