

Increasing Uniformity in Sprinkler Irrigation Through Hexagonal Layout Design In lieu of Conventional Grid Arrangement

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Abstract - Sprinkler irrigation plays a crucial role in promoting water-efficient agriculture in semi-arid areas like Banaskantha District, Gujarat. However, traditional square-grid configurations have geometric flaws that lead to uneven water distribution, particularly at the edges of fields. A field trial carried out on a 0.53-hectare groundnut farm in Kodram Village, Banaskantha District, Gujarat, observed an overall Christiansen's Uniformity Coefficient (CU) of just 66.30% with a standard 9 m × 9 m square grid, although the central interior zones performed better at 92.7%. This paper presents the theoretical foundation for substituting the square-grid layout with a hexagonal one. Based on circle-packing mathematics validated by Joseph Louis Lagrange, the hexagonal design addresses issues of edge concentration, reduces wind drift, and is anticipated to improve the overall field CU to 84.7%. Supported by state micro-irrigation subsidies, this transition offers a highly efficient and economically viable solution to maximize water utilization and crop yields.

Keywords: *Sprinkler Irrigation, Uniformity Coefficient, Hexagonal Layout Sprinkler System, Water Distribution, Micro-Irrigation, Christiansen's Coefficient, Semi-Arid Agriculture, Irrigation Engineering*

INTRODUCTION TO AGRONOMIC WATER SCARCITY AND MICRO-IRRIGATION DEFICITS

Water scarcity is a pressing systemic challenge in semi-arid regions like Gujarat, India, making efficient micro-irrigation systems essential for water-sensitive cash crops like groundnut (*Arachis hypogaea*). While micro-irrigation technologies supply water directly to the root zone—eliminating conveyance losses and evaporation inherent in surface irrigation—standard field installations frequently fall short of their potential due to geometric design flaws, environmental variables, and hydraulic inefficiencies.

An extensive field study was carried out on a 0.53-hectare plot in Kodram Village to assess a traditional square-grid sprinkler pattern in order to quantify this. Testing showed serious problems with spatial uniformity: the measured Christiansen's Uniformity Coefficient (CU) was just 66.30%, much below the industry standard of 85%. Nonetheless, an outstanding 92.7% CU was attained by central collecting nodes that received overlap from four sprinklers. A basic mathematical issue is revealed by this 26.4 percentage-point difference: traditional square grids routinely compromise field-edge and corner regularity in order to attain sufficient interior coverage.

To resolve the spatial coverage limitations at their geometric foundation, this study proposes substituting the conventional square grid layout with a hexagonal configuration. By integrating principles of dense circle packing, hydraulic pressure optimization, and environmental drift control, the hexagonal arrangement is expected to reduce edge-related coverage deficiencies, improve overall field uniformity to nearly 84.7%, and considerably strengthen the system's resistance to wind-induced drift.

LITERATURE REVIEW

The pursuit of optimized spatial water distribution dates back to early agricultural engineering efforts of the twentieth century. In 1926, Wadsworth proposed the use of uniformly cross-sectioned test receptacles placed at equidistant intervals to measure application equality, establishing the foundational methodology for modern catch-can auditing. *Staebner (1935)* later ran extensive tests on sprinklers, judging performance based on maximum and minimum depth ratios to highlight distribution inequalities.

The defining standard for measuring this specific distribution was introduced by *J.E. Christiansen* in 1942. Through the execution of over two hundred rigorous field tests designed to determine the uniformity of distribution for various emitter spacings and geometrical configurations, Christiansen formalized the numerical uniformity coefficient that remains the global standard today. Christiansen additionally classified the cross-sectional water distribution profiles into specific geometric patterns, such as "Pattern Type A," which describes a conical water distribution where the maximum application depth occurs adjacent to the sprinkler and decreases linearly toward the perimeter.

Subsequent research expanded on these foundational metrics. *Wilcox and Swailes (1947)* proposed a modified procedure for determining uniformity to better evaluate the complex effects of pressure, wind, spacing, and nozzle size. *Wiersma (1952)* and *Korven (1952)* further investigated the severe influences of wind velocity and pressure drops on the distribution pattern of slow-revolving sprinklers. *Molenaar and associates (1954)* evaluated these overlapping factors under actual field conditions, concluding that the pronounced effect of wind drift could be largely overcome by calculating and implementing the correct spatial spacing of the sprinklers across the field.

THEORETICAL FRAMEWORK OF SPRINKLER UNIFORMITY METRICS

The ability of a pressurized sprinkler system to distribute irrigation water uniformly across a topographical plane is critical; uneven distribution generates localized drought zones that stunt crop root development, alongside waterlogged zones that lead to nutrient leaching and anaerobic root asphyxiation.

