Improvement of CIC Filter Characteristics

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***Abstract*— This paper presents comparison between CIC filter, narrowband CIC filter and wideband CIC filter. It also gives comparison between RS filter, modified RS filter and compensated modified RS filter for a decimation factor of 64. Techniques like narrowband compensation, wideband compensation and RS filters improve the performance characteristics of CIC filters. Narrowband and wideband compensation techniques improve the passband characteristics, and RS filters improve the stopband characteristics. Multistage realization furthur improves the performance characteristics of the enhancement techniques.**

Keywords—CIC filters, Narrowband compensation, Wideband compensation, RS filter

I. INTRODUCTION

Cascaded integrator-comb (CIC) filters are multirate filters used for realizing large sample rate changes in digital systems. They are also known as Hogenauer filters [1]. CIC filters are typically employed in applications that have a large excess sample rate. That is, the system sample rate is much larger than the bandwidth occupied by the signal. They are frequently used in digital down-converters and digital up-converters. The transfer function of the CIC filter is given by

$$H(z) = \left(\frac{1}{D}\frac{1-z^{-D}}{1-z^{-1}}\right)^{K} \qquad (1)$$

where D is the decimation ratio, and K is the number of the stages. CIC filters are multiplierless structures, consisting of only adders and delay elements which is a great advantage when aiming at low power consumption. They first perform the averaging operation then follow it with the decimation.

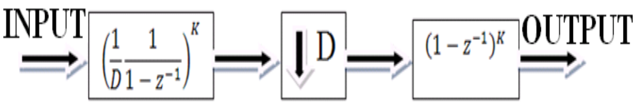

Fig. 1 CIC decimation filter

The magnitude characteristic of CIC filter has a passband droop in the desired passband that is dependent upon the decimation factor D and the cascade size K. The authors, G. J. Dolecek and S. K. Mitra, proposed an idea for improving the passband using compensating filters. Narrowband and wideband compensation techniques improve the passband characteristics [2], [3]. The Rotated Sinc (RS) filter was proposed by G. J. Dolecek and S. K. Mitra to increase the attenuations and widths in the folding bands i.e stopband characteristics [5]-[6].

In this paper, we give a comparison between CIC filter, narrowband CIC filter and wideband CIC filter. Also a comparison between RS filter, Modified RS filter and Compensated modified RS filter is performed . The paper is organized as follows. Section 2 gives the introduction of CIC filters along with its gain responses. Section 3 gives the method to improve passband characteristics. Section 4 gives the method to improve stopband characteristics. Section 5 gives the analysis of the comparison done for the decimation factor of 64.

II. CIC FILTERS

The term CIC filter we use only for the cascade-integrator-comb implementation scheme. The frequency response $H(e^{jw})$following from (1) is given by,

$$H(e^{jw}) = \left\{\frac{1}{D}\left(\frac{sin(wD/2)}{sin(w/2)}\right)\right\}^{K} e^{-jwK\,[(D-1)/2]} \quad (2)$$

Example 1: Consider D = 16 and K = 1, 2, 3, 4. Figure 2(a) shows the magnitude responses and Figure 2(b) shows the passband zoom. Figure 2(a) illustrates, the $H(e^{jw})$ exhibits the comb-like magnitude response. The natural nulls of the comb filter occur exactly at the integer multiples of F_x/D thus providing the maximum alias suppression at those frequencies. The aliasing bandwidths around the nulls are narrow, and usually too small to provide sufficient suppression of aliasing in the entire baseband of the signal. A very poor magnitude characteristic of the comb filter is improved by cascading several identical comb filters.

