Influence of Some Vital Structural Parameters on the Dynamic Characteristics of Axially Prestressed Beam Under Moving Masses.

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Abstract

This study concerns the problem of the longitudinal vibrations of beams resting on elastic foundation and under the actions of travelling exponentially varying load with a constant velocity type of motion. A mathematical formulation representing the transverse motions of the engineering structure which is valid for all variants of classical boundary conditions is set up. An analytical method of analysis is presented for investigating the effects of some vital structural parameters on the dynamic response characteristic of the structurally prestressed elastic beam with rotatory inertia correction factor. The theory proposed is applied to a beam having simple supports at both ends. However, the theory and application is not limited to the beam with simply supported boundary condition and without much mathematical difficulties, it can also be used to treat dynamic analysis of continuous beam structures including energy dissipation. Closed form solutions of the equation of motion describing the beam-load interactions are obtained. Analysis is carried out and various results are presented in plotted curve.

1. Introduction

The vibration analysis (linear or non-linear) of structural members has been and continues to be the subject of numerous studies in Engineering Science and related fields; this is due largely to the fact that it embraces a wide class of problems with great relevance in many engineering applications. For example, the analysis and design of highway and railway bridges, cable-railways, cableways and overhead cranes to mention a few.

Thus, the effect of the nature of the complexity of the interactions between beams or beam-like structural elements and the load traversing them at various velocities on the dynamic characteristics of such structures have been investigated by many researchers [1-10].

Historically, three types of problems have been considered in the open literature. If the inertia effect of the moving subsystem is neglected, the problem reduces to the vibration of elastic structure under the actions of an external moving force and this is termed the moving force problem. When the inertia effect of the mass of the moving load is taken into account and assuming infinite stiffness of the coupling between the continuous system and the moving subsystem, we have the moving mass. The dynamical problem involving finite coupling stiffness leads to the moving oscillator's problem. Depending on the type of model used and assumptions adopted, great numbers of publications have evolved during the past years [11-26].

In spite of the enormous investigations and great amount of work that have been devoted to the dynamic response of beams under moving load, it unarguably remains a major topic for future scientific research because of the continuing advancements in design technology and emergence of new materials with improved quality which enable the construction of lighter and more slender structures, vulnerable to fast traveling heavy loads [27].

It is well known that a considerable amount of work dealing with the vibration of beams under the effect of moving load has been found in the open literature, however, to the authors' knowledge, the vibration analysis of beams incorporating a rotatory inertia correction factor and subjected to exponentially varying load is not common. Thus, this work concerns the dynamic response of axially prestressed elastic beam resting on elastic foundation and traversed by masses traveling at constant velocity. The specific aims of this study is to classify the effects of some parameters namely prestressed, foundation subgrade, rotatory inertia correction factor, mass ratio etc on the flexural motions and critical velocity of elastic beams subjected to moving masses.

2.0 Formulation of the problem

The governing differential equation for isotropic beam of length L on an elastic foundation and traversed by a moving load of mass M travelling with constant velocity v is given by

$$EI\frac{\partial^4 Q(x,t)}{\partial x^4} - N\frac{\partial^2 Q(x,t)}{\partial x^2} + \mu\frac{\partial^2 Q(x,t)}{\partial t^2} - \mu R^o\frac{\partial^4 Q(x,t)}{\partial x^2 \partial t^2} + K_o e^{-\lambda x}Q(x,t) = P(x,t) \quad (1)$$

where EI is the flexural rigidity of the beam, $Q(x,t)$ is the transverse deflection, μ is the mass per unit length of the beam, N is the constant axial force, K_o is the elastic foundation, R^o is the rotatory inertia, $P(x,t)$ is the transverse load, x is the spatial coordinate taken along the axis of the beam and t is the time variable.

When the inertia of the moving load is taken into consideration, the transverse load can be expressed in the form

$$P(x,t) = P_f(x,t)\left[1 - \frac{1}{g}\frac{d^2Q(x,t)}{dt^2}\right] \quad (2)$$

where the continuous moving force $P_f(x,t)$ acting on the beam model is given by

$$P_f(x,t) = e^{\omega t}M_i g\delta(x - v_i t) \quad (3)$$

and the convective acceleration operator $\frac{d^2}{dt^2}$ is defined as

$$\frac{d^2}{dt^2} = \frac{\partial^2}{\partial t^2} + \frac{2v_i\partial^2}{\partial x\partial t} + \frac{v_i\partial^2}{\partial x^2} \quad (4)$$

The boundary condition is arbitrary and without loss of generality, the initial conditions are of the form

$$Q(x,0) = 0, \qquad \frac{\partial Q(x,0)}{\partial t} = 0 \quad (5)$$

Substituting equations (2) to (4) in equation (1) one obtains

$$EI\frac{\partial^4 Q(x,t)}{\partial x^4} - N\frac{\partial^2 Q(x,t)}{\partial x^2} + \mu\frac{\partial^2 Q(x,t)}{\partial t^2} - \mu R^o\frac{\partial^4 Q(x,t)}{\partial x^2\partial t^2} + K_o e^{-\lambda x}Q(x,t) + M\delta(x - v_i t)\left[\frac{\partial^2 Q(x,t)}{\partial t^2} + 2v_i\frac{\partial^2 Q(x,t)}{\partial x\partial t} + v_i^2\frac{\partial^2 Q(x,t)}{\partial x^2}\right] = e^{\omega t}\sum_{i=1}^{m} M_i g\delta(x - v_i t) \quad (6)$$

which describes the flexural motions of axially prestressed beam resting on elastic foundation and subjected to variable magnitude moving load. It is remarked at this juncture that equation (6) is valid for all variants of classical boundary conditions.

