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Insight of Oblique Inelastic Collisions of Two Smooth Round Objects

P Srinivas Kalyan,
U.G Student, Department of Mechanical Engineering
Anil Neerukonda Institute of Technology and Sciences,
Visakhapatnam,India

***Abstract*:- The disperse angles and speeds obtained from an inelastic collision of a smooth ball or sphere when hit on a second sphere at rest on a no friction surface or table are studied and analyzed based on impact parameter and the ratio of masses of two. A special interest is given for finding simple special cases for scattering angles that could find use in some homework problems or in lab experiments. The level of scattering is understandable to an introductory undergraduate physics and engineering mechanics course.**

Keywords:- Two dimensional inelastic collision, impact parameter, disperse angles, coefficient of restitution

INTRODUCTION

The detailed study of inelastic collisions between two smooth spheres is basic and standard concept in the introductory mechanics course [1]. In general we have four unknowns, the final speeds and angles of the two objects in the plane of collision but only three equations that is, conservation of linear momentum in the line of impact and the line perpendicular to it, lying in the plane of contact and the formula of coefficient of restitution, are known. In inelastic collision the kinetic energy is not conserved and hence law of conservation of kinetic energy is not considered. Thus one can only find relationships between them but not unique solutions for each of them unless other additional quantity such as impact parameter [2] is mentioned. Despite efforts [3], However, simple expressions have not been found when the two objects are of unequal masses. The basic and primary goal and outcome of the present paper is to derive basic expression for the scattering angles and then to deduce some special cases from them that could provide help to develop homework problems or laboratory experiments.

RELATIONSHIPS BETWEEN TWO DISPERSE ANGLES

Two round spheres make an oblique inelastic collision on a frictionless table [4]. The surface of the spheres are smooth enough that friction can be neglected and thus there is no force or mechanism to change the rotational kinetics energies, in particular, if the spheres are started with zero angular speeds, they will obviously remain same way after the collision.

Let the two spheres have masses m and M where $M \geq m$. Suppose the heavier sphere is initially at rest and the origin is at the Centre of mass M. Choose the x-axis so that the final velocity of M is in that direction and choose the y-axis perpendicular to the line of impact so that the y-coordinate of lighter sphere is positive before collision and the impact parameter is b.

Let the initial velocity of lighter sphere is υ in the direction making an angle (ϕ) in the counter clockwise direction. And the final velocity of it is at an angle (θ) with the positive x-axis in counter clockwise direction.

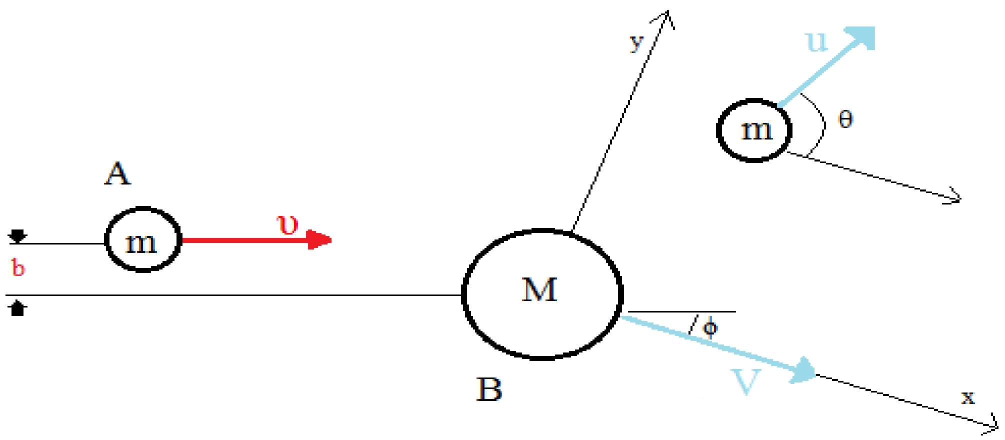

Figure 1. Initial velocity vector of magnitude υ (in red) of a light ball of mass m making an oblique collision with a heavy ball of mass M where the heavy ball is initially at rest. The final velocity vectors of the two balls are indicated in blue, with magnitude u and angle θ for the light ball and magnitude V at angle ϕ for the heavy ball. The impact parameter is b and the x-coordinate axis is indicated in the direction of M and the y-axis perpendicular to it.