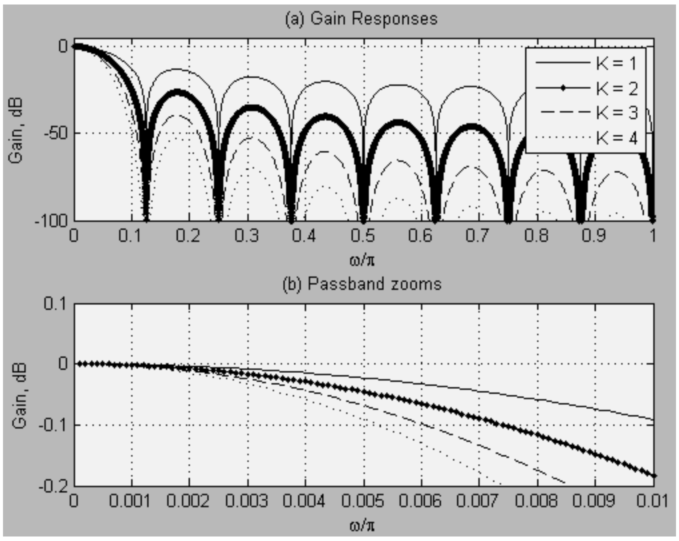

Fig. 2 Illustratiion of magnitude response in Example 1

The multistage realization improves the selectivity and the stop-band attenuation of the overall filter: the selectivity and the stopband attenuation are augmented with the increase of the number of comb filter sections. The filter has multiple nulls with multiplicity equal to the number of the sections. Consequently, the stopband attenuation in the null intervals is very high. For modifying the comb filter magnitude response one can use only two parameters: the comb filter order (D), and the number of the comb filter sections (K) [2]. Figure 2(b) illustrates a monotonic passband characteristic produces an inevitable passband droop, which for many applications should be compensated.

III. METHOD TO IMPROVE PASSBAND

A. Narrowband Compensation Technique

Consider a second order compensation filter [3]

$$|G(e^{jw})| = |1 + 2^{-b}\sin^2(wD/2)| \qquad (3)$$

where *b* is a integer parameter. Using the well known relation

$$\sin^2\alpha = (1-\cos(2\alpha))/2 \qquad (4)$$

The corresponding transfer function can be expressed as

$$G(z^D) = -2^{-(b+2)}[1-(2^{b+2}+2)z^{-D}+z^{-2D}] \qquad (5)$$

Denoting

$$A = -2^{-(b+2)}; \quad B = -(2^{b+2}+2); \qquad (6)$$

Substituting in the above equation we get,

$$G(z^D) = A[1 - Bz^{-D} + z^{-2D}] \qquad (7)$$

We use a simple MATLAB program based on (2) to find the corresponding value b for a given K.
Example 2: Consider D = 16, K = 2 and b = 2. Using (1) and (5), we get,

$$H_{comp}(z) = H(z)G(z^{16}) = \left(\frac{1}{16}\frac{1-z^{-16}}{1-z^{-1}}\right)^2 \times\{-2^{-4}[1-(2^4+2)z^{-16}+z^{-32}]\} \quad (8)$$

TABLE I

TYPICAL VALUES OF COMPENSATION FILTER PARAMETER

Number of stages (K)	Parameter (b)
2	2
3,4	1
5,6,7,8	0
9,10,11	-1

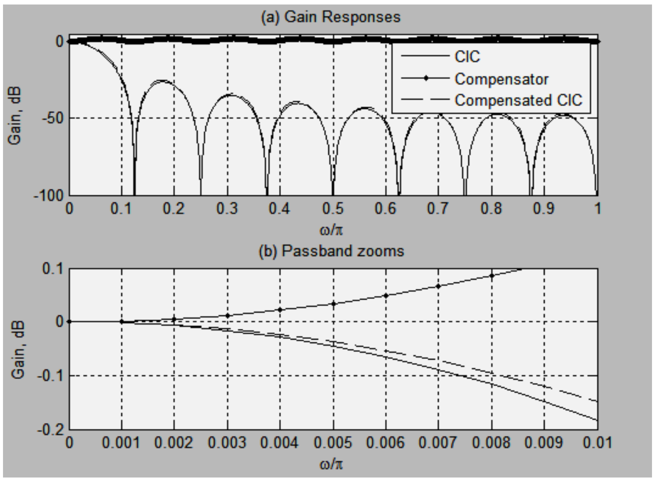

Fig. 3 Illustratiion of magnitude response in Example 2

B. Wideband Compensation Technique

Consider the transfer function of the three coefficient FIR filter. [4]