3.0 General Solution Procedures

It is evident that an exact closed form solution of the above partial differential equation (6) is not feasible. Consequently, an approximate solution is sought. Thus, the Galerkin technique described in [24] is employed. By this technique the partial differential equation (6) is reduced to a sequence of ordinary differential equations in the first instance and the resulting set of second order ordinary differential equations is further simplified using an asymptotic method of solution due to Struble and these set of equations are finally solved completely by the method of integral transformations. This versatile technique requires that the solution of equation (6) takes the form

$$Q_n(x,t) = \sum_{i=1}^{n} W_i(t) U_i(x) \qquad (7)$$

where $U_i(x)$ is chosen such that pertinent boundary conditions are satisfied. Equation (7) when substituted into the equation (6) yields

$$\sum_{i=1}^{n} \Big\{ EIW_i(t)U_i^{iv}(x) - NW_i(t)U_i''(x) + \mu \ddot{W}_i(t)U_i(x) - \mu R^o \ddot{W}_i(t)U_i''(x)$$
$$+ K_o e^{-\lambda x} W_i(t)U_i(x) + \sum_{i=1}^{n} M_i \delta(x - v_i t)\left[\ddot{W}_i(t)U_i(x) + 2v_i \dot{W}_i(t)U_i'(x)\right.$$
$$\left. + v_i^2 W_i(t)U_i''(x)\right] \Big\} - e^{\omega t} \sum_{i=1}^{n} M_i g \delta(x - v_i t) = 0$$

(8)

An appropriate selection of functions for beam problems are beam mode shapes. Thus, the m[th] normal mode of vibration of a uniform beam

$$U_i(x) = \sin\frac{\lambda_i x}{L} + A_i \cos\frac{\lambda_i x}{L} + B_i \sinh\frac{\lambda_i x}{L} + C_i \cosh\frac{\lambda_i x}{L}$$

(9)

is chosen such that the pertinent boundary conditions are satisfied. In equation (9), λ_i is the mode frequency, A_i, B_i, C_i are constants which can be determined by using the boundary conditions associated with the beam structure.

4.0 Operational simplification

To obtain $W_i(t)$ from equation (8), it is required that the expression on the left hand side of equation (8) be orthogonal to the function $U_j(x)$. Thus

$$\sum_{i=1}^{n} \Big\{ \left[EIH_1(i,j) - NH_2(i,j) + K_o H_4(i,j)\right] W_i(t) + \left[\mu H_3(i,j) - \mu R^o H_2(i,j)\right] \ddot{W}_i(t)$$
$$+ \sum_{i=1}^{n} M_i \left[\ddot{W}_i(t) \int_0^L \delta(x - v_i t) U_i(x) U_j(x) dx + 2v_i \dot{W}_i(t) \int_0^L \delta(x - v_i t) U_i'(x) U_j(x) dx \right.$$
$$\left. + v_i^2 W_i(t) \int_0^L \delta(x - v_i t) U_i''(x) U_j(x) dx \right] \Big\} = e^{\omega t} \sum_{i=1}^{m} M_i g U_j(v_i t)$$

(10)

we note that the dirac delta function as an even function can be expressed as

$$\delta(x - v_i t) = \frac{1}{L} + \frac{2}{L} \sum_{n=1}^{\infty} \cos\frac{n\pi v_i t}{L} \cos\frac{n\pi x}{L}$$

(11)

In view of equation (7), using equation (11) in equation (10), after some simplifications and rearrangement, one obtains

$$\sum_{i=1}^{n} \Bigg\{ \ddot{W}_i(t) + \frac{\Delta_1(i,j)}{\Delta_o(i,j)} W_i(t) + \sum_{i=1}^{n} \frac{\Gamma^*}{\Delta_o(i,j)} \Bigg[\left(H_a(i,j) + 2\sum_{n=1}^{\infty} \cos\frac{n\pi v_i t}{L} H_b(i,j,n) \right) \ddot{W}_i(t)$$
$$+ \left(2v_i H_c(i,j) + 4v_i \sum_{n=1}^{\infty} \cos\frac{n\pi v_i t}{L} H_d(i,j,n) \right) \dot{W}_i(t)$$
$$+ \left(v_i^2 H_e(i,j) + 2v_i^2 \sum_{n=1}^{\infty} \cos\frac{n\pi v_i t}{L} H_f(i,j,n) \right) W_i(t) \Bigg] \Bigg\} = \frac{e^{\omega t}}{\mu \Delta_o(i,j)} \sum_{i=1}^{m} M_i g U_j(v_i t)$$

(12)

where

$$H_1(i,j) = \int_0^L U_i^{iv}(x) U_j(x) dx \qquad H_2(i,j) = \int_0^L U_i''(x) U_j(x) dx$$

$$H_3(i,j) = \int_0^L U_i(x) U_j(x) dx \qquad H_4(i,j) = \int_0^L e^{-\lambda x} U_i(x) U_j(x) dx$$

$$H_a(i,j)=\int_0^L U_i(x)U_j(x)dx \quad H_b(i,j,n)=\int_0^L \cos\frac{n\pi x}{L}U_i(x)U_j(x)dx$$

$$H_c(i,j)=\int_0^L U_i'(x)U_j(x)dx \quad H_d(i,j,n)=\int_0^L \cos\frac{n\pi x}{L}U_i'(x)U_j(x)dx$$

$$H_e(i,j)=\int_0^L U_i''(x)U_j(x)dx \quad H_f(i,j,n)=\int_0^L \cos\frac{n\pi x}{L}U_i''(x)U_j(x)dx$$

$$\Delta_o(i,j)=H_3(i,j)-R^o H_2(i,j)$$

$$\Delta_1(i,j)=\frac{EI}{\mu}H_1(i,j)-\frac{N}{\mu}H_2(i,j)+\frac{K_o}{\mu}H_4(i,j) \quad (13)$$

5.0 Application and Illustrative examples

For the purpose of analysis, an elastic beam with simply supported boundary conditions, carrying fast traveling masses is considered. However, the analysis and formulation presented in this work are not limited to just simply supported boundary condition. The analysis in its general form may well be applied to beams with various boundary conditions. For beams with simple supports at both ends x = 0 and x = L, it can be shown that $A_i = B_i = C_i = 0$ and $\lambda_i = \frac{i\pi}{L}$. To this effect, the transverse displacement response of beams having simple supports at both ends can be given taking into account (7) as

$$Q_n(x,t)=\sum_{i=1}^{n}W_i(t)\sin\frac{i\pi x}{L} \quad (14)$$

and

$$U_j(x)=\sin\frac{j\pi x}{L},\ \ U_j(v_i t)=\sin\frac{j\pi v_i t}{L} \quad (15)$$

Substituting equation (14) into the transformed governing equation (12) and after some simplifications and rearrangements one obtains

$$\sum_{i=1}^{n}\Bigg\{\ddot{W}_i(t)+\frac{\Delta_1(i,j)}{\Delta_o(i,j)}W_i(t) + \sum_{i=1}^{n}\frac{\Gamma^*}{\Delta_o(i,j)}\Bigg[\Bigg(H_a(i,j)+2\sum_{n=1}^{\infty}\cos\frac{n\pi v_i t}{L}H_b(i,j,n)\Bigg)\ddot{W}_i(t) + \Bigg(2v_iH_c(i,j)+4v_i\sum_{n=1}^{\infty}\cos\frac{n\pi v_i t}{L}H_d(i,j,n)\Bigg)\dot{W}_i(t) + \Bigg(v_i^2H_e(i,j)+2v_i^2\sum_{n=1}^{\infty}\cos\frac{n\pi v_i t}{L}H_f(i,j,n)\Bigg)W_i(t)\Bigg]\Bigg\} = \frac{e^{\omega t}}{\mu\Delta_o(i,j)}\sum_{i=1}^{m}M_i g\sin\frac{j\pi v_i t}{L} \quad (16)$$

Equation (16) represents the transformed equation of a uniform Rayleigh beam on a constant elastic foundation. Evidently, an exact closed form solution to this problem is not possible. Consequently, in what follows two cases of the coupled equation are considered.

