Abstract - Measurements of the magnetic field of the Earth and low frequency magnetic field disturbance require a small size, highly sensitive, low noise, and stable magnetic sensor with directional capabilities. Fluxgate magnetometer design problems usually involve a large number of design variables with multiple objectives under complex nonlinear constraints. The methods for solving fluxgate multi-objective optimization problems can be significantly different from the methods for fluxgate single objective optimization. No matter how simple the problem may be, finding the optimal solution for a nonlinear multi-objective fluxgate optimization problem requires complex numerical effort. Meta-heuristic algorithms start to show their advantages in dealing with nonlinear multi-objective optimization problems. In this paper, the recently developed single-objective Firefly Optimization Algorithm (FOA) was modified to solve fluxgate multi-objective optimization problems. A complete magnetometer based on fluxgate principle for magnetic field measurement has been developed using a ferrite ring core with wire-wound excitation and pick-up coils. The fluxgate magnetometer consists of a fluxgate sensor with electronic circuitry based on second-harmonic detection. The sensing method is based on the conventional type of fluxgate magnetometer with detection of second harmonics by a phase sensitive detector. The sensor realizes a linear full scale in the range of ±49.44 μT with a sensitivity of 97.08 mV/μT. In addition, when compared to the existing sensors, the modified FOA sensor exhibited a reduction of the core dimension by 38.9%, the reduction in the pick-up coil winding turns by 66%, increased magnetic field range by 64.8%, and increased sensitivity by a factor of 8.5.

Key words: Fluxgate, ferrite magnetic material, phase sensitive detector.

Significance
This paper finds the optimum dimensions of the magnetic core, sensing coil, and detection circuit elements. The dimensions of the core and the number of excitations and sensing coils are significant for matching the excitation and detection circuits.

1. INTRODUCTION
Fluxgate magnetometers are preferred to other magnetic sensors because of their low cost, directionality, easy of construction, reliability, ruggedness and their capability to operate in harsh environment where endurance against magnetic, thermal and mechanical shocks is required (Kaluza et al., 2003; Ripka, 2003). The main drawback of fluxgate magnetometer is the complicated construction of the magnetic core, the excitation and pick-up coils.

Fluxgate magnetometer design problems usually involve a large number of design variables with multiple objectives under complex nonlinear constraints. The methods for solving fluxgate multi-objective optimization problems can be significantly different from the methods for fluxgate single objective optimization. To find the optimal solution for a nonlinear multi-objective fluxgate optimization problem may require complex numerical effort, no matter how simple the problem may be. Meta-heuristic algorithms start to show their advantages in dealing with nonlinear multi-objective optimization problems (Yang, 2013). In this paper, the recently developed single-objective firefly algorithm was modified to solve multi-objective fluxgate optimization problems.

No single optimal solution to multi-objective optimization problem exists, but instead a set of solutions defined as the Pareto optimal solutions (Talbi, 2009). One way to solve multi-objective optimization problem is to extend the FOA to produce Pareto optimal front directly (Yang, 2013). A solution is Pareto optimal if a given objective cannot be improved without degrading other objectives (Mladineo et al., 2015). However, this set of solutions represents a compromise between different conflicting objectives (Mladineo et al., 2015). Another way to solve multi-objective optimization problem is by modifying the single objective FOA. In this case, all other objectives are combined into a single objective so that algorithms for single objective optimization can be used without complex modifications (Yang, 2013).

FOA can be used directly to solve multi-objective optimization problems in this way (Apostolopoulos & Vlachos, 2011). In this study, modified FOA is combined with systematic optimization approach where the systematic optimization is used for simultaneously obtaining the matching between the sensor parameters and modified FOA is used for optimizing the sensor parameters along with its performances.

2. THEORY OF OPERATION
Fluxgate magnetometers are commonly used magnetic field sensors for measuring DC or low frequency magnetic field vectors (Lu and Huang, 2015). Fluxgate magnetometer has very high sensitivity and spans a wide range, from 100 pT to 100 μT (Lv and Liu, 2013). It also has low noise, small size, small power requirement, and high temperature stability (Frydrych et al., 2014).
Fluxgates consist of a ferromagnetic core material and two coils wound around the core (Ripka, 2010). One of the coils acts as the excitation coil which produces the excitation magnetic field to periodically saturate the core when certain magnitude and frequency of excitation current is applied through it, while the other coil acts as the pick-up coil to detect changes in the flux through the core.