$$G(z^D) = bz^{-D} + az^{-2D} + bz^{-3D} \qquad (9)$$

with the corresponding magnitude characteristic

$$|G(e^{jDw})| = |2b\cos(Dw) + a| \qquad (10)$$

The condition that the magnitude characteristic has the value 1 for w= 0, gives

$$a = 1 - 2b \qquad (11)$$

Replacing Eq (3.2.2.3) into Eq (3.2.2.2) we arrive at

$$|G(e^{jDw})| = |2b[\cos(Dw) - 1] + 1| \quad (12)$$

The value of b is estimated minimizing the squared error in the passband

$$\min_b \int_0^{w_p} E^2(w)dw \qquad (13)$$

where

$$E^2 = \left\langle \left| \frac{\sin\left(\frac{Dw}{2}\right)}{D\sin\left(\frac{w}{2}\right)} \{2b[\cos(Dw)-1]+1\} \right| - 1 \right\rangle^2 \quad (14)$$

The estimated value of b is rounded using the rounding constant r = 2^{-6} resulting in

$$b = -4 \times 2^{-6} \qquad (15)$$

From (11) and (15), we have

$$a = 72 \times 2^{-6} \qquad (16)$$

Using (9), (15) and (16), we arrive at

$$G(z^D) = 2^{-6}[-4z^{-D} + 72z^{-2D} - 4z^{-3D}\} \qquad (17)$$

Finally from (17) the proposed filter is given as

$$G(z^D) = -2^{-4}[z^{-D} - (2^4+2)z^{-2D} + z^{-3D}\} \quad (18)$$

The proposed filter is multiplier-free with only three adders and can be implemented at a lower rate after down sampling by D by making use of the multirate identity. The corresponding magnitude characteristic approximates inverse magnitude characteristic of (1), for K = 1, in the passband. The same is confirmed for K > 1 [4].

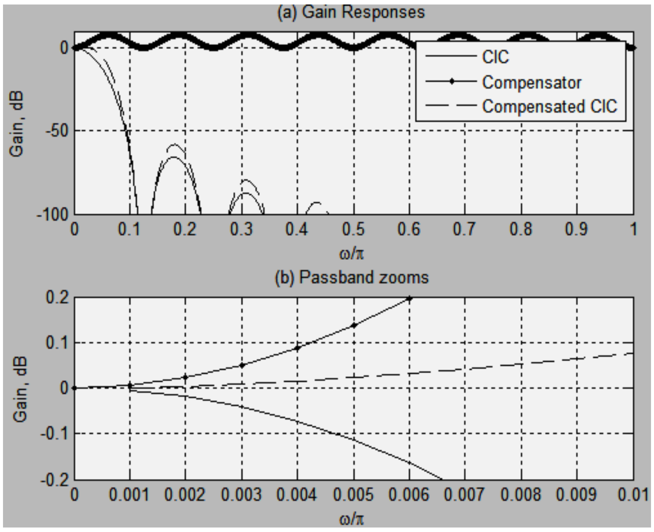

Fig. 4 Illustratiion of magnitude response in Example 3

$$H_{comp}\,(z^D) = \begin{cases} G^K(z^D) & for\ 1 < K \le 3 \\ G^{K-1}(z^D) & for\ K > 3 \end{cases} \qquad (19)$$

where K is the CIC parameter. The total number of additions depends on K, as given by

$$N_{add} = \begin{cases} 3K & for\ K \le 3 \\ 3K-3 & for\ K > 3 \end{cases} \qquad (20)$$

