Improved Channel Estimation for Slow to Moderate Fading Channel in OFDM Systems using Random Walk Based Kalman Filter

K. Vidhya
ECE Department, JCET
Trichy, India

M. Geetha
HOD, ECE Department, JCET
Trichy, India

***Abstract*— This project deals with estimating the multi path channel condition in orthogonal frequency division multiplexing system. Many estimation algorithms exploit the time-domain correlation of the channel by employing a Kalman filter based on an approximation of the time-varying channel. Least square estimator explores the frequency domain correlation of the channel and the knowledge of the delays to convert the pilot frequencies into a primary estimate of the path Complex Amplitude. It also defines an error signal for each path. The lower dimensional RW-KF that estimates the complex amplitude of each path separately from the LS estimated signal. We demonstrate that this amounts to a simplification of the joint multi-path Kalman gain formulation through the Woodbury's identities. Hence, this new algorithm consists of a superposition of independent single-path single-carrier KFs. This observation allows us to adapt the optimization to the actual multi-path multi-carrier scenario, to provide analytic formulas for the mean-square error performance and the optimal tuning of the proposed estimator directly as a function of the physical parameters of the channel (Doppler frequency, signal-to-noise-ratio). These analytic formulae are given for the first-, second-, and third-order RW models used in the KF. The proposed per-path KF is shown to be as efficient as the exact KF (i.e., the joint multi-path KF).**

Keywords—Orthogonal frequency division multiplexing; Channel estimation; Least square estimator; Random walk model; Kalman filter.

I. INTRODUCTION

Orthogonal frequency division multiplexing (**OFDM**) is a method of encoding digital data on multiple carrier frequencies. OFDM has developed into a popular scheme for wideband digital communication, whether wireless or over copper wires, used in applications such as digital television and audio broadcasting, DSL Internet access, wireless networks, power line networks, and 4G mobile communications. OFDM is a frequency-division multiplexing (FDM) scheme used as a digital multi-carrier modulation method. A large number of closely spaced orthogonal sub-carrier signals are used to carry data on several parallel data streams or channels. Each sub-carrier is modulated with a conventional modulation scheme (such as quadrature amplitude modulation or phase-shift keying) at a low, symbol rate maintaining total data rates similar to the conventional Single-carrier modulation schemes in the same bandwidth.

The primary advantage of OFDM over single-carrier schemes is its ability to cope with severe channel conditions (for example, attenuation of high frequencies in a long copper wire, narrowband interference and frequency-selective fading due to multipath) without complex equalization filters. Channel equalization is simplified because OFDM may be viewed as using many slowly modulated narrowband signals rather than one rapidly modulated wideband signal. The low symbol rate makes the use of a guard interval between symbols affordable, making it possible to eliminate inter symbol interference (ISI) and utilize echoes and time-spreading (on analogue TV these are visible as ghosting and blurring, respectively) to achieve a diversity gain, i.e. a signal-to-noise ratio improvement. This mechanism also facilitates the design of single frequency networks (SFNs), where several adjacent transmitters send the same signal simultaneously at the same frequency, as the signals from multiple distant transmitters may be combined constructively, rather than interfering as would typically occur in a traditional single-carrier system.

Orthogonal Frequency-Division Multiplexing (OFDM) is an effective technique for alleviating frequency-selective channel effects in wireless communication systems. In this technique, a wideband frequency-selective channel is converted to a number of parallel narrow-band flat fading sub channels which are free of Inter-Symbol-Interference (ISI) and free of Inter-Carrier Interference (ICI) (for negligible channel time variation within one OFDM symbol period T). For coherent detection of the information symbols, reliable estimation of the channel in the OFDM system is crucial.