THE CHRISTIANSEN UNIFORMITY COEFFICIENT

The Christiansen Uniformity Coefficient (Cu) is defined by the following mathematical equation:

$$Cu = 100 \left(1 - \frac{\sum X}{mn} \right)$$

In this formula, m is the average value of all observations, n shows total number of observation points and X is numerical deviation of individual observations from the avg.application rate. A calculated Cu of 100% indicates absolute, perfect uniformity, while a coefficient of 85% or greater is conventionally considered satisfactory by entities such as the Soil Conservation Service and the American Society of Agricultural Engineers.

FIELD STUDY OPERATIONS: THE KODRAM FARM EXPERIMENT

To establish an empirical baseline demonstrating the severe deficiencies of conventional grid layouts, an exhaustive evaluation was executed on an active groundnut farm in Kodram Village, Vadgam Taluka, Banaskantha District.

Groundnut is a highly water-sensitive cash crop typically cultivated with a tight plant spacing of 0.76 m × 0.4 m. The total farm area of 2.49 hectares was divided into five distinct operational shifts to manage available flow, and a 0.53-hectare section operating under a single shift was isolated for this rigorous study.

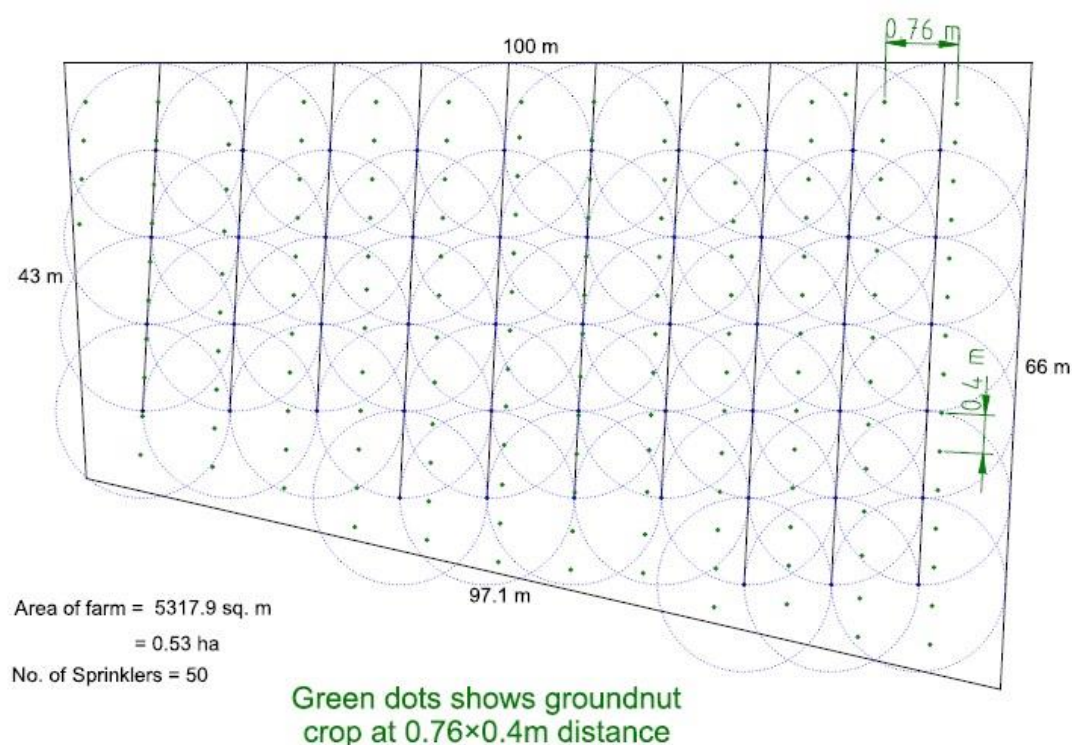


Fig. 1 : Sprinkler layout for groundnut with wetting circles

SYSTEM ARCHITECTURE AND HYDRAULIC SPECIFICATIONS

The installed micro-irrigation system consisted of fifty rotating impact mini-sprinklers initially designed for a 9 m X 9 m square grid layout. The primary hardware infrastructure supporting this layout included a main conveyance line consisting of 90 mm PVC pipe and lateral pipelines constructed from 32 mm LLDPE Class II tubing (2.0 mm wall thickness). The sprinklers were elevated above the mature groundnut canopy using 12 mm diameter riser pipes measuring 1330 mm in length.

Hydraulic power was supplied by a 25 horsepower (HP) submersible turbine pump drawing groundwater from a deep bore well at a dynamic head depth of 137.17 meters, possessing a tested available flow capacity of 8.2 liters per second (lps). According to the manufacturer's engineering specifications, the system was designed to operate at an optimal nozzle pressure of 2.0 bar to deliver a discharge of 450 liters per hour (lph) with an effective wetting radius of 9.0 meters.