Example 3: Consider D = 16 and K = 5. Using (1), (18) and (19), we get,

$$H_{comp}\,(z) = H(z)G(z^{16}) = \left(\frac{1}{16}\frac{1-z^{-16}}{1-z^{-1}}\right)^5 \times\{-2^{-4}[z^{-16}-(2^4+2)z^{-32} + z^{-48}]\}(21)$$

According to (19) and (20), the compensator needs four stages requiring a total of 12 adders [4]. Magnitude characteristic zoom in the passband is shown in Figure 4(b), thus confirming the good passband compensation.

IV. Method To Improve Stopband

A. RS Filter

The Rotated Sinc (RS) filter was proposed to increase the attenuations and widths in the folding bands [5]. By applying a clockwise rotation of β radians to any zero of CIC filter, we obtain the following transfer function

$$H_u(z) = \frac{1}{D}\frac{1-z^{-D}e^{j\beta D}}{1-z^{-1}e^{j\beta}} \qquad (21)$$

An expression equivalent to (21) is obtained by applying the opposite rotation

$$H_d(z) = \frac{1}{D}\frac{1-z^{-D}e^{-j\beta D}}{1-z^{-1}e\ -^{j\beta}} \qquad (22)$$

These two filters have complex coefficients, but they can be cascaded, thus obtaining a filter $H_r(z)$ with real coefficients.

$$H_r(z) = H_u(z)H_d(z) = \frac{1}{D^2}\frac{1-2\cos(\beta D)\,z^{-D}+z^{-2D}}{1-2\cos(\beta)\,z^{-1}+z^{-2}} \quad (23)$$

The cascade of CIC filter and the filter $H_R(z)$is given as follows,

$$H_R(z) = H_{comb}\,(z)\,H_r(z) \qquad (24)$$

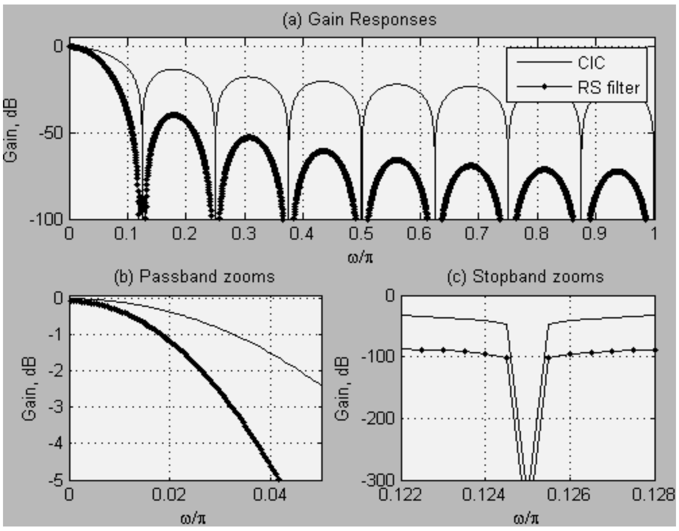

Fig. 5 Illustratiion of magnitude response in Example 4

The magnitude response of this filter is given as $|H_R(e^{jw})|$

$$=\left\{\left|\frac{1}{D^3}\frac{\sin\left(\frac{wD}{2}\right)}{\sin\left(\frac{w}{2}\right)}\right|\left|\frac{\sin\left(\frac{(w+\beta)D}{2}\right)}{\sin\left(\frac{w+\beta}{2}\right)}\right|\left|\frac{\sin\left(\frac{(w-\beta)D}{2}\right)}{\sin\left(\frac{w-\beta}{2}\right)}\right|\right\}^K \qquad (25)$$

Example 4: Consider D=16, K=1 and β=0.0184. The magnitude response and passband zooms for the RS filter is shown in Figure 5(a) and (b) respectively.
It is observed that the folding band widths are wider and the attenuations are increased in comparison with the CIC filter. However, the passband droop is increased and additionally RS filter needs two multipliers, one working at high input rate [5].