II. PROBLEM FORMULATION

Most of the conventional methods work in a symbol-by symbol scheme [1]–[3] using the correlation of the channel only in the frequency domain; i.e., the correlation between the sub channels. More advanced algorithms are based on the Kalman Filter (KF), to also exploit the time-domain correlation. This paper deals with channel multi-path Complex Amplitude (CA) estimators based on KFs. An approximation often used in the literature consists of approaching the fading process as auto-regressive [4]. Hence, a widely used channel approximation is based on a first-order Auto-Regressive model (AR1), as recommended by [4], combined with a Correlation- Matching (CM) criterion to fix the AR1 coefficient. The KF channel estimator that results from this choice, hereafter called $AR1_{CM}$ -KF, has been used in several studies concerning various systems, such as in multiple-input-multiple- output systems [4], and in OFDM systems [4]–[6]. The $AR1_{CM}$ -KF appears to be convenient for the very high mobility case, which leads to quasi-optimal channel estimation performance compared to lower bounds, as seen, for example, in [5], [6] (in these studies, the AR1 -KF is actually used to track the basis extension model coefficients of the high-speed channel). However, here we consider moderate normalized Doppler frequency (f_dT) values; i.e., $f_dT \leq 10^{-2}$. This corresponds to low mobility (≤ 50 km/h) with the actual systems such as Worldwide Interoperability for Microwave Access (WiMAX) Mobiles. However, with the development of the cognitive radio, lower carrier frequencies are investigated for future systems.

For instance, VHF/UHF television broadcast bands from 54 MHz to 862 MHz [8] and aeronautical bands from 960 MHz to 1215 MHz are planned to be deployed. For a given f_dT, as the speed is inversely proportional to the carrier frequency, f_dT values around 10^{-2} can correspond to a relative high mobility with such systems (hundreds of km/h). This prompts the need for a comprehensive study of channel estimation for $f_dT \leq 10^{-2}$.

For this scenario, whereby the channel variation within one symbol duration can be neglected ([3]–[5], [7]), the $AR1_{CM}$ -KF estimator usually exploited in the literature is far from being effective [7]. A better tuning of the AR1 coefficient can focus on minimizing the estimation variance in the output of the KF, as proposed by [8] (with the analytic mean-square error (MSE) performance for a given Doppler and signal-to-noise ratio (SNR) scenario. This performance can be obtained by a first-order Random Walk (RW)-model based KF (RW1-KF) ([8]).

On the other hand, it has been shown recently that the MSE performance of a KF can still be improved by switching from the AR1 model to an *integrated* RW model (also called the integrated Brownian model) for the approximation model. A second-order RW model and a third-order RW model have been respectively considered in [8] and [9]. They take into account that the exact path CA continues in a given direction during several symbols for low f_dT, and shows strong trend behaviour. The Kalman estimators based on these second order and third order models are here called the RW2-KF and RW3-KF estimators, respectively. The RW-KF estimators of the previously cited studies were designed for single-path channel estimation in single-carrier systems. In the present study, we consider multi-path channel estimation in multi carrier systems (i.e., OFDM systems). In this context, we are interested in devising simplified methods compared to the high-dimensional KFs that perform joint estimation of the path CAs. Some simplified methods have lately been proposed in [10]–[12]. Reference [12] converts the vector of pilot subcarriers into L multi-path values where L is the number of multi-paths and applies a KF to each path.

III. SYSTEM DESCRIPTION

A. *OFDM System Model*

Orthogonal frequency division multiplexing (OFDM) is a parallel transmission scheme, where a high-rate serial data stream is split up into a set of low-rate sub streams, each of which is modulated on a separate sub-carrier (SC) (frequency division multiplexing). Let us consider an OFDM system with N subcarriers, and cyclic prefix length N_g. The duration of an OFDM symbol is $T = N_T T_S$, where T_S is the sampling time and $N_T = N+N_g$. Let X(k) is the sequence of transmitted elementary symbols of the k-th OFDM symbol. X(k) transmitted symbol is modulated by (M-PSK). Modulate symbol transmitted on the subcarrier with indice n-1-N/2. The sequence of transmitted symbol is assumed to be zero mean and stationary with normalized variance. After transmission over a slowly time-varying multi-path channel and fast Fourier transform demodulation, the k-th received OFDM symbol Y(k) is given by

$$Y(k) = H(k)\, X(k) + W(k) \qquad (1)$$

Where W(k) is a complex circular Gaussian noise vector and H(k) is a diagonal matrix.

$$[H(k)] = 1/N \sum_{l=1}^{L} \alpha_{(k)}^{(l)}\, e^{-j2\pi\left(\frac{n-1}{N}-\frac{1}{2}\right)} \qquad (2)$$

Where L is the total number of propagation paths, $\alpha_{(k)}^{(l)}$ is a CA of the l-th path at k-th OFDM symbol.