EMPIRICAL CATCH-CAN AUDITING PROTOCOL

To calculate the actual Uniformity Coefficient under real-world conditions, a physical catch-can audit was executed with high precision using sixty-seven identical tin catch cans (14.5 cm in height and 10 cm in diameter). These receptacles were systematically placed across the field in a secondary grid intersecting exactly halfway between the primary sprinklers—specifically at 4.5-meter intervals.

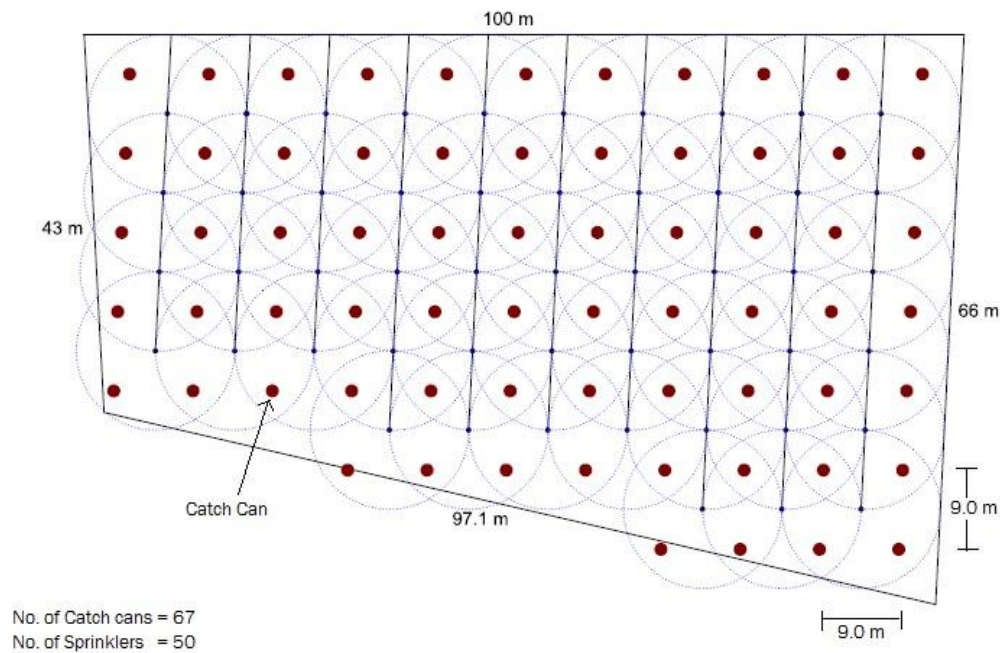


Fig. 2 : Catch can layout

The pressurized system was operated continuously for exactly 2.0 hours. Environmental meteorological conditions were actively recorded, noting an ambient, sustained wind speed of 7.11 meters per second (25.6 km/hr) blowing predominantly in a North/East direction. Following the strict two-hour test duration, the water volume in each can was measured using a 100 ml graduated cylinder (3.6 cm internal diameter).

EMPIRICAL DATA ANALYSIS: IDENTIFYING GRID DEFICIENCIES

The empirical data extracted from the Kodram farm experiment exposed a series of cascading, interconnected inefficiencies that are endemic to square-grid layouts operating in harsh, semi-arid field conditions.

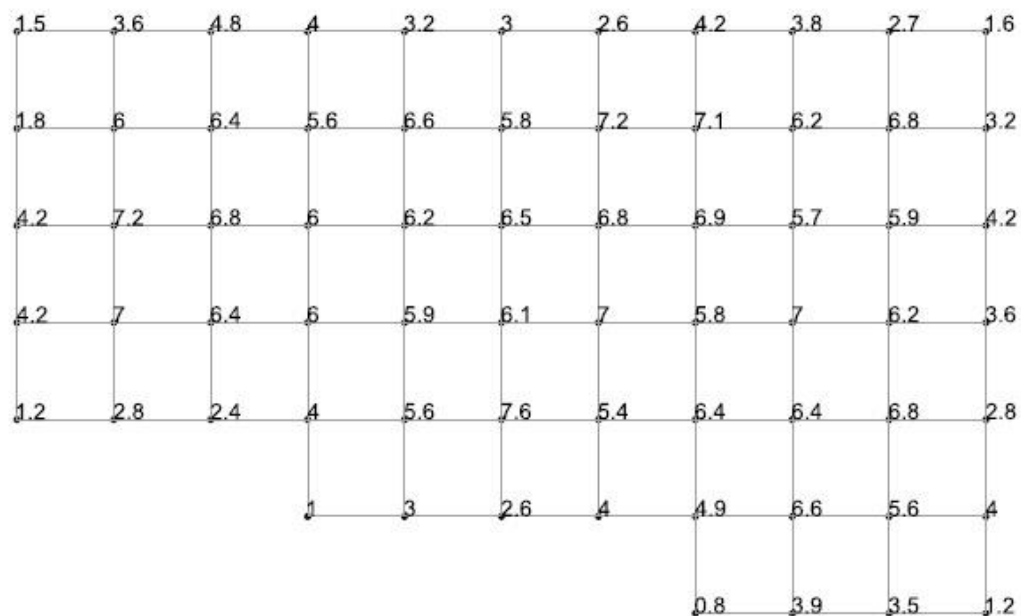


Fig. 3 : Water depth in each can (All readings are in mm)

The processed depth measurements from the sixty-seven catch cans varied widely, ranging from a severe minimum of 0.8 mm at the outer boundary edges to a maximum of 7.6 mm in the overlapping interior zones.