B. Modified RS filter

Considering that the downsampling factor D can be expressed as a product of integers,

$$D = D_1 D_2 \ldots . D_n \qquad (26)$$

We can rewrite as

$$H(z) = [H_1(z)H_2(z^{D_1})]^K \qquad (27)$$

where

$$H_1[z] = \frac{1}{D_1}\frac{1-z^{-D_1}}{1-z^{-1}};\, H_2[z] = \frac{1}{D_2}\frac{1-z^{-D_1D_2}}{1-z^{-1}} \qquad (28)$$

The corresponding frequency responses are, respectively,

$$\left|H_1(e^{jw\,D_1})\right| = \left|\left(\frac{1}{D_2}\frac{sin(wD/2)}{sin(wD_1/2)}\right)\right|;\ \left|H_2(e^{jw})\right| = \left|\left(\frac{1}{D_1}\frac{sin(wD_1/2)}{sin(w/2)}\right)\right| \quad (29)$$

Therefore the decimation filter can be constructed using different number of stages for the two sections resulting in a modified transfer function $H_2(z)$ given by:

$$H_m(z) = [H_1(z)]^{K_1}[H_2(z^{D_1})]^{K_2} \qquad (30)$$

The filter $H_2(z)$ can be moved to a low rate which is D_2 times lesser than the high input rate. Additionally, the polyphase decomposition of the filter H_1 (z) moves all filtering to a lower rate.

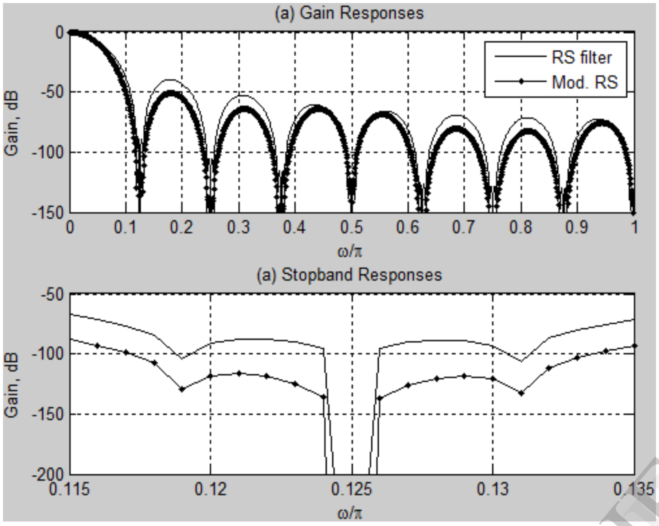

Fig. 6 Illustratiion of magnitude response in Example 5

The corresponding RS filter is modified in such way that it can also be moved to a lower rate [5]. Using (23) and (30), we get,

$$H_{rm}(z) = H_{um}(z)\,H_{dm}(z)$$
$$= \frac{1}{(D/D_1)^2}\frac{1-2\,cos(\beta D)\,z^{-D}+z^{-2D}}{1-2\,cos(\beta D_1)\,z^{-D_1}+z^{-2D_1}} \quad (31)$$

The corresponding modified RS filter is obtained as the cascade of the modified comb filter (30) and the modified rotated filter (31).

$$H_{RM}(z) = H_{comb}(z)\,H_{rm}(z) \qquad (32)$$

The corresponding magnitude response is

$$\left|H_{RM}(e^{jw})\right| = \left|H_m(e^{jw})\right|\left|\frac{1}{D_2}\frac{\sin\left(\frac{(w+\beta)D}{2}\right)}{\sin\left(\frac{(w+\beta)D_1}{2}\right)}\right|\left|\frac{1}{D_2}\frac{\sin\left(\frac{(w-\beta)D}{2}\right)}{\sin\left(\frac{(w-\beta)D_1}{2}\right)}\right| \quad (33)$$

Example 5: Consider D=16, K=1, K_1=3, K_2=2, D_1=D_2=4 and β=0.0184. The magnitude response and passband zooms for the RS filter for is shown in Figure 6(a) and (b) respectively. The magnitude responses along with the zoom in the first folding band are shown in Figure 6(a) and (b). It is observed that the attenuation in the all folding bands except the last one, are improved. Additionally, the filter H_r (z) works at a lower rate [5].