6.0 Solution of the transform equation

(6.1) The moving force problem

If the inertia effect of the moving mass is considered as negligible, we shall have the classical case of a moving force problem. Under this assumption $\Gamma^* = 0$ and after some simplifications and rearrangements and considering only the ith particle of the system, equation (16) becomes

$$\ddot{W}_i(t)+\omega_{mf}^2 W_i(t)=\frac{e^{\omega t}}{\mu\Delta_o(i,j)}Mg\sin\frac{j\pi v_i t}{L} \quad (17)$$

where

$$\omega_{mf}^2=\frac{\Delta_1(i,j)}{\Delta_o(i,j)} \quad (18)$$

It can be shown that the general solution of equation (17) can be written in the form

$$W_i(t)=C_1\cos\omega_{mf}t+C_2\sin\omega_{mf}t+P_1(t)\cos\omega_{mf}t+P_2(t)\sin\omega_{mf}t \quad (19)$$

where

$$P_1(t) = -\frac{P_{mf}}{\omega_{mf}} \int e^{\omega t} \sin\frac{\pi v_i t}{L} \sin\omega_{mf} t dt \quad (20)$$

and

$$P_2(t) = \frac{P_{mf}}{\omega_{mf}} \int e^{\omega t} \sin\frac{\pi v_i t}{L} \cos\omega_{mf} t dt \quad (21)$$

C_1 and C_2 are constants to be determined and $P_{mf} = \dfrac{Mg}{\mu\Delta_0(i,j)}$.

Evaluating integrals (20) and (21) leads to

$$P_1(t) = -\frac{P_{mf}}{2\omega_{mf}}\left[\frac{a-\omega_{mf}}{\omega^2+(a-\omega_{mf})^2}e^{\omega t}\sin(a-\omega_{mf})t + \frac{\omega}{\omega^2+(a-\omega_{mf})^2}e^{\omega t}\cos(a-\omega_{mf})t \right.$$
$$\left. -\frac{a+\omega_{mf}}{\omega^2+(a+\omega_{mf})^2}e^{\omega t}\sin(a+\omega_{mf})t - \frac{\omega}{\omega^2+(a+\omega_{mf})^2}e^{\omega t}\cos(a+\omega_{mf})t\right] \quad (22)$$

and

$$P_2(t) = -\frac{P_{mf}}{2\omega_{mf}}\left[\frac{\omega}{\omega^2+(a+\omega_{mf})^2}e^{\omega t}\sin(a+\omega_{mf})t - \frac{a+\omega_{mf}}{\omega^2+(a-\omega_{mf})^2}e^{\omega t}\cos(a+\omega_{mf})t \right.$$
$$\left. -\frac{\omega}{\omega^2+(a-\omega_{mf})^2}e^{\omega t}\sin(a-\omega_{mf})t - \frac{a-\omega_{mf}}{\omega^2+(a+\omega_{mf})^2}e^{\omega t}\cos(a-\omega_{mf})t\right] \quad (23)$$

substituting equations (22) and (23) into equation (19) yields

$$W_i(t) = C_1\cos\omega_{mf}t + C_2\sin\omega_{mf}t - e^{\omega t}\left[\cos\omega_{mf}t\left(\frac{(a-\omega_{mf})\sin(a-\omega_{mf})t}{\omega^2+(a-\omega_{mf})^2}\right.\right.$$
$$\left. + \frac{\omega\cos(a-\omega_{mf})t}{\omega^2+(a-\omega_{mf})^2} - \frac{(a+\omega_{mf})\sin(a+\omega_{mf})t}{\omega^2+(a+\omega_{mf})^2} - \frac{(a+\omega_{mf})\cos(a+\omega_{mf})t}{\omega^2+(a+\omega_{mf})^2}\right)$$
$$+\sin\omega t\left(\frac{\omega\sin(a+\omega_{mf})t}{\omega^2+(a+\omega_{mf})^2} - \frac{(a+\omega_{mf})\cos(a+\omega_{mf})t}{\omega^2+(a+\omega_{mf})^2} + \frac{\omega\sin(a-\omega_{mf})t}{\omega^2+(a-\omega_{mf})^2}\right.$$
$$\left.\left.\left. - \frac{(a-\omega_{mf})\cos(a-\omega_{mf})t}{\omega^2+(a-\omega_{mf})^2}\right)\right]\right\} \quad (24)$$

When use is made of the initial conditions (5) in conjunction with equation (24), one obtains

$$C_1 = \frac{P_{mf}\,\omega}{2\omega_{mf}}\left[\frac{1}{\omega^2+(a-\omega_{mf})^2} - \frac{1}{\omega^2+(a+\omega_{mf})^2}\right] \quad (25)$$

and

$$C_2 = -\frac{P_{mf}}{2\omega_{mf}}\left[\frac{a+\omega_{mf}}{\omega^2+(a+\omega_{mf})^2} - \frac{a-\omega_{mf}}{\omega^2+(a-\omega_{mf})^2}\right] \quad (26)$$

substituting equations (25) and (26) into equation (24), simplifying and inverting yield