Obs. (O)	Freq. (F)	O×F	Deviation (D)	F×D	Obs. (O)	Freq. (F)	O×F	Deviation (D)	F×D
0.8	1	0.8	4.02	4.02	4.9	1	4.9	0.08	0.08
1	1	1	3.82	3.82	5.4	1	5.4	0.58	0.58
1.2	2	2.4	3.62	7.24	5.6	3	16.8	0.78	2.34
1.5	1	1.5	3.32	3.32	5.7	1	5.7	0.88	0.88
1.6	1	1.6	3.22	3.22	5.8	2	11.6	0.98	1.96
2.4	1	2.4	2.42	2.42	5.9	2	11.8	1.08	2.16
2.6	2	5.2	2.22	4.44	6	3	18	1.18	3.54
2.7	1	2.7	2.12	2.12	6.1	1	6.1	1.28	1.28
2.8	2	5.6	2.02	4.04	6.2	3	18.6	1.38	4.14
3	2	6	1.82	3.64	6.4	4	25.6	1.58	6.32
3.2	2	6.4	1.62	3.24	6.5	1	6.5	1.68	1.68
3.5	1	3.5	1.32	1.32	6.6	2	13.2	1.78	3.56
3.6	2	7.2	1.22	2.44	6.8	4	27.2	1.98	7.92
3.8	2	7.6	1.02	2.04	6.9	1	6.9	2.08	2.08
3.9	1	3.9	0.92	0.92	7	3	21	2.18	6.54
4	4	16	0.82	3.28	7.1	1	7.1	2.28	2.28
4.2	4	16.8	0.62	2.48	7.2	2	14.4	2.38	4.76
4.8	1	4.8	0.02	0.02	7.6	1	7.6	2.78	2.78
						ΣF =	mn =	ΣX =	
						67	323.8	108.9	

Table 1: Observation Table for Coefficient of Uniformity - Whole Field

Based on the aggregated data, the arithmetic mean and the final Uniformity Coefficient were calculated as follows:

$$Mean = \frac{mn}{\Sigma F} = \frac{323.08}{67} = 4.82$$

Christiansen's uniformity coefficient,

$$Cu = 100 \left(1 - \frac{\Sigma X}{mn} \right) = 100 \left(1 - \frac{108.9}{323.08} \right)$$

Cu = 66.30 %

This CU of 66.30% is categorically classified as "poor" to "worse" under standard irrigation evaluation metrics, falling drastically short of the 85% minimum threshold required for agricultural viability.

THE EDGE-EFFECT PHENOMENON

To mathematically isolate the primary cause of this poor field-wide uniformity, the dataset was carefully filtered to examine only the thirty-five catch cans located in the interior "central zones" of the field, which are fully bounded by the overlapping radii of four distinct sprinklers.

Obs. (O)	Freq. (F)	O×F	Deviation (D)	F×D
5.4	1	5.4	0.99	0.99
5.6	3	16.8	0.79	2.37
5.7	1	5.7	0.69	0.69
5.8	2	11.6	0.59	1.18
5.9	2	11.8	0.49	0.98
6	3	18	0.39	1.17
6.1	1	6.1	0.29	0.29
6.2	3	18.6	0.19	0.57
6.4	4	25.6	0.01	0.04
6.5	1	6.5	0.11	0.11
6.6	2	13.2	0.21	0.42
6.8	4	27.2	0.41	1.64
6.9	1	6.9	0.51	0.51
7	3	21	0.61	1.83
7.1	1	7.1	0.71	0.71
7.2	2	14.4	0.81	1.62
7.6	1	7.6	1.21	1.21
$\Sigma F = 35$		$mn = 223.5$	$\Sigma X = 16.33$	

Table 2: Observation Table for Coefficient of Uniformity for center cans

Recalculating the Christiansen coefficient exclusively for this isolated, well-supported zone yielded the following result:

$$Mean = \frac{mn}{\Sigma F} = \frac{223.5}{35} = 6.39$$

Christiansen's uniformity coefficient,

$$Cu = 100 \left(1 - \frac{\Sigma X}{mn} \right) = 100 \left(1 - \frac{16.33}{223.5} \right)$$

$Cu = 92.7 \%$

This exceptionally high value confirms an important hypothesis: the sprinklers themselves function properly, but the square grid layout structurally sacrifices field perimeters. A boundary point receives water from only two contributing sprinklers, and a corner point receives water from only a single sprinkler. Generated water contours

definitively illustrated that the central part of the farm receives uniform, heavy water application, while the borders remain critically dry. This geometric reality guarantees that the perimeter of any square-grid field will always suffer from chronic, systemic under-watering.