Typically, fluxgate sensors work on the second harmonic principle (Lv and Liu, 2013) and close-loop configuration (Matsuoka et al., 2013). When the excitation current is applied to the excitation coil, a lock-in amplifier (phase sensitive detector and amplifier) is used to obtain the second harmonic of the induced voltage the output of the pick-up measuring coil of the fluxgate (Miles et al., 2013; Lv and Liu, 2014). The amplitude of the induced second harmonic signal in the pick-up coil is proportional to the magnetic field to be measured (Lv and Liu, 2013; Lu and Huang, 2015).

The design of fluxgate magnetometers is typically a nonlinear multi-objective optimization problem. Different objectives often conflict with each other (Can and Topal, 2015) and sometimes an optimal magnetometer performance is difficult to achieve (Grosz and Paperno, 2012). The sensitivity of the sensor decreases with an increase of noise level while trying to reduce the sensor dimension (Can and Topal, 2015).

Different optimization techniques had been developed for the structures and core materials of fluxgate magnetometer. For instance, the conventional approach was based on Part-by-Part Optimization (PPO) technique. However, PPO technique is difficult, slow, time consuming, expensive, and does not produce optimal magnetometer performance (Grosz and Paperno, 2012). Another, technique used for the optimization of the magnetometer parameters was based on an analytical model, which was numerically solved to obtain an improved large set of parameters such as volume and weight of pick-up coil at reduced power consumption and noise of the detection circuit. However, the analytical optimization technique becomes unnecessarily complex when performing a large number of numerical calculations to optimize the magnetometer, thus introducing difficulty in interpreting the results obtained (Grosz and Paperno, 2012).

Hence, there is need for a systematic optimization approach for fluxgate magnetometer design to find its optimum performance. The combined modified FOA and systematic optimization technique are used to improve ring core parallel-type fluxgate magnetometer design in this research. These techniques optimize the sensitivity and reduce noise of a fluxgate while the sensor core, pick-up coil, and detection circuit are minimized. Such a modified FOA is powerful in dealing with fluxgate magnetometer design problems with a large number of design variables and multiple objectives under complex nonlinear constraints (Yang, 2013). Therefore, this research proposes such an algorithm as a tool for optimizing the design of a ring core parallel-type fluxgate magnetometer.

The algorithm starts by placing the fireflies in random locations. The location of a firefly corresponds to the values of the parameters (dimensions of the core, pick-up coil, and detection circuit elements) for the objective function (sensor sensitivity) to be solved. The multiple objectives in this research using modified FOA are implemented using the following steps:

1. Initializing number of fireflies, \( n \), biggest attraction \( \beta_0 \), absorption coefficient of light intensity \( \gamma \), step size factor \( a \), and maximum number of iterations or generations \( l_{\text{max}} \).
2. Initializing the positions of fireflies (namely design variables of the fluxgate parameters) randomly, the values of objective functions of fireflies are set as their maximum brightness of fluorescence \( I_0 \).
3. Calculating relative brightness and attractiveness of fireflies belonging to the population. The direction of movement depends on the relative brightness of fireflies. An expression for this maximum brightness of fluorescence is (Yang, 2013):

\[
I = I_0 \times e^{-\gamma r_{ij}} \quad (1)
\]

\[
\beta = \beta_0 \times e^{-\gamma r_{ij}} \quad (2)
\]

where \( \beta_0 \) is the maximum attractiveness at \( r = 0 \), \( \gamma \) is the absorption coefficient of the light intensity, and \( r_0 \) is the spatial distance between fireflies \( i \) and \( j \). The attractiveness of a firefly is proportional to its brightness and they both decrease with distance.
4. Updating the spatial positions of fireflies. Random perturbations are injected to the firefly with the best position. The updated equation is:

\[
x_i = x_i + \beta \times (x_j - x_i) + a \times (\text{rand} - 0.5) \quad (3)
\]

where \( x_i, x_j \) represent the spatial positions of firefly \( i \) and \( j \), respectively. \( a \) is the step size factor. \( \text{rand} \) is random factor distributed uniformly in [0,1].
5. Recalculating the brightness of fireflies according to the updated positions.
6. Returning to Step 3 until the search precision is met or the maximum number of generations is achieved.

3. METHODOLOGY

In order to develop fluxgate magnetometer using modified FOA technique, the dimensions of the sensor core and number of excitation and pick-up coil turns play an important role in matching the excitation and detection circuits (Zorlu et al., 2010; Lei et al., 2011). The magnetic core material is deeply saturated to avoid permeing effect (Ripka, 2010). The commercially available manganese zinc ferrite ring core material is chosen because of its high resistivity, low saturation flux density, and high relative magnetic permeability.