B. *Pilot Pattern*

We use N_p pilot sub-carriers, they are evenly inserted into the N sub-carriers. The received pilot sub-carriers can be written as

$$Y_p(k) = \mathrm{diag}\{X_p(k)\}F_p\, \alpha(k) + W_p(k) \qquad (3)$$

Where X_p, Y_p and W_p which correspond to the sent and received data symbol, and the channel noise on the pilot sub-carriers, respectively. The $N_p \times L$ matrix F_p is the Fourier matrix of the pilot sub-carriers, with elements given by

$$[F_p]_{np,l} = e^{-j2\pi\left(\frac{np-1}{N}-\frac{1}{2}\right)} \qquad (4)$$

C. *Least Square Estimator*

To define an error signal for each path, we use the least-square (LS) estimator. The error signal for each path can be reduced by RLS filter and also it can estimate the path CAs obtained from the current OFDM symbol $Y_p(k)$. The LS estimate is obtained by

$$\hat{\alpha}_{LS}(k) = (F_P^H F_P)^{-1} F_P^H Y_p(k) \qquad (5)$$

D. *Single Path RW-KF*

We use the LS estimation of α instead of $Y_p(k)$ as the input signal to reformulate the KF, and impose independent processing of the L paths. The l-th element of $\hat{\alpha}_{LS}(k)$, denoted by $\hat{\alpha}^{(l)}_{LS}(k)$, corresponds to the LS estimation of the l-th path CA. Also, let us define the LS estimation error as the loop noise applied on the per-path KF, denoted by w_{LS}. Then, the state-space del of per- h KF for the l-th path is given by

$$\hat{\alpha}^{(l)}_{LS}(k) = \alpha^{(l)}_{(k)} + w^{(l)}_{LS}(k) \qquad (6)$$

Where, $w^{(l)}_{LS}(k) = (F^H_P F_P)^{-1} F^H_P W_p(k)$ (7)

The state-space model of the per-path KF for the l-th path CA $\hat{\alpha}^{(l)}_{LS}(k)$ ca $\hat{\alpha}^{(l)}_{(k/k)}$ applied to single-path RW-KF which estimates $\hat{\alpha}_{(k/k)}$. The state model of the L-path CAs can be expressed in vector form as

$$a(k) = Ma(k-1)+u(k) \qquad (8)$$

Where, $a(k) = [a^{(1)T}_{(k)} \dots a^{(L)T}_{(k)}]^T$ with $a^{(l)}_{(k)}$ as the state vector of the l-th path, $u(k) = [u^{(1)T}_{(k)} \dots u^{(L)T}_{(k)}]^T$ with $u^{(l)}_{(k)}$ the state noise vector of the l-th path, $\mathcal{M} = I_L \times M$ is the channel state evolution matrix.

TABLE I. List of Terms in the KF State Space Model

Variables	RW1	RW2	RW3
[illegible]	[illegible]	[illegible]	[illegible]
$u^{(l)}_{(k)}$	[illegible]	[0 [illegible]]	[0 0 [illegible]]
M	1	[1 1] [0 1]	[1 1 ½] [0 1 1] [0 0 1]
S	1	[1 0]	[1 0 0]
U[illegible]	$\sigma^{2}_{u}{}^{(l)}$	[0 0] [0 $\sigma^{2}_{u}{}^{(l)}$]	[0 0 0] [0 0 0] [0 0 $\sigma^{2}_{u}{}^{(l)}$]