$$Q_n(x,t) = \sum_{i=1}^{n}\frac{Mg}{2\omega_{mf}\Delta_o(i,j)}\left\{\cos\omega_{mf}t\left(\frac{\omega}{\omega^2+(a-\omega_{mf})^2} - \frac{\omega}{\omega^2+(a+\omega_{mf})^2}\right)\right.$$
$$-\sin\sigma t\left(\frac{(a+\omega_{mf})}{\omega^2+(a+\omega_{mf})^2} - \frac{(a-\omega_{mf})}{\omega^2+(a-\omega_{mf})^2}\right) - e^{\omega t}\left[\cos\omega_{mf}t\left(\frac{(a-\omega_{mf})\sin(a-\omega_{mf})t}{\omega^2+(a-\omega_{mf})^2}\right.\right.$$
$$\left. + \frac{\omega\cos(a-\omega_{mf})t}{\omega^2+(a-\omega_{mf})^2} - \frac{(a+\omega_{mf})\sin(a+\omega_{mf})t}{\omega^2+(a+\omega_{mf})^2} - \frac{(a+\omega_{mf})\cos(a+\omega_{mf})t}{\omega^2+(a+\omega_{mf})^2}\right)$$
$$+\sin\omega_{mf}t\left(\frac{\omega\sin(a+\omega_{mf})t}{\omega^2+(a+\omega_{mf})^2} - \frac{(a+\omega_{mf})\cos(a+\omega_{mf})t}{\omega^2+(a+\omega_{mf})^2} + \frac{\omega\sin(a-\omega_{mf})t}{\omega^2+(a-\omega_{mf})^2}\right.$$
$$\left.\left.\left. - \frac{(a-\omega_{mf})\cos(a-\omega_{mf})t}{\omega^2+(a-\omega_{mf})^2}\right)\right]\right\}\times\sin\frac{i\pi x}{L}$$

Equation (27) represents the transverse displacement response to a moving force of a uniform Rayleigh beam resting on an elastic foundation.

(6.2) The moving mass problem

In this case, $\Gamma^* \neq 0$ that is, the mass of the moving load is commensurable with that of the structure, the inertia effect of the moving mass is taken into consideration. This is termed the moving mass problem. To this end equation (16) is rearranged to take the form

$$\ddot{W}_i(t) + \omega_{mf}^2 W_i(t) + \Gamma^*\left\{H_A(i,j)\ddot{W}_i(t) + 2\sum_{n=1}^{\infty}\cos\frac{nv_i t}{L}H_B(i,j,n)\ddot{W}_i(t)\right.$$
$$+2v_i H_C(i,j)\dot{W}_i(t) + 4v_i\sum_{n=1}^{\infty}\cos\frac{nv_i t}{L}H_D(i,j)\dot{W}_i(t) + v_i^2 H_E(i,j,n)W_i(t)$$
$$\left.+2v_i^2\sum_{n=1}^{\infty}\cos\frac{nv_i t}{L}H_F(i,j,n)W_i(t)\right\} = \frac{e^{\omega t}}{\mu\Delta_o(i,j)}M_i g\sin\frac{j\pi v_i t}{L}$$

where

$$H_A(i,j)=\frac{H_a(i,j)}{\Delta_o(i,j)} \quad H_B(i,j,n)=\frac{H_b(i,j,n)}{\Delta_o(i,j)}$$

$$H_C(i,j)=\frac{H_c(i,j)}{\Delta_o(i,j)} \quad H_D(i,j,n)=\frac{H_d(i,j,n)}{\Delta_o(i,j)}$$

$$H_E(i,j)=\frac{H_e(i,j)}{\Delta_o(i,j)} \quad H_F(i,j)=\frac{H_f(i,j,n)}{\Delta_o(i,j)} \tag{29}$$

Equation (28) after some simplifications and rearrangements take the form

$$\ddot{W}_i(t)+\frac{\Gamma^*\left(2v_iH_c(i,j)+4v_i\sum_{n=1}^{\infty}\cos\frac{n\pi v_it}{L}H_D(i,j,n)\right)}{\left[1+\Gamma^*\left(H_A(i,j)+2\sum_{n=1}^{\infty}\cos\frac{n\pi v_it}{L}H_B(i,j,n)\right)\right]}\dot{W}_i(t)$$
$$+\frac{\left[\omega_{mf}^2+\Gamma^*\left(v_i^2H_E(i,j)+2v_i^2\sum_{n=1}^{\infty}\cos\frac{n\pi v_it}{L}H_F(i,j,n)\right)\right]}{\left[1+\Gamma^*\left(H_A(i,j)+2\sum_{n=1}^{\infty}\cos\frac{n\pi v_it}{L}H_B(i,j,n)\right)\right]}W_i(t)$$
$$=\frac{e^{\omega t}}{\mu\Delta_o(i,j)}\frac{MgU_i(v_it)}{\left[1+\Gamma^*\left(H_A(i,j)+2\sum_{n=1}^{\infty}\cos\frac{n\pi v_it}{L}H_B(i,j,n)\right)\right]} \tag{30}$$

In equation (30), unlike in the case of moving force problem an exact analytical solution is not feasible. Though the equation yields readily to numerical technique, an analytical approximate method is desirable as solutions so obtained often shed light on vital information about the vibrating system.

Here we seek the modified frequency corresponding to the frequency of the free system due to the presence of the effect of the mass of the system. An equivalent free system operator defined by the modified frequency then replaces equation (30). To this end, we set the right hand side of (30) to zero and consider a parameter $\eta<1$ for any arbitrary mass ratio Γ^* define as

$$\eta=\frac{\Gamma^*}{1+\Gamma^*} \tag{31}$$

So that

$$\Gamma^*=\eta+O(\eta^2) \tag{32}$$

and

$$\frac{1}{\left[1+\Gamma^*\left(H_A(i,j)+2\sum_{n=1}^{\infty}\cos\frac{n\pi v_it}{L}H_B(i,j,n)\right)\right]}$$

$$=\left[1-\eta\left(H_A(i,j)+2\sum_{n=1}^{\infty}\cos\frac{n\pi v_it}{L}H_B(i,j,n)\right)\right] \tag{33}$$

where

$$\left|\eta\left(H_A(i,j)+2\sum_{n=1}^{\infty}\cos\frac{n\pi v_it}{L}H_B(i,j,n)\right)\right|<1 \tag{34}$$

Substituting equations (32) and (33) into equation (30), one obtains

$$\ddot{W}_i(t)+\eta\left[2v_iH_C(i,j)+4v_i\sum_{n=1}^{\infty}\cos\frac{n\pi v_it}{L}H_D(i,j,n)\right]\dot{W}_i(t)$$
$$+\left[\omega_{mf}^2+\eta\left(v_i^2H_E(i,j)+2v_i^2\sum_{n=1}^{\infty}\cos\frac{n\pi v_it}{L}H_F(i,j,n)\right)\right]W_i(t)$$
$$-\omega_{mf}^2\eta\left(H_A(i,j)+2\sum_{n=1}^{\infty}\cos\frac{n\pi v_it}{L}H_B(i,j,n)\right)$$
$$=\frac{e^{\omega t}}{\mu\Delta_o(i,j)}M_ig\sin\frac{j\pi v_it}{L} \tag{35}$$

to $O(\eta)$ only.