C. Compensated Modified RS filter

The generalized form of modified comb is given as follows

$$H_m(z) = [H_1(z)]^{K_1}[H_2(z^{D_1})]^{K_2}\ \ldots .\,[H_2(z^{D_1\ldots D_{N-1}})]^{K_N} \quad (34)$$

where

$$H_i^{K_i}(z^{L_2}) = \left[\frac{1}{D_i}\frac{1-z^{L_1}}{1-z^{L_2}}\right]^{K_i} \qquad (35)$$

$$L_1 = \prod_{j=1}^{i} D_j \quad ; L_2 = \prod_{j=1}^{i-1} D_j;\ D_0 = 1 \qquad (36)$$

and the corresponding magnitude response is

$$\left|H_i^{K_i}(e^{jw})\right| = \left|\left[\frac{1}{D_i}\left(\frac{sin(wL_1/2)}{sin(wL_2/2)}\right)\right]^{K_i}\right| \qquad (37)$$

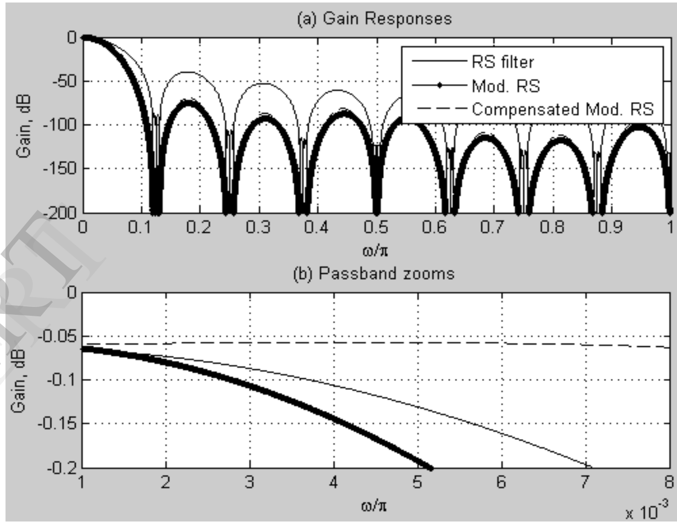

Fig. 7 Illustratiion of magnitude response in Example 6

It should be noted that by using different values for the number Ki of factors in each stage, the magnitude characteristic of the modified comb can be improved over that of the original comb decimator. Using the results from above sections, later a decimator filter was proposed in the following form [6].

$$H_{COMP,RS}(z) = G(z^D)H_m(z)H_{rm}(z) \qquad (38)$$

The polyphase decomposition of the comb filters in the first section allows the sub filters to operate at a lower rate, which is D_1 time lower than the input rate. In that way there is no filtering at high input rate.

Example 6: Consider K = 3; D=16, D_1 =D_2 = 2, D_3 = 4, K_1 = K_2 = K_3 =4, β=0.0184, b=1. The magnitude response and passband zooms for the RS filter, Modified RS filter and Compensated Modified RS filter is shown in Figure 7(a) and (b) respectively [6]. It is observed that not only stopband but also passband is improved.