We define the multi path selection matrix $\mathcal{S} = I_L \times S$. This matrix allows us to pass from the vector a(k) to α(k) using α(k) = $\mathcal{S}$a(k). By defining $F_S = F_P\mathcal{S}$, we obtain the equation of the joint multi-path KF. The path variables $a^{(l)}_{(k)}$, $u^{(l)}_{(k)}$, and the path state evolution matrix M, path selection matrix S are defined in following table according to the model order. The single path RW-KF for the l-th path can thus be written as

$$\hat{\alpha}^{(l)}_{(k/k)} = \hat{\alpha}^{(l)}_{(k/k-1)} + K^{(l)}_{(k)}\left(\hat{\alpha}^{(l)}_{LS(k)} - S\,\hat{\alpha}^{(l)}_{(k/k-1)}\right) \qquad (9)$$

Where,

$$\hat{\alpha}^{(l)}_{(k/k-1)} = M\,\hat{\alpha}^{(l)}_{(k-1/k-1)} \qquad (10)$$

$$\hat{P}^{(l)}_{(k/k-1)} = M\,\hat{P}^{(l)}_{(k-1/k-1)} M^T + U^{(l)} \qquad (11)$$

$$K^{(l)}_{(k)} = P^{(l)}_{(k/k-1)} S^T / S\,P^{(l)}_{(k/k-1)} S^T + \sigma^2_{LS(l)} \qquad (12)$$

E. Joint Multi-Path RW-KF

Joint multi-path RW-KF super positioning the per-path CA to estimates $\hat{\alpha}_{(k/k)}$ by using Woodbury's identity.

$$\hat{\alpha}_{(k/k)} = \hat{\alpha}_{(k/k-1)} + K_{pp(k)} \cdot v_{\epsilon(k)} \qquad (10)$$

Where, $\hat{\alpha}_{(k/k-1)} = M\,\hat{\alpha}_{(k-1/k-1)}$ (11)

$$\hat{P}_{(k/k-1)} = M\,\hat{P}_{(k-1/k-1)} M^T + U \qquad (12)$$

$$K_{pp(k)} = P_{(k/k-1)} S^H \left(S P_{(k/k-1)} S^H + \mathrm{diag}\{[\sigma^2_{LS(1)} \cdots \sigma^2_{LS(l)}]\}\right)^{-1} \qquad (13)$$

$$v_{\epsilon(k)} = \hat{\alpha}_{LS}(k) - \hat{\alpha}_{(k/k-1)} \qquad (14)$$

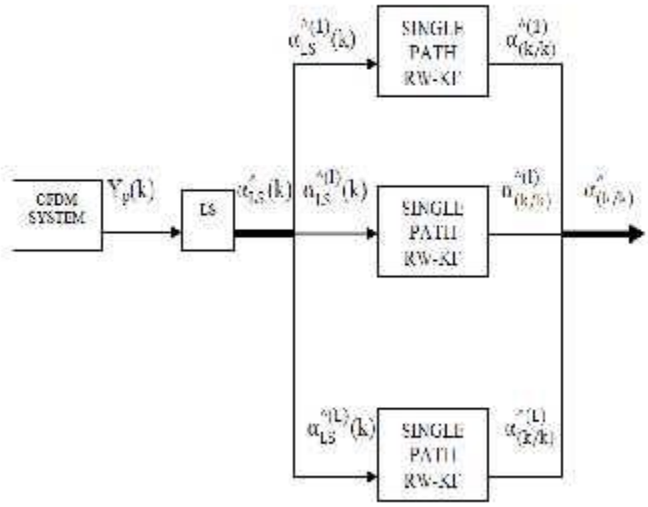

Fig. 1. Per-path KF structure.

This CA is compared with the CA of OFDM transmitted symbol X(k). If the CA of joint multi path RW-KF is less than the CA of OFDM transmitted symbol X(k), then we can know that the channel is slow to moderate fading channel. If the CA of joint multi path RW-KF is higher than the CA of OFDM transmitted symbol X(k), then we can know that the channel is fast fading channel. If the CA of joint multi path RW-KF is equal to the CA of OFDM transmitted symbol X(k), then we can know that there is no fading in multipath channel.