When $\eta=0$ in equation (35), a case corresponding to the case when the inertia effect of the mass of the system is neglected, then the solution of (35) can be written as

$$\overline{W}(x,t)=A_{mf}\cos(\omega_{mf}t-\beta_m) \tag{36}$$

where A_{mf} and β_m are constants.

Furthermore as $\eta < 1$ struble's technique required that the asymptotic solutions of the homogeneous part of the equation (28) can be written as

$$\overline{W}(x,t) = C(m,t)\cos\left[\omega_{mf} t - \phi(m,t)\right] + \eta\varphi(1,t) + O(\eta^2) \tag{37}$$

where $C(m,t)$ and $\phi(m,t)$ are slowly varying function of time or equivalent,

where $\rightarrow$ implies " is of "

To obtain the modified frequency, equation (37) and its derivatives are substituted into the homogeneous part of equation (35). We extract only the variational part of the equation describing the behaviour of $C(m,t)$ and $\phi(m,t)$ during the motion of the mass. Thus, making this substitution and taken into account the following trigonometric identities

$$\sin\left[\omega_{mf} t - \phi(m,t)\right]\cos\tfrac{n\pi v_i t}{L} = \frac{1}{2}\left\{\sin\left[\omega_{mf} t - \phi(m,t) + \tfrac{n\pi v_i t}{L}\right] + \sin\left[\omega_{mf} t - \phi(m,t) - \tfrac{n\pi v_i t}{L}\right]\right\}$$

$$\cos\left[\omega_{mf} t - \phi(m,t)\right]\cos\tfrac{n\pi v_i t}{L} = \frac{1}{2}\left\{\cos\left[\omega_{mf} t - \phi(m,t) + \tfrac{n\pi v_i t}{L}\right] + \cos\left[\omega_{mf} t - \phi(m,t) - \tfrac{n\pi v_i t}{L}\right]\right\} \tag{38}$$

and neglecting terms which do not contribute to the variational equations we obtain

$$\begin{aligned} &-2\dot{C}(m,t)\omega_{mf}\sin\left[\omega_{mf} t - \phi(m,t)\right] + 2C(m,t)\dot{\phi}(m,t)\omega_{mf}\cos\left[\omega_{mf} t - \phi(m,t)\right] \\ &-2\eta v_i H_C(i,j)C(m,t)\omega_{mf}\sin\left[\omega_{mf} t - \phi(m,t)\right] + \eta C(m,t)v_i^2 H_E(i,j)\cos\left[\omega_{mf} t - \phi(m,t)\right] \\ &-\eta\omega_{mf}^2 H_A(i,j)\cos\left[\omega_{mf} t - \phi(m,t)\right] = 0 \end{aligned}$$

Retaining terms to $O(\eta)$ only.

The variational equations are obtained by equating the coefficients of $\sin\left[\omega_{mf} t - \phi(m,t)\right]$ and $\cos\left[\omega_{mf} t - \phi(m,t)\right]$ and setting them to zero, thus

$$-2\dot{C}(m,t) - 2\eta v_i H_C(i,j)C(m,t)\omega_{mf} = 0 \tag{40}$$

and

$$-2C(m,t)\dot{\phi}(m,t)\omega_{mf} + \eta C(m,t)v_i^2 H_E(i,j) - \omega_{mf}^2 \eta H_A(i,j) = 0 \tag{41}$$

solving equations (40) and (41) respectively gives

$$C(m,t) = A_m^* e^{-Zt} \tag{42}$$

and

$$\phi(m,t) = \frac{\eta}{2}\left[-v_i^2\frac{H_E(i,j)}{\omega_{mf}} + \omega_{mf} H_A(i,j)\right]t + \varphi_m \tag{43}$$

where φ_m is a constant.

Therefore, when the inertia effect of the moving mass is considered, the first approximation to the homogeneous system is

$$W_i(t) = A_m^* e^{-Zt}\cos\left[\omega_{kf} t - \varphi_m\right] \tag{44}$$

where

$$\omega_{mm} = \omega_{mf}\left\{1 - \frac{\eta}{2}\left[H_A(i,j) - v_i^2\frac{H_E(i,j)}{\omega_{mf}^2}\right]\right\} \quad 45)$$

represents the modified natural frequency due to the presence of the moving mass. It is observed that when $\eta = 0$, we recover the frequency of the moving force problem when the inertia effect of the moving mass is neglected. Thus, to solve the non-homogeneous equation (28), the differential operator which acts on $Q(m,t)$ and $Q(k,t)$ is replaced by the equivalent free system operator defined by the modified frequency ω_{kf}. Using equation (45), the homogeneous part of equation (30) can be written as

$$\frac{d^2 W_i(t)}{dt^2} + \omega_{mm}^2 W_i(t) = 0 \tag{46}$$

Thus, the entire equation (30) becomes

$$\frac{d^2 W_i(t)}{dt^2} + \omega_{mm}^2 W_i(t) = \frac{e^{\omega t}}{\mu\Delta_o(i,j)} Mg\sin\frac{j\pi v_i t}{L} \tag{47}$$

retaining $O(\eta)$ only.