Optimization of the entire fluxgate magnetometer is carried out using the analytical model that includes the sensor core, pick-up coil, and detection circuit. The analytical model is numerically solved using modified FOA to find the optimum fluxgate magnetometer configuration subject to a large set of parameters such as, sensor core, pick-up coil, and detection circuit (Yang, 2013).
In order to demonstrate how the modified FOA works, it was implemented in MATLAB with ten design variables presented in Table 1. The geometric constraints and operational limits are shown in Table 2.

### Table 1: Fluxgate Sensor Design Variables and Ranges

<table>
<thead>
<tr>
<th>Variables</th>
<th>Range</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core outside diameter</td>
<td>10 – 20</td>
<td>mm</td>
</tr>
<tr>
<td>Core inside diameter</td>
<td>8 – 18</td>
<td>mm</td>
</tr>
<tr>
<td>Core height</td>
<td>1 – 4</td>
<td>mm</td>
</tr>
<tr>
<td>Number of Layers of Pick-up coil</td>
<td>5 – 150</td>
<td>-</td>
</tr>
<tr>
<td>Pick-up Coil bobbin thickness</td>
<td>1 – 10</td>
<td>mm</td>
</tr>
<tr>
<td>Pick-up coil axial length turns</td>
<td>5 – 150</td>
<td>-</td>
</tr>
<tr>
<td>Amplifier Feedback resistor</td>
<td>1 – 270</td>
<td>kΩ</td>
</tr>
<tr>
<td>Pick-up coil inductance</td>
<td>4 – 10</td>
<td>mH</td>
</tr>
<tr>
<td>Amplifier feedback capacitor</td>
<td>100-180</td>
<td>nF</td>
</tr>
<tr>
<td>Amplifier input resistor</td>
<td>1 – 5</td>
<td>kΩ</td>
</tr>
</tbody>
</table>

### Table 2: Fluxgate Sensor Design Constraints

<table>
<thead>
<tr>
<th>Variables</th>
<th>Range</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core thickness</td>
<td>≤ 3.0</td>
<td>mm</td>
</tr>
<tr>
<td>Pick-up wire diameter</td>
<td>≤ 0.361</td>
<td>mm</td>
</tr>
<tr>
<td>Sensor winding turns</td>
<td>≤ 2000</td>
<td>-</td>
</tr>
<tr>
<td>Core aspect (diameter to height) ratio</td>
<td>≤ 10</td>
<td>-</td>
</tr>
<tr>
<td>Coil aspect (length to height) ratio</td>
<td>≤ 20</td>
<td>-</td>
</tr>
<tr>
<td>Amplifier output voltage</td>
<td>≤ 5.0</td>
<td>-</td>
</tr>
</tbody>
</table>

In this study, fluxgate multi-objective optimization problem was solved by combining all other objectives into a single objective so that algorithms for single objective optimization can be used without complex modifications (Yang, 2013). FOA can be used directly to solve fluxgate multi-objective problems in this way (Apostolopoulos & Vlachos, 2011). By extending the basic ideas of FOA, multi-objective FOA can be developed, which can be summarized as the pseudo code listed below (Yang, 2013).

Define objective functions \( f_i(x), \ldots, f_k(x) \) where \( x = (x_1, \ldots, x_d)^T \)

Initialize a population of \( n \) fireflies \( x_i (i = 1, 2, \ldots, n) \)
while \( t < \text{MaxGeneration} \)
For \( i, j = 1 \) to \( n \) (all \( n \) fireflies)
For \( x_i = 1 \) to \( r_x \)
For \( x_j = 1 \) to \( r_y \)
For \( x_j = 1 \) to \( h_z \)
Calculate the effective cross section area of ring core material
Calculate the effective magnetic path length of ring core material
If core thickness violates the geometric constraint, then the solution is rejected and the next loop is executed
For \( x_k = 1 \) to \( nT_w \)
For \( x_5 = 1 \) to \( T_b \)
For \( x_6 = 1 \) to \( nT_w \)
If core aspect ratio violates the core geometric constraint, then the candidate solution is rejected and the next loop is executed
Calculate pick-up coil dimensions
Calculate the total length of pick-up coil winding
Calculate the resistance of the pick-up coil
Calculate the height of pick-up coil winding
Calculate the length of pick-up coil winding
Calculate the apparent permeability of the magnetic core
If coil aspect ratio violates the coil geometric constraint, then the candidate solution is rejected and the next loop is executed
Calculate the mutual inductance of sensor coils
If the number of turns of pick-up coil winding is violated, then the solution is rejected and the next loop is executed
Calculate the induced output voltage of pick-up coil and sensitivity
For \( x_7 = 1 \) to \( R_I \)
For \( x_8 = 1 \) to \( L_w \)
For \( x_9 = 1 \) to \( C_f \)
For \( x_{10} = 1 \) to \( R_0 \)
Calculate the overall output voltage of sensor
If the overall output voltage of sensor specified is violated, then the candidate solution is rejected and the next loop is executed
Calculate the total sensitivity of sensor
Calculate overall sensor voltage noise
Calculate the total magnetic field noise of sensor
Optimum fluxgate sensor is the one with the minimum magnetic field noise.