Slow fading arises when the coherence time of the channel is large relative to the delay constraint of the channel. In this regime, the amplitude and phase change imposed by the channel can be considered roughly constant over the period of use. Slow fading can be caused by events such as shadowing, where a large obstruction such as a hill or large building obscures the main signal path between the transmitter and the receiver. The received power change caused by shadowing is often modeled using a log-normal distribution with a standard deviation according to the log-distance path loss model.

F. Asymptotic Mean Square Error of the Per-Path KF

The variance of estimation error is then comprised of two parts, one of which comes from the variation of the parameter α, and the other comes from the input loop noise w_{LS}. Thus for a given path l, we have

$$\sigma^2_{\epsilon(l)} = \sigma^2_{\epsilon\alpha(l)} + \sigma^2_{\epsilon w(l)} \qquad (15)$$

The component $\sigma^2_{\epsilon\alpha(l)}$ results from the high pass filtering of the input CA $\alpha^{(l)}_{(k)}$, which can be expressed in the frequency domain. The component $\sigma^2_{\epsilon w(l)}$ results from the low pass filtering of the input loop noise. The global mean MSE (per path) of the channel estimation is calculated by

$$\sigma^2_{\epsilon min} = 1/L \sum_{l=1}^{L} \sigma^2_{\epsilon(l)min} \qquad (16)$$

The integrated RW-KF allows us to provide analytic formulas for the mean square error performance. The MSE performance can still be improved by proposed estimator.

IV. SIMULATIONS

In this simulation section, The PSK-OFDM system with N=128 sub-carriers used to validate the proposed approximate method and the analytic results. By default, the OFDM system has N_g=0.5 samples and N_p=16 pilot sub-carriers in each transmitted OFDM symbol. After that this OFDM symbol is applied to RLS filter and LS estimator to define the error signal and to calculate the CA. Then, it can be applied to Single Path RW-KF which estimates CA of each path separately. Finally each separate path CA's are super positioned to get multi path CA which is compared with OFDM symbol CA to estimate the channel condition. And also the MSE performance of per path and joint multi path RW-KF, MSE of Per Path KF Versus SNR, MSE comparison between Per Path and Joint Multi-path RW-KF are shown.

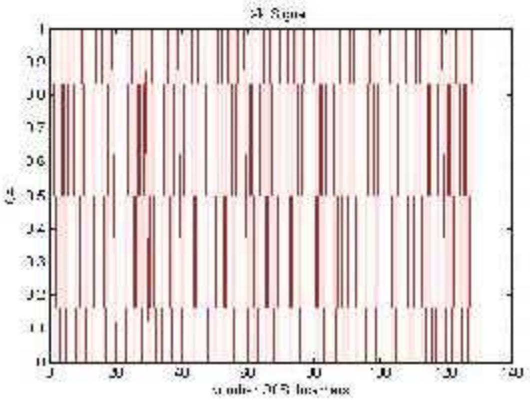

Fig. 2. Input signal.

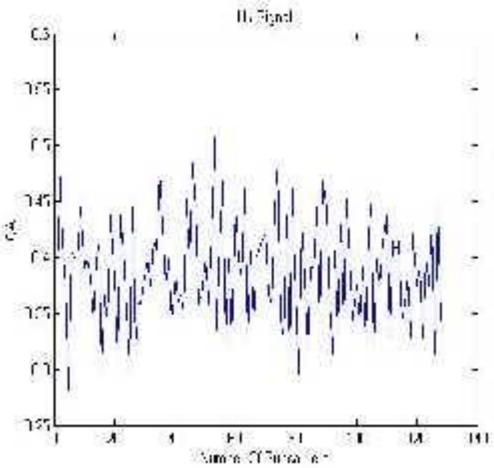

Fig. 3. Hk signal.