Equation (47) is analogous to equation (17). Thus, using similar argument as in moving force problem, $W_i(t)$ can be obtained which on inversion gives

$$
\begin{aligned}
Q_m(x,t) = \sum_{i=1}^{n} \frac{\eta L g}{2\omega_{mm}\Delta_o(i,j)} \Bigg\{ & \cos\omega_{mm}t\left(\frac{\omega}{\omega^2+(a-\omega_{mm})^2} - \frac{\omega}{\omega^2+(a+\omega_{mm})^2}\right) \\
& - \sin\omega_{mm}t\left(\frac{(a+\omega_{mm})}{\omega^2+(a+\omega_{mm})^2} - \frac{(a-\omega_{mm})}{\omega^2+(a-\omega_{mm})^2}\right) - e^{\omega t}\Bigg[\cos\omega_{mm}t\Bigg(\frac{(a-\omega_{mm})\sin(a-\omega_{mm})t}{\omega^2+(a-\omega_{mm})^2} \\
& + \frac{\omega\cos(a-\omega_{mm})t}{\omega^2+(a-\omega_{mm})^2} - \frac{(a+\omega_{mm})\sin(a+\omega_{mm})t}{\omega^2+(a+\omega_{mm})^2} - \frac{(a+\omega_{mm})\cos(a+\omega_{mm})t}{\omega^2+(a+\omega_{mm})^2}\Bigg) \\
& + \sin\omega_{mm}t\Bigg(\frac{\omega\sin(a+\omega_{mm})t}{\omega^2+(a+\omega_{mm})^2} - \frac{(a+\omega_{mm})\cos(a+\omega_{mm})t}{\omega^2+(a+\omega_{mm})^2} + \frac{\omega\sin(a-\omega_{mm})t}{\omega^2+(a-\omega_{mm})^2} \\
& - \frac{(a-\omega_{mm})\cos(a-\omega_{mm})t}{\omega^2+(a-\omega_{mm})^2}\Bigg)\Bigg]\Bigg\} \times \sin\frac{i\pi x}{L}
\end{aligned}
\tag{48}
$$

which represents the transverse-displacement response to a moving mass at a constant velocity of a uniform Rayleigh beam resting on elastic foundation.

7.0 Results and Discussions

In order to illustrate the analytical results, the uniform Bernoulli-Euler beam is taken to be of the length L=15.192. Other values used are $M = 8407.27Kg$, $E = 2.10924\times10^9$ and $V = 3.128m/s$. The transverse deflections of the beam are calculated and plotted against time for various values of foundation constant (moduli), axial force and the rotatory inertia factor. Values of K between $0N/m^3$ and $4.0\times10^6 N/m^3$ were used while the values of N were varied between $N = 0N$ and $N = 2.0\times10^6 N$.

Figure 1 illustrates the displacement response of the simply supported Bernoulli

-Euler beam for a moving force problem for fixed values of N and various values of K. Clearly, the results show that as the foundation modulus increases, the transverse displacement of the beam decreases. Similar results are obtained when the simply supported beam is transverse by a concentrated masses moving at constant speed as shown in figure 4.

The deflection profile for various values of applied axial force N for both cases of moving force and moving mass problems of the uniform beam are displayed in figures 2 and 5 respectively. It is observed that as the applied axial force N increases the transverse displacement of the beam decreases.

In Figures 3 and 6, the dynamic response of Bernoulli-Euler beam simply supported at both ends for various values of rotatory inertia R^o are showcased for both cases of moving force and moving mass problems. These figures depict that as the rotatory inertia correction factor R^o increases, the response amplitudes of vibration of the elastic beam decreases.

The comparison of the transverse displacement of moving force and moving mass cases for the simply supported beam transverse by a moving load travelling at constant velocity for $K = 40{,}000N/m^3$ and

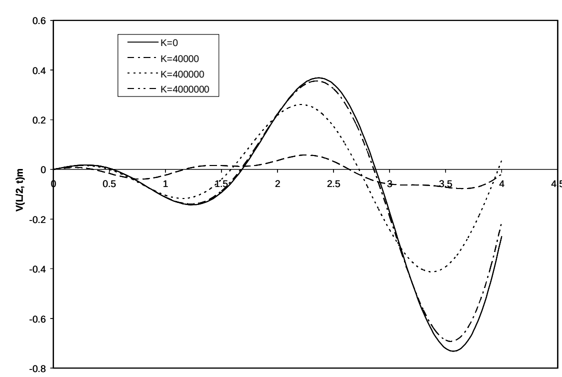

Fig 1: Transverse displacement of a simply supported moving force for various values of foundation modulus and fixed values of axial force $N = 20000N$ and rotatory inertia $R^0 = 0.8$

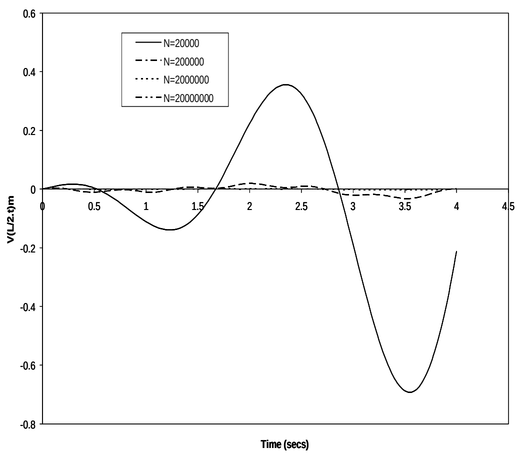

Fig 2: Deflection profile of simply supported moving force for various values of axial force and fixed values of foundation modulus $K = 40000N/m^3$ and rotatory inertia $R^0 = 0.8$

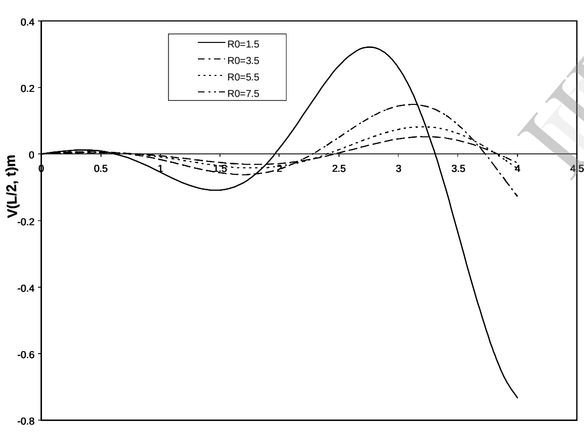

Fig 3: The deflection of simply supported moving force for various values of rotatory inertia and fixed values of foundation modulus $K = 40000N/m^3$ and axial force $N = 20000N$

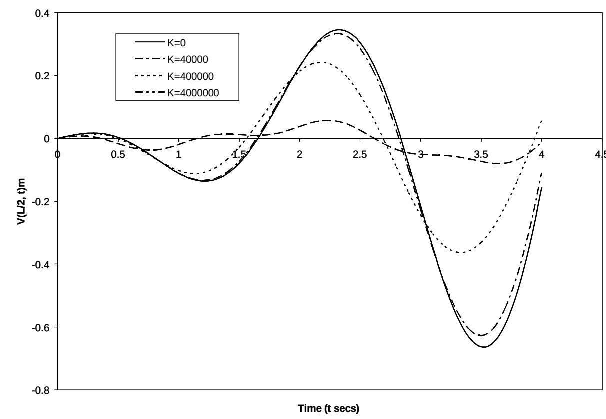

Fig 4: Transverse displacement of a simply supported moving mass for various values of foundation modulus and fixed axial force $N = 20000N$ and rotatory inertia $R^0 = 0.8$.