The procedure starts with an appropriate definition of objective functions with associated non-linear constraints. A population of \( n \) fireflies was first initialized so that they could be distributed among the search space as uniformly as possible. The amplitude of the equivalent magnetic field noise, \( H_b \), is found as (Richard and Kenneth, 1989):

\[
\text{Minimize } H_n(x) = \frac{V_{tot}}{S_{tot}} \tag{4}
\]

Where \( H_n \) is the equivalent magnetic field noise in \( \text{T} / \sqrt{\text{Hz}} \), \( V_{tot} \) is the total voltage noise of the sensor in \( \text{V} / \sqrt{\text{Hz}} \), and \( S_{tot} \) is the total sensor sensitivity in \( \text{V} / \text{T} \).
4. RESULTS AND DISCUSSIONS

The optimum fluxgate sensor has the following technical characteristics presented in Table 3, obtained from the sensitivity analysis results.

Table 3: Optimization Sensitivity Analysis Results

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Modified FOA Model</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core outside diameter</td>
<td>12.22</td>
<td>mm</td>
</tr>
<tr>
<td>Core height</td>
<td>1.95</td>
<td>mm</td>
</tr>
<tr>
<td>Pick-up Coil bobbin thickness</td>
<td>5.18</td>
<td>mm</td>
</tr>
<tr>
<td>Amplifier Feedback resistor</td>
<td>105.35</td>
<td>kΩ</td>
</tr>
<tr>
<td>Amplifier input resistor</td>
<td>1.75</td>
<td>kΩ</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>76.95</td>
<td>mV/µT</td>
</tr>
<tr>
<td>Noise level at 1 Hz</td>
<td>3.465</td>
<td>pT/√Hz</td>
</tr>
</tbody>
</table>

As shown in Tables 3, the optimum solution has a sensitivity of 76.95 mV/µT and a magnetic field noise of 3.465 pT/√Hz. This means that with a specific variation of the geometric dimensions of core, pick-up coil, and detection circuit elements, a higher sensitivity and lower field noise solution are found.

The electronic circuit is mounted on a PCB, and the performance of the sensor is tested. The complete construction of the ring cores with excitation coils wound circumferentially are placed inside a pick-up coil bobbin to hold it firm. Finally, the pick-up coil with 646 turns is wound diametrically on the core with copper wire having 0.2 mm diameter. The complete fluxgate sensor prototype is shown in Figure 1, while the legend showing the sensor stages is shown in Table 4.

Figure 1: Complete Fluxgate Sensor System

Table 4: Description of Fluxgate Sensor System

<table>
<thead>
<tr>
<th>S/N</th>
<th>Legend</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fluxgate sensor</td>
</tr>
<tr>
<td>2</td>
<td>Detection circuit</td>
</tr>
<tr>
<td>3</td>
<td>Synchronous detector</td>
</tr>
<tr>
<td>4</td>
<td>Frequency generator</td>
</tr>
<tr>
<td>5</td>
<td>Frequency divider</td>
</tr>
<tr>
<td>6</td>
<td>Voltage-to-current converter</td>
</tr>
</tbody>
</table>

The result of the calibration of the fabricated modified FOA fluxgate magnetometer is shown in Figure 2, which shows that the output of the fluxgate magnetometer is approximately linear with the increase in external magnetic field effect.
In order to determine the sensitivity of the developed sensors, the measured maximum output voltage (4.8 V) of the sensor is divided by the maximum magnetic field strength of 49.44 μT. The sensitivity $S_{sen}$ of the sensor, which is determined from the shape of the line, is calculated as 97.08 mV/μT by using the relation (Tumanski, 2013):

$$S_{sen} = \frac{dV_{out}}{dH_{ext}}$$  \hspace{1cm} (5)

where $V_{out}$ is the output voltage of the sensor, and $H_{ext}$ is the external magnetic field.

The results of the developed, modified, and fabricated optimal FOA fluxgate sensor design is compared with the reference fluxgate sensor (Can & Topal, 2015) as presented in Table 5.