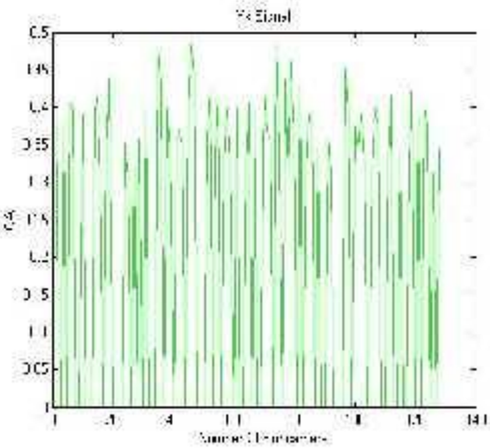

Fig. 4. Yk signal.

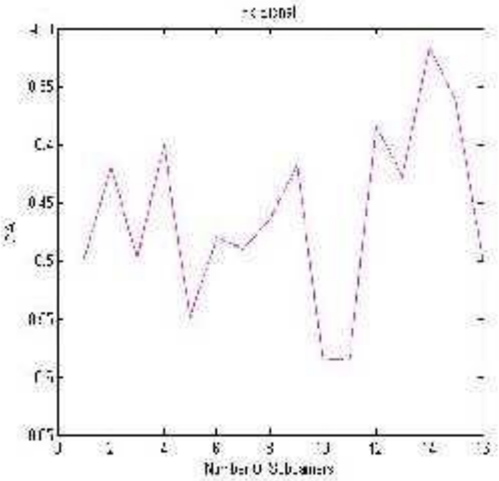

Fig. 5. Fp signal.

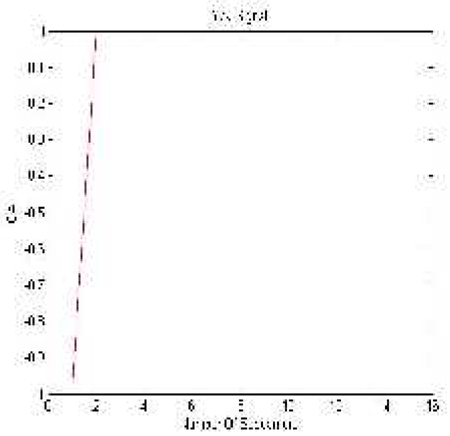

Fig. 6. Ypk signal.

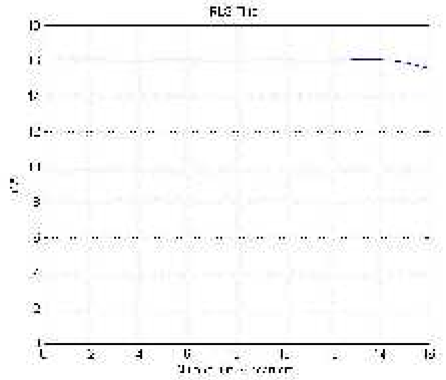

Fig. 7. RLS Filtered signal.

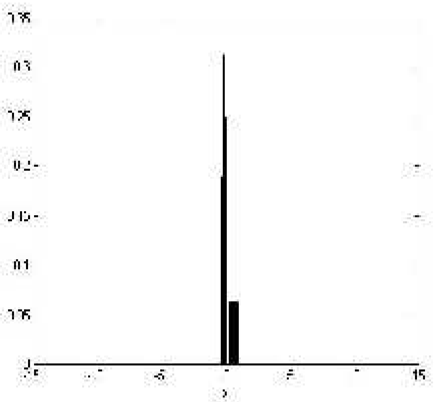

Fig. 8. Single path RW-KF signal.

Fig. 2 shows the input (Xk) signal with 128 sub-carriers and cyclic prefix length is 0.5 samples. Fig. 3 is the carrier signal for PSK modulation. Fig. 4 shows the OFDM symbol with 128 sub-carriers which is generated by input signal modulated with carrier signal and summed with white Gaussian noise signal. Fig. 5 represents the Fourier matrix of the pilot sub-carriers. Fig. 6 presents the OFDM symbol with pilot sub-carriers which is generated by the Fourier matrix and diagonal elements of input signal with pilot sub-carriers.