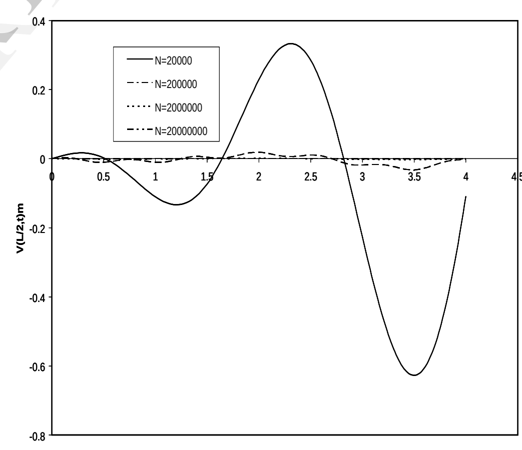

Fig 5: Deflection profile of simply supported moving mass for various values of axial force and fixed values of foundation odulus $K = 40000N/m^3$ and rotatory inertia $R^0 = 0.8$

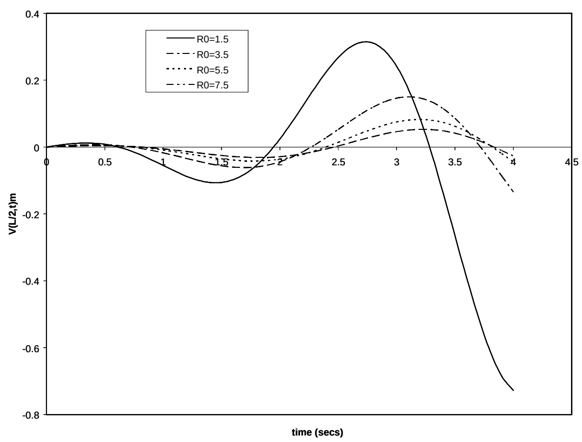

Fig 6: The deflection of simply supported moving mass for various values of rotatory inertia and fixed values of foundation modulus $K = 40000N/m^3$ **and axial force** $N = 20000N$.

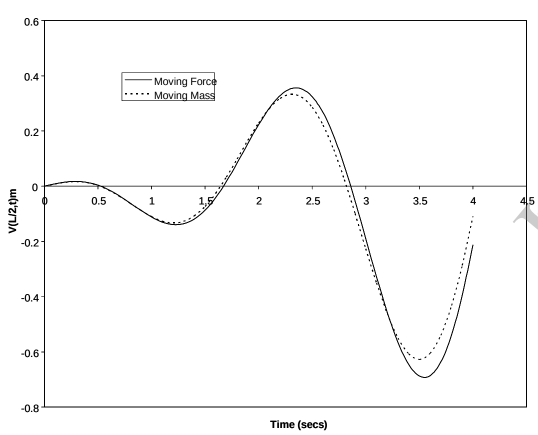

Fig 7: Comparison of the displacement response of moving force and moving mass cases of a simply supported fixed values $N = 20000N$, $K = 40000N/m^3$ **and** $R^0 = 0.8$

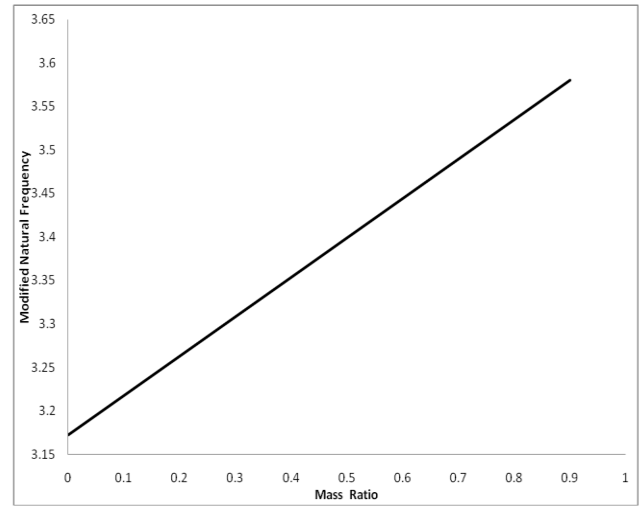

Fig 8: The graph of modified natural frequency against mass ratio for fixed value of foundation modulus $K = 40000N/m^3$ **and axial force** $N = 20000N$.

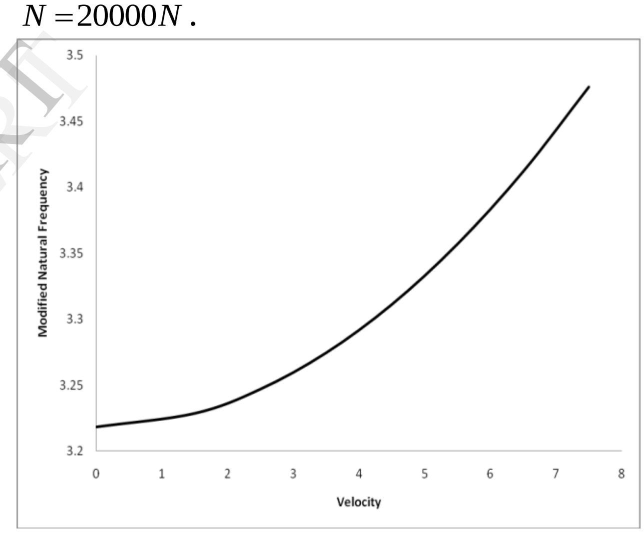

Fig 9: The graph of modified natural frequency against velocity for fixed value of foundation modulus $K = 40000N/m^3$ **and axial force** $N = 20000N$.