Table 5: Comparison of Reference Fluxgate Sensor and Modified FOA Fluxgate Sensor

<table>
<thead>
<tr>
<th>Variable Comparison</th>
<th>Reference Model</th>
<th>Modified FOA Model</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core outside diameter</td>
<td>20.0 mm</td>
<td>12.22 mm</td>
<td>mm</td>
</tr>
<tr>
<td>Core height</td>
<td>3.2 mm</td>
<td>2.0 mm</td>
<td>mm</td>
</tr>
<tr>
<td>Number of Pick-up coil turns</td>
<td>1900</td>
<td>646</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Performance Comparison</th>
<th>Reference Model</th>
<th>Modified FOA Model</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensitivity</td>
<td>11.40</td>
<td>97.08</td>
<td>mV/μT</td>
</tr>
<tr>
<td>Noise level at 1 Hz</td>
<td>2720</td>
<td>4.94</td>
<td>pT/√Hz</td>
</tr>
<tr>
<td>Magnetic field range</td>
<td>30</td>
<td>49.44</td>
<td>μT</td>
</tr>
</tbody>
</table>

From Table 5, significant reduction in core dimensions (outside diameter and height) and number of pick-up coil turns are achieved with the modified FOA fluxgate sensor. Most importantly, there is significant increase in sensitivity and magnetic field range, as well as reduction in noise level. The field sensitivity of sensor with 20 mm ring core outside diameter and 3.2 mm core height (Can & Topal, 2015) was 11.40mV/μT, while the field sensitivity of modified FOA sensor with 12.22 mm ring core outside diameter and 2.0 mm core height increased to a maximum value of 97.08mV/μT.

The percentage (%) decrease in core dimension, $C_{dec}$ from 20 mm to 12.22 mm with respect to the reference fluxgate magnetometer (Can & Topal, 2015) is obtained using:

$$\% C_{dec} = \frac{C_{ref} - C_{FOA}}{C_{ref}} \times 100\%$$  \hspace{1cm} (6)

where $C_{FOA}$ is the modified FOA sensor core ring core diameter and $C_{ref}$ is the reference sensor ring core diameter.

Hence, the modified FOA sensor core dimension reduced by 38.9%.

Despite the decrease in modified FOA sensor dimension to 12.22 mm from 20 mm (Can & Topal, 2015) with respect to the reference fluxgate magnetometer (Can & Topal, 2015), the sensitivity of modified FOA sensor increased by a factor of 8.5 with respect to the reference fluxgate magnetometer (Can & Topal, 2015).

Also, the percentage (%) decrease in pick-up winding turns, $w_{inc}$ with respect to Can & Topal, (2015) is obtained using:

$$\% w_{inc} = \frac{w_{ref} - w_{FOA}}{w_{ref}} \times 100\%$$  \hspace{1cm} (7)

where $w_{FOA}$ is the pick-up winding turn required by the modified FOA sensor, and $w_{ref}$ is the pick-up winding turn required by the reference sensor.
Hence, the magnetic fields range of modified FOA sensor increased by 66%.

Finally, the percentage (%) increase in magnetic field, $B_{inc}$ range with respect to Can & Topal, (2015) is obtained using:

$$\% B_{inc} = \frac{B_{ref} - B_{FOA}}{B_{ref}} \times 100\% \quad (8)$$

where $B_{FOA}$ is the maximum magnetic field range obtained from modified FOA sensor, and $B_{ref}$ is the maximum magnetic field range of the reference sensor.

Hence, the magnetic fields range of modified FOA sensor increased by 64.80%.

5. CONCLUSION

This paper demonstrates the feasibility of finding the correct values of sensor core, pick-up coil, and detection circuit elements required to match the excitation and detection circuits for better results. Fluxgate sensor design is a complex task that includes many variations in design variables so as to maximize the sensitivity and satisfying fluxgate sensor constraints (specifications) with respect to power consumption, excitation current, and temperature stability due to thermal resistances of windings in the event of long-term application. The efficiency of the proposed fluxgate sensor design optimization algorithm is presented through a design example. The simulation results indicated that the sensitivity and noise of the modified sensor are 76.95 mV/μT and 3.465 pT/√Hz at 1 Hz, respectively in the range from ±49.44 μT. To verify the validity of the design, a prototype sensor has been fabricated, and the experimental sensitivity and noise are 97.08 mV/μT and 4.94 pT/√Hz at 1 Hz, respectively in the range from ±49.44 μT. When compared with the developed modified FOA sensor to the existing sensors, the modified FOA sensor shows a reduction of the core dimension by 38.9%, the reduction in the pick-up coil winding turns by 66%, increased magnetic field range by 64.8%, and increased sensitivity by a factor of 8.5.

REFERENCES