ENVIRONMENTAL DISTORTION: THE PHYSICS OF WIND DRIFT

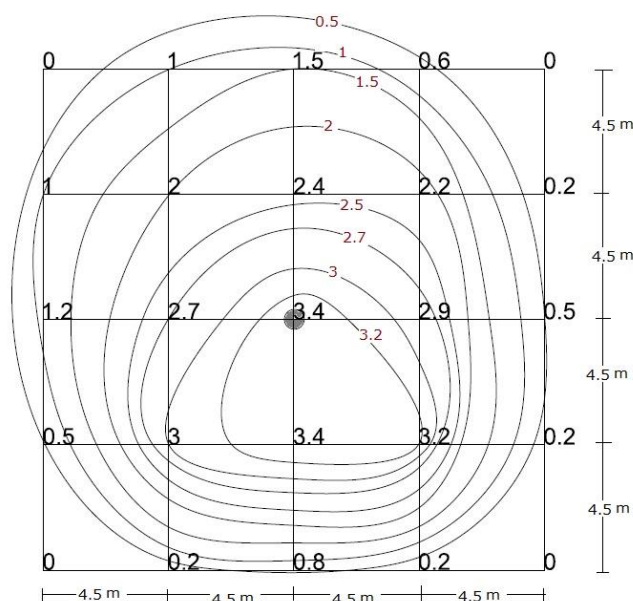


Fig. 4 : Effect of wind on water contour (All depth readings are in mm)

Wind velocity profoundly and immediately affects sprinkler application uniformity by physically altering droplet trajectories. Wind drift distorts the inherently circular wetting pattern into an elongated oval, violently shifting the precipitation volume away from the windward side.

During the field test, the sustained 7.11 m/s wind speed created stark, observable dry zones precisely between the 9-meter laterals, compounding the boundary deficits and severely fracturing overlap integrity.

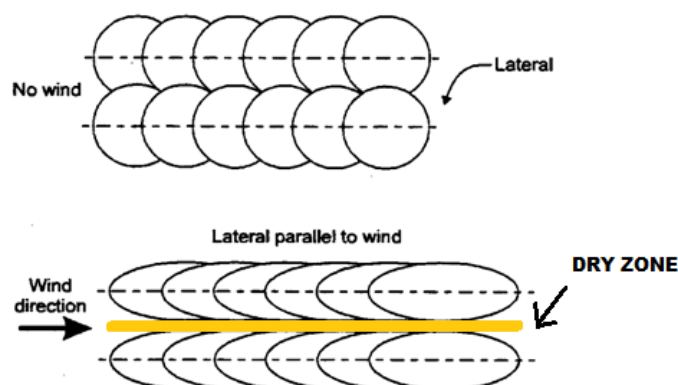


Fig.5 : Dry zone due to wind drift

MATHEMATICAL PRINCIPLES OF SPATIAL COVERAGE AND CIRCLE PACKING

The geometric distribution of overlapping sprinkler trajectories across a two-dimensional agricultural plane is fundamentally an applied iteration of the classical mathematical "circle packing" and "covering" problems.

THE TRUE HEXAGONAL (HONEYCOMB) STRUCTURAL COVERING

In the two-dimensional Euclidean plane, the mathematician Joseph Louis Lagrange definitively proved in 1773 that the highest-density lattice packing of identical circles is the hexagonal arrangement. In this staggered, honeycomb-like lattice structure, the center of each circle serves as the vertex of a regular hexagon, and each circle is intimately surrounded by and tangent to six neighboring circles. A true hexagonal (honeycomb) lattice

offers a unique, highly efficient structural covering strategy for overlapping sprinkler trajectories. In a honeycomb lattice, sprinklers are positioned strictly at the vertices of regular hexagons, mimicking natural cellular structures.

The mathematical packing density of a standard square grid arrangement is structurally limited to $\frac{\pi}{4}$, which equates to approximately 78.54%. Conversely, the hexagonal lattice achieves a mathematical packing density of $\frac{\pi}{2\sqrt{3}}$, which equates to approximately 90.69%. When applying this pure geometry to agricultural irrigation, the large geometric gaps inherent in a square grid mandate massive, highly inefficient overlaps, inevitably leading to extreme precipitation concentration in the geometric center of the field and severe, unmitigated deficits at the boundaries.