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Resolving components of velocities of each of them in x and y axis :

$V_{bx1} = 0$ $\quad V_{bx2} = V$

$V_{by1} = 0$ $\quad V_{by2} = 0$

$V_{ax1} = \upsilon \cos\phi$ $\quad V_{ax2} = u \cos\theta$

$V_{ay1} = \upsilon \sin\phi$ $\quad V_{ay2} = u \sin\theta$

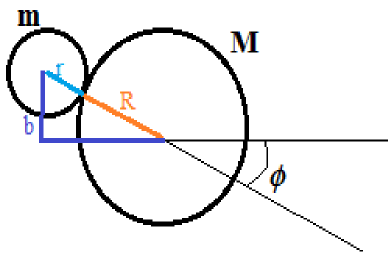

Figure 2. At the time of impact when two balls collide

$$Sin\ \phi = \frac{b}{D} \qquad (1)$$

$$Cos\ \phi = b/(D^2-b^2)^{1/2} \qquad (2)$$

here D= r + R where r and R are the two radii of the lighter mass and the heavier mass respectively and D is the average of radii of the two .

Conservation of linear momentum of each sphere in y-direction gives :

$M(V_{by1}) = M(V_{by2})=0$

$m(V_{ay1}) = m(V_{ay2})$

$\upsilon \sin\phi = u \sin\theta$

$$\upsilon = \frac{Du\sin\theta}{b} \qquad (3)$$

Conserving momentum of spheres in x- direction gives [5]:

$m(V_{ax1})+M(V_{bx1})=m(V_{ax2})+M(V_{bx2})$

$$\frac{m}{M}\upsilon \cos\phi = \frac{m}{M} u \cos\theta + V \qquad (4)$$

Substitute equation (1) and (3) in (4) and solve to get

$$\frac{m}{M} u(D^2-b^2)^{1/2} \sin\theta - \frac{m}{M} (bu \cos\theta) = bV \quad (5)$$

From Coefficient of restitution formula [6] :

$e = [V_{bx2}- V_{ax2}]/[V_{ax1}- V_{bx1}]$

Substitute velocity components and equations (2) and (3) and solve to get

$$eu(D^2-b^2)^{1/2} \sin\theta +bu \cos\theta = bV \qquad (6)$$

Equating L.H.S'^s of equations (5) and (6) and solving to get θ

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$$\theta = \begin{cases} \tan^{-1} \dfrac{b\left(\frac{m}{M}+1\right)}{(D^2-b^2)^{1/2}\left(\frac{m}{M}-e\right)} \\[2ex] 180^{o} - \tan^{-1} \dfrac{b\left(\frac{m}{M}+1\right)}{(D^2-b^2)^{1/2}\left(e-\frac{m}{M}\right)} \end{cases}$$

Now,

e= 0 for perfectly inelastic collision

e =0.5 for wood

e=0.56 for steel

e=0.89 for ivory

e=0.94 for glass

e=1 for perfectly elastic collision

A series of contour plots are drawn in the order of increasing e :

e = {0, 0.5, 0.56, 0.89, 0.94, 1}

Where, the z– slices or the iso-response values are at an angle interval of 20^{o} that is

$\theta \in \{0^{o},20^{o},40^{o},60^{o},80^{o},100^{o},120^{o},140^{o},160^{o},180^{o}\}$

On x axis, we take $0 \leq \frac{b}{D} \leq 1$

On y axis, we take $0\leq \frac{m}{M} \leq 1$

The red curves are for angles below 90^{o}and the black curves are for angles above 90^{o}.