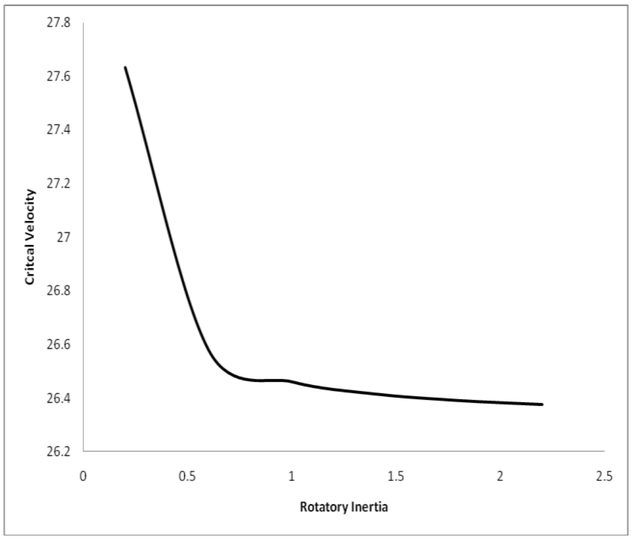

Fig 10: The graph of modified natural frequency against rotatory inertia for fixed value of foundation modulus $K = 40000 N/m^3$ and axial force $N = 20000N$.

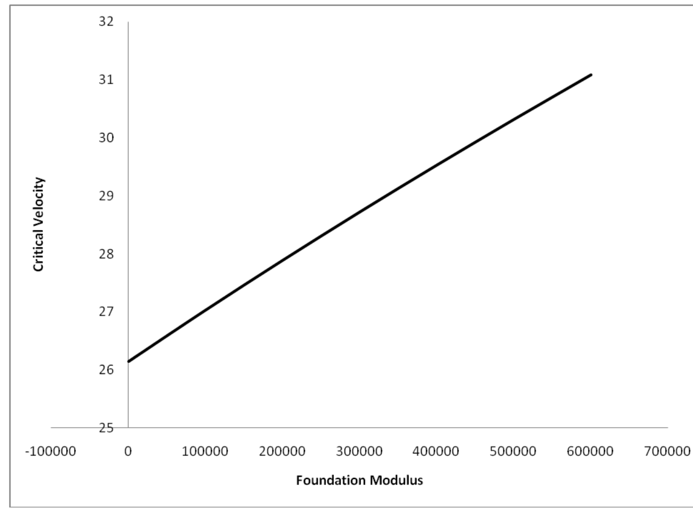

Fig 12: The graph of critical velocity against foundation modulus for fixed value of axial force $N = 20000N$.

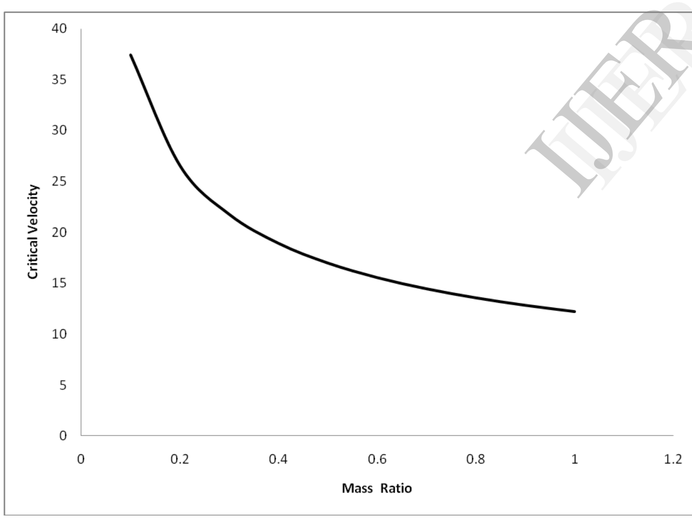

Fig 11: The graph of critical velocity against mass ratio for fixed value of foundation modulus $K = 40000 N/m^3$ and axial force $N = 20000N$.

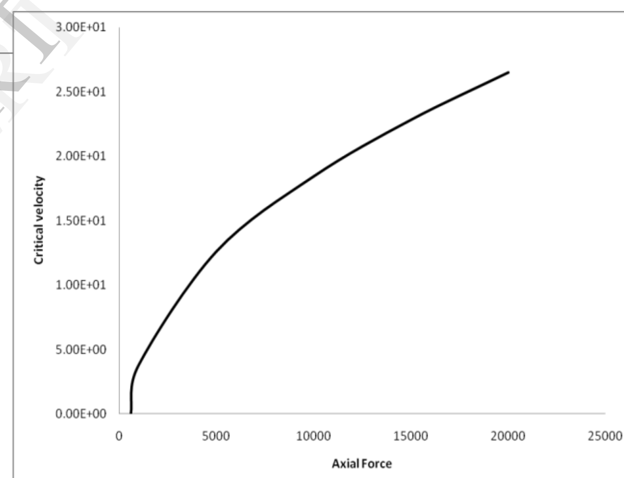

Fig 13: The graph of critical velocity against axial force for fixed value of foundation modulus $K = 40000 N/m^3$.

$N = 20,000N$ is displayed in figure 7. It is observed from this figure that relying on moving force solution as a safe approximation to moving mass problem is quite misleading.

In figure 8, the relationship between the natural frequency and the mass ratio is displayed. It is shown from the figure that as the mass ratio increases the natural frequency of the system also increases. Similar result is obtained in figure 9 which depicts that as the velocity of the traversing load increases the natural frequency of the system also increases. For fixed values of foundation modulus K and axial force N figure 10 clearly shows that as the values of rotatory inertia correction factor increases, the natural frequency of the system decreases.

In figure 11, it is clearly shown that as the mass ratio increases the critical velocity of the dynamical system decreases. While figure 12 depicts that as the values of foundation modulus K increases for fixed values of other parameters the critical velocity of the system also increases. Similar result is obtained in figure 13 which show that for fixed value of foundation modulus K and rotatory inertia correction factor R^0 the critical velocity of the system increases as the value of axial force N increases. These interesting results confirm that the presence of these structural parameters in appropriate measures in the design of engineering structures will enhance safety and reliability in the design of such structures.

7.0 Conclusion

The dynamic behaviour of a beam simply supported at both ends and carrying moving concentrated varying magnitude loads has been analyzed. An approximate method of solution has been employed to treat the governing differential equations of motion describing the dynamic interactions of the continuous system and the moving sub-systems. The system response in series form has been obtained with the inclusion of the inertial effect of the moving mass in the governing differential equations of motion. It was observed that the moving mass inertial effect is very significant. Results further show that structural parameters such as the foundation modulus, axial force, mass ratio and the rotatory inertia correction factor have significant effects on the flexural motions and critical velocity of elastic structures carrying moving load. Thus, in the design of engineering structures such as railway bridges, overhead cranes, cableways and tunnels effects of these important parameters should put into considerations to guarantee the safety and reliability of the design.

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