TRIGONOMETRIC DERIVATIONS OF OVERLAP AREA

To mathematically quantify the superiority of the hexagonal layout, one must calculate the exact area of precipitation overlap. The exact area of overlap between two intersecting circular wetting patterns of radius R , separated by a linear distance d between their origin points, is derived from the area of the circular sectors minus the area of the central kites formed by the radii and the intersecting chord. The formula is expressed as:

$$\text{Overlap Area} = 2R^2 \times \arccos\left(\frac{d}{2R}\right) - \frac{d}{2}\sqrt{4R^2 - d^2}$$

Applying this to a honeycomb field spacing where the effective sprinkler radius $R = 9$ m and the target hexagonal side length $a = 7.5$ m, the trigonometric calculation guarantees a deep, multi-layered precipitation overlap. In a true hexagonal packing lattice, every interior sprinkler is geometrically tangent to three equidistant nearest neighbors at distance $d=a$. Crucially, this specific geometric arrangement guarantees that any interior point in the field receives simultaneous precipitation from an absolute minimum of three overlapping sprinklers, completely eliminating the single-sprinkler corner deficits that plague conventional square designs.

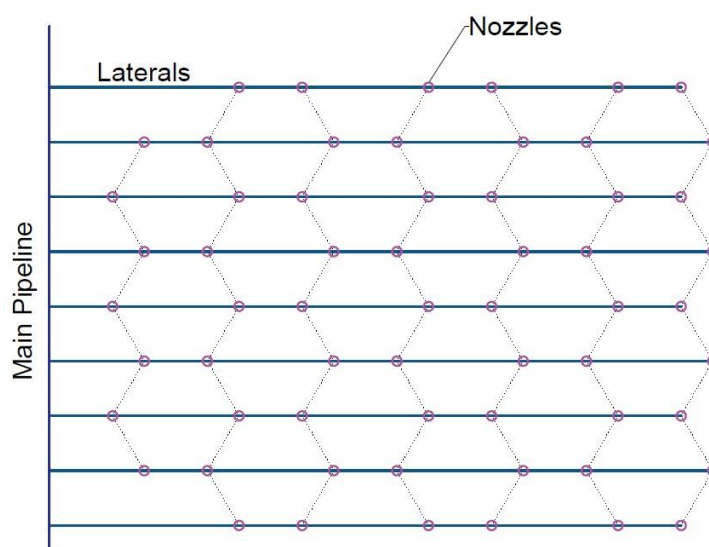


Fig. 6 : sprinklers (nozzles) positioned at the vertices of regular hexagons

THE TRUE HEXAGONAL LAYOUT SOLUTION: GEOMETRIC CORRECTIONS

To structurally and permanently eliminate the edge-effect deficits and improve the system's environmental resilience against wind drift, a complete paradigm shift from the square grid to a true hexagonal (honeycomb) layout is necessary. In this advanced arrangement, sprinklers are placed strictly at the vertices of regular hexagons, forming an interlocking honeycomb lattice.

Area Allocation and Row Offset Adjustments

If a represents the fixed side length of the regular hexagon (set to 7.5 meters for optimization), the geometry of a true hexagonal layout drastically alters the spatial relationship on the lateral pipelines.

- **Along-Lateral Spacing:** Unlike a square grid where nozzles are equidistant, a true hexagonal layout requires the distance between consecutive nozzles on a single lateral to alternate. To form the hexagons, the nozzles are clustered in pairs separated by a distance of a (7.5 m), with a larger gap of $2a$ (15.0 m) between adjacent pairs. Thus, the spacing sequence along any single horizontal pipe is $a, 2a, a, 2a$.
- **Lateral-to-Lateral Spacing:** The perpendicular distance between the lateral rows is mathematically defined as the altitude spanning the slanted sides of the hexagon: $a \times \frac{\sqrt{3}}{2} \approx 0.866a$. For $a=7.5$ m, the lateral spacing physically reduces from 9.0 meters down to approximately 6.5 meters.
- **Row Offset:** Alternate lateral rows in the field are physically shifted down the line so that the paired nozzles (gap a) align perfectly with the wider gaps ($2a$) of the adjacent rows above and below them. Because a single hexagon spans across three laterals (top edge on Lateral 1, middle vertices on Lateral 2, bottom edge on Lateral 3), this offset creates the staggered, interlocking honeycomb effect.