V. ANALYSIS

A. Comparison between CIC filter, Narrowband CIC filter and Wideband CIC filter

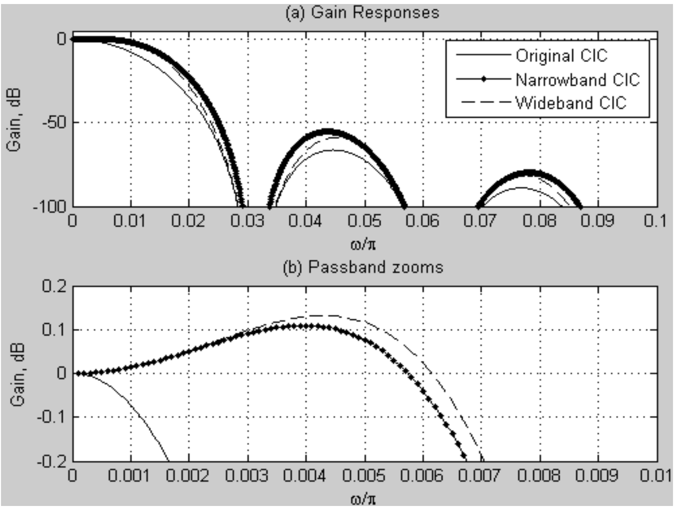

Fig. 8 Comparison between CIC filter, Narrowband CIC filter and Wideband CIC filter

TABLE II

Comparison between CIC filter, Narrowband CIC filter and Wideband CIC filter

Decimation technique	Specification	Passband Droop (dB)
CIC filter	D = 64, K = 5	-0.2929
Narrowband CIC filter	D = 64, b = -2, K = 5	0.05178
Wideband CIC filter	D = 64, b = -2, K = 5	0.05015

It is observed that wideband CIC filter has better passband characteristics but narrowband CIC filter requires only one compensation filter section to be cascaded at the end whereas wideband CIC filter requires more compensation filters to be cascaded. Table II gives the values of passband and stopband attenuation.

B. Comparison between RS filter, Modified RS filter and Compensated modified RS filter

It is observed that the RS filter has better passband characteristic while the modified RS filter has higher attenuations in the folding bands. Compensation techniques for the same improve their respective characteristics. Table III gives the values of passband and stopband attenuation.

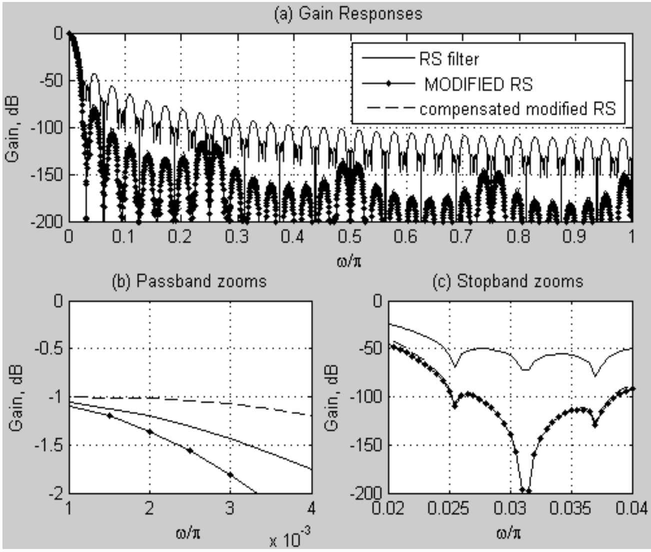

Fig. 9 Comparison between RS filter, Modified RS filter and Compensated modified RS filter

TABLE III

Comparison between RS filter, Modified RS filter and Compensated modified RS filter

Decimation technique	Specification	Passband Droop (dB)	Alias Rejection (dB)
RS filter	D = 64; K = 1; B = 0.0184	-1.2	-54.23
Modified RS filter	D_1 = 8; D_2 = 8; K_1 = 4; K_2 = 4; B = 0.0184	-1.358	-127.3
Compensated Modified RS filter	D_1 = 8; D_2 = 8; K_1 = 4; K_2 = 4; B = 0.0184; b =0	-1.019	-127.3

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