For e= 0:

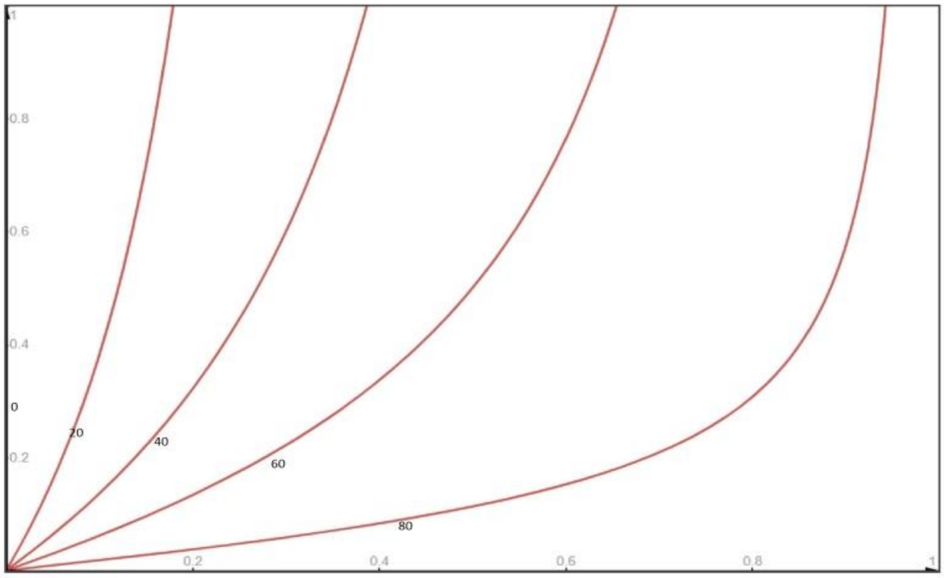

Figure 3

For e = 0.5:

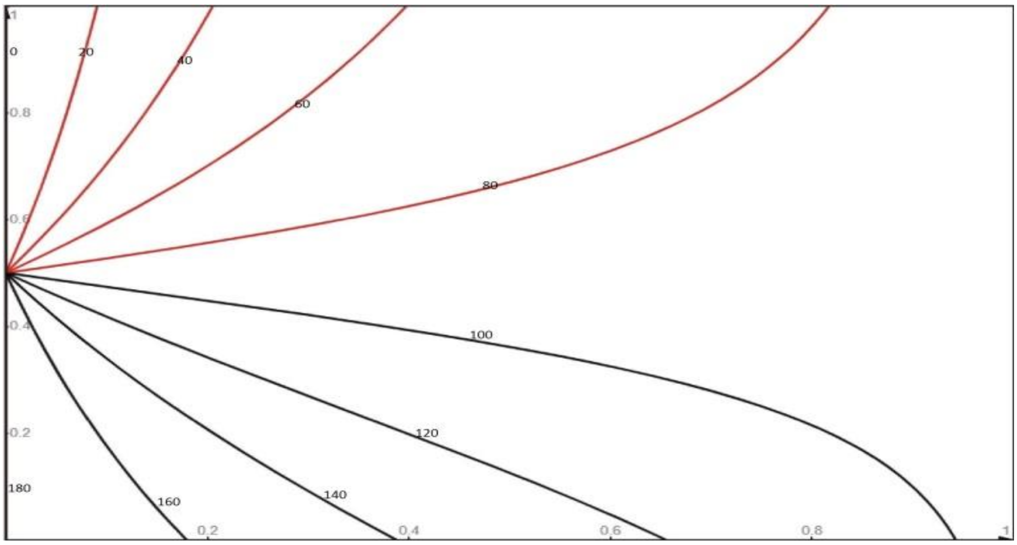

Figure 4

For e = 0.56:

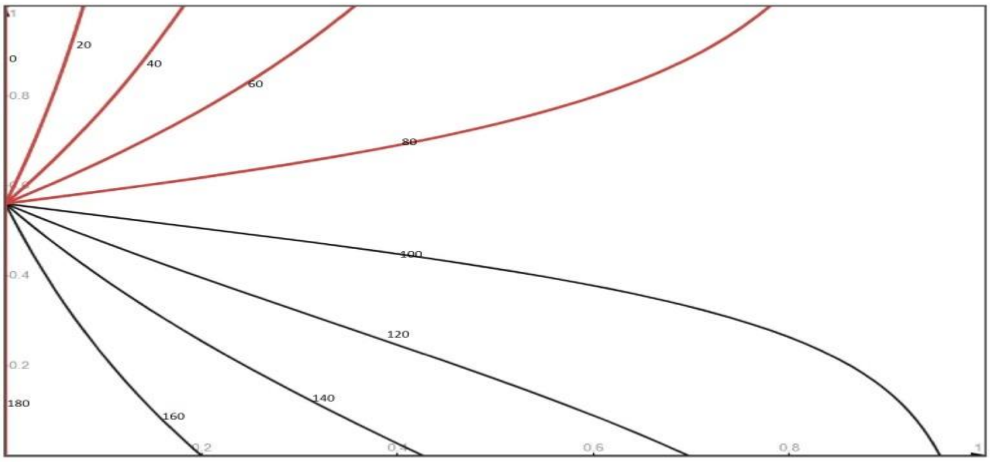

Figure 5

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For e = 0.89:

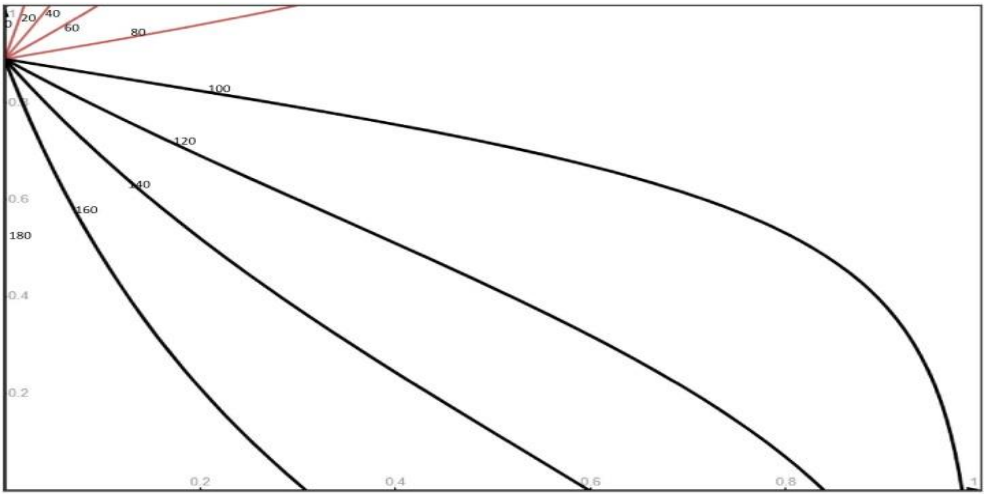

Figure 6

For e = 0.94:

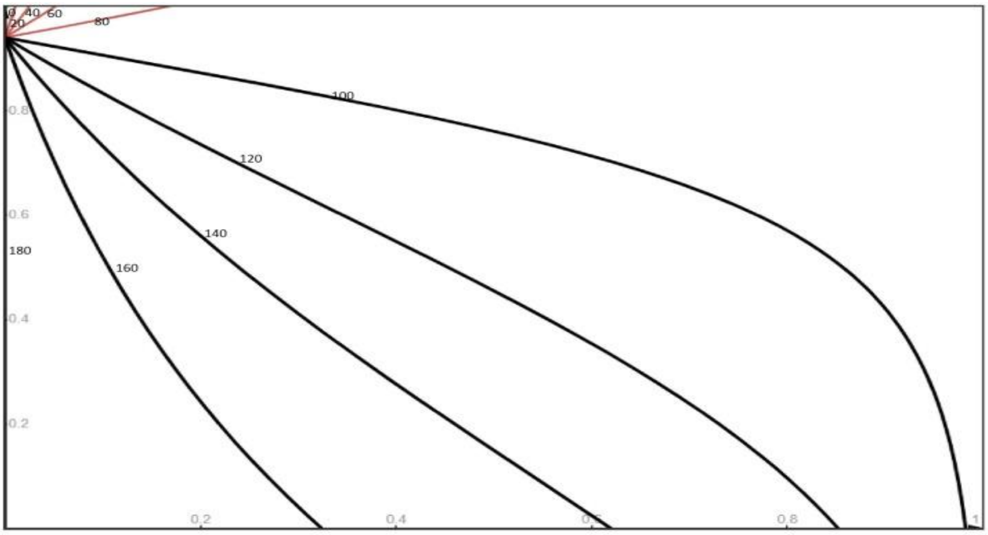

Figure 7

For e = 1:

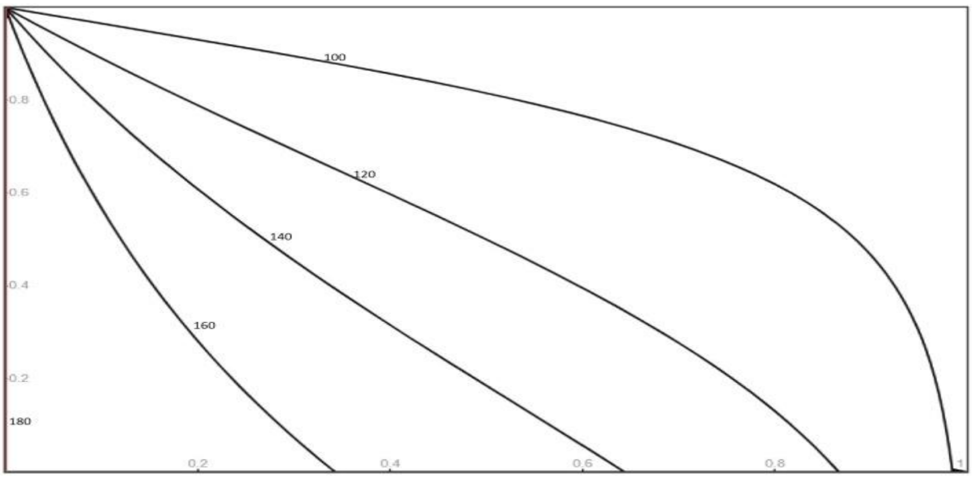

Figure 8

For any contour plot from above **figures 3,4,5,6,7,8** we see, 0^o and 180^o lines are vertical at x=0 that is $\frac{b}{D} = 0$ which implies b=0 that is head on collision.

Therefore,

When, m/M ≥e, one scatters in the same as initial direction or forward =0^o when x=0.

When, m/M< e, one scatters in the backward direction or is backscattering =180^o when x=0. [6]

SPECIAL CASES

1. For any collision, if the mass ratio is e, that is m=Me; then for any impact parameter, the two masses move perpendicular to each other after collision [since tan θ = ∞]

2. Now consider ,m=M and e=1 like a game of billiards where balls are made of phenolic resin which make nearly elastic collisions , in that case also tan θ = ∞ or θ=90^o

From equations (1) and (3), for any given initial velocity of lighter mass, we have

$$u = \left(\frac{b}{D}\right) v = v \sin\phi \qquad (7)$$

$$V = v\left[1-\left(\frac{b}{D}\right)^2\right]^2 \qquad (8)$$

CONCLUSION

Many of these special cases could make good help in lab experiments using carbon paper[7] or video analysis [8] for colliding marbles in rolling. Alternatively, this would be gratifying to simulate these motions on the computer when the two scattering angles have simple values or relations to each other. Such simulative methods will also show that the angles depend only on the impact parameter and mass ratio, and not, on the incident speed of the lighter mass.

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