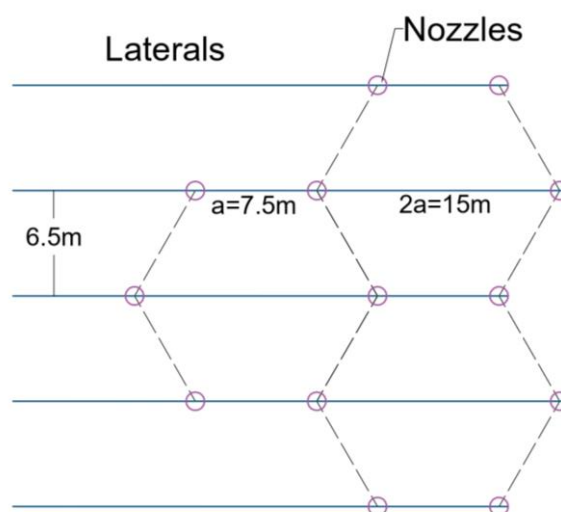


Fig. 7 : Spacing along laterals and lateral to lateral spacing

- **Area per Emitter:** In a true hexagonal layout, the area serviced by one sprinkler is calculated as $\frac{3\sqrt{3}}{4}a^2 \approx 1.299a^2$. For a base hexagon side length of 7.5 meters, the required area per sprinkler reduces from 81 square meters to 73.06 square meters, indicating a moderately denser, more efficient application grid.

Geometric Property	Square Grid (9 × 9 m)	True Hexagonal (a = 7.5 m)
Lateral Spacing (Row to Row)	9.0 meters	6.5 meters
Along-Lateral Spacing (Emitter to Emitter)	9.0 meters	Alternating 7.5 m and 15.0 m
Land Area per Sprinkler Head	81.0 square meters	73.06 square meters
Number of Tangent Neighbors per Sprinkler	4 units	3 units (Honeycomb structure)
Minimum Overlapping Sprinklers (Interior)	4 units	3 units (Geometrically guaranteed)
Minimum Overlapping Sprinklers (Boundary)	1 to 2 units	2 to 3 units
Number of Sprinklers required for 0.53 Ha	50 units	~73 units

Table 3: Geometric Comparison — Square Grid vs. True Hexagonal Layout Architecture

PROJECTED UNIFORMITY CALCULATION FOR TRUE HEXAGONAL LAYOUT

Based directly on Christiansen's observed Pattern Type A (the conical distribution profile), adopting this denser true hexagonal layout ensures that the extreme low-precipitation boundary zones (which previously recorded application depths as low as 0.8 mm to 2.4 mm) are physically eliminated by the multi-lateral overlap. The true hexagonal matrix guarantees that almost every boundary coordinate intersects with the wetting radius of at least 2 to 3 sprinklers, lifting the baseline moisture depth.

To mathematically project the impact of this geometric correction across the entire 0.53-hectare parcel, we map the expected variables onto Christiansen's formula. By eliminating the dry boundary extremes, the field's new mean application depth rises to match the highly supported central zone measurements from the original field test (approximately 6.45 mm), and the absolute deviations between overlapping zones tighten substantially.

For an equivalent sample size of $n = 67$ observation points across the field:

- Projected Mean Application Depth = 6.45 mm
- Projected Total Sum of Observations $mn = 432.15$ mm
- Projected Total Sum of Deviations $\sum X = 66.12$ mm (Dropping dramatically due to consistent 3-point minimum overlap)

Applying these projected metrics into the governing Uniformity Coefficient formula:

Christiansen's uniformity coefficient,

$$Cu = 100 \left(1 - \frac{\sum X}{mn} \right) = 100 \left(1 - \frac{66.12}{432.15} \right) = 84.7\%$$

This calculation demonstrates that by transitioning to the true hexagonal layout, the overall CU rises substantially to a projected 84.7%, successfully satisfying the industry-standard minimum uniformity threshold of 85%.

HYDRAULIC OPTIMIZATION AND NETWORK RESILIENCE

Transitioning a commercial farm from a square grid to a denser true hexagonal layout inherently requires more lateral pipelines to cover the identical acreage. For the 0.53-hectare Kodram field, the total number of laterals increases from six to approximately eight, and the sprinkler head count surges from 50 to approximately 73.

CAPACITY, FLOW DEMANDS, AND FRICTION HEAD LOSS

The required system flow rate (Q) is calculated using the established equation:

$$Q = \frac{2.778 \times i_g \times A}{N_{op} \times T_{op}}$$

Where i_g represents the gross irrigation requirement in millimeters, A represents the total irrigated area in hectares, N_{op} is the number of operation days per interval, and T_{op} represents the operating hours per day.

With the actual measured output at the nozzles being only 240 lph (0.067 lps), the total fluid demand of the 73-sprinkler honeycomb array is strictly $73 \times 0.067 = 4.89$ lps. This falls well within the available 8.2 lps capacity of the existing 25 HP turbine pump, indicating that the source water supply architecture does not need an expensive upgrade to support the denser grid.

Furthermore, the addition of two extra lateral rows redistributes the heads. This reduction significantly lowers the peak flow rate traversing any single 32 mm pipe, drastically reducing friction head losses. This critical hydraulic stabilization ensures that the distal nozzles in the hexagonal layout will operate at pressures much closer to the inlet nozzles, directly combating the progressive reduction in wetting circle radii observed during field tests.

Infrastructure Component	Conventional Grid Layout	Proposed True Hexagonal Layout
Main Conveyance Line Diameter	90 mm PVC	90 mm PVC (Unchanged)
Lateral Pipe Diameter	32 mm LLDPE Class II	32 mm LLDPE Class II (Unchanged)
Lateral Pipeline Spacing	9.0 meters	6.5 meters (Reduced by ~27.8%)
Total Number of Laterals	~6 lines	~8 lines (Increased)
Average Sprinklers per Lateral	~9 units	~9-10 units (Similar/Grouped)
Peak Volumetric Flow per Lateral	~1.125 liters/second	~0.90 liters/second (Reduced)

Table 4: Hydraulic Infrastructure Shifts — Grid Layout vs. True Hexagonal Layout

INFILTRATION RATES, TOPOGRAPHICAL RUNOFF AND WIND MITIGATION

In the Kodram region, the loamy to sandy loam soil features infiltration limits ranging safely from 13 to 50 mm/hr. While the average application rate of 2.41 mm/hr is well below this threshold, a flawed square layout with a CU of 66.30% masks destructive local variations.

Ambient wind speed critically exacerbates this issue; wind variability physically distorts the spray patterns, shifting and concentrating the precipitation volume into much smaller, localized areas. Because these wind-concentrated application rates frequently exceed the soil's infiltration limits in some localized overlapping zones, irrigated water on sloped ground tends to run off from higher elevations to lower elevations, causing waterlogging, fertilizer leaching, and dry high spots. By elevating the overall uniformity to 84.7% via a dense true hexagonal layout, the variance between the highest and lowest precipitation points is tightly minimized, and the layout inherently resists wind-driven pattern distortion, entirely preventing micro-runoff and localized erosion.

ECONOMIC FEASIBILITY AND POLICY INTEGRATION

Transitioning from a 9 m x 9 m square grid to a 7.5 m true hexagonal grid necessitates increasing the physical density of equipment per hectare, posing a capital expenditure challenge for smallholder farmers.

CAPITAL EXPENDITURE AND SUBSIDIES

Adopting the hexagonal arrangement requires twenty-three additional impact mini-sprinklers, additional 32 mm LLDPE lateral pipelines, and various fittings, totaling an estimated raw capital expenditure between INR 8,500 and INR 17,600.

However, under the national *Pradhan Mantri Krishi Sinchayee Yojana* (PMKSY) program, operationalized in Gujarat by the Gujarat Green Revolution Company (GGRC), aggressive subsidies cushion this impact. Under the progressive 2024-2025 subsidy norms, general category farmers receive up to 70% assistance, while small, marginal, SC, and ST farmers can receive between 80% and 90% assistance in designated "dark zones". When these subsidies are applied, the net out-of-pocket investment for the individual farmer drops drastically to between INR 850 and INR 7,000 for a 0.53-hectare field.

YIELD DYNAMICS AND RETURN ON INVESTMENT (ROI)

By successfully elevating the Uniformity Coefficient from 66.30% to 84.7% via the true hexagonal shift, the effective agronomic yield from the outer 10% to 15% of the field is fully restored to optimal baseline levels. This translates to an additional, highly profitable revenue generation of INR 3,000 to INR 6,000 per hectare, per cropping season. Combining the revenue gains from the increased boundary harvest and the operational savings from reduced pumping times, the minimal net out-of-pocket investment is mathematically projected to be fully recovered within two to four cropping seasons.

CONCLUSION

The traditional 9 m × 9 m square-grid sprinkler configuration contains a significant geometric limitation. Although it can achieve high uniformity within the central portion of the field, it consistently underperforms along the boundaries, resulting in a comparatively low overall Christiansen's Uniformity Coefficient (CU) of 66.30%. Replacing this arrangement with a true hexagonal (honeycomb) pattern presents a mathematically efficient alternative. In the hexagonal configuration, sprinklers are positioned at alternating distances of a and $2a$ along laterals spaced at $0.866a$, ensuring that each area receives overlapping coverage from a minimum of three sprinklers. This design is projected to improve the overall field uniformity to nearly 84.7%. In addition, the closer lateral spacing of approximately 6.5 m improves resistance to wind drift and promotes more balanced hydraulic pressure distribution at distant nozzles. With the support of government-backed micro-irrigation subsidy programs such as those implemented by the Gujarat Green Revolution Company, the additional installation cost associated with the extra sprinklers can be recovered through improved edge-field crop productivity and lower pumping energy demands. Consequently, the true hexagonal layout emerges as a practical, economically sustainable, and scientifically optimized solution for enhancing water-use efficiency in semi-arid agricultural regions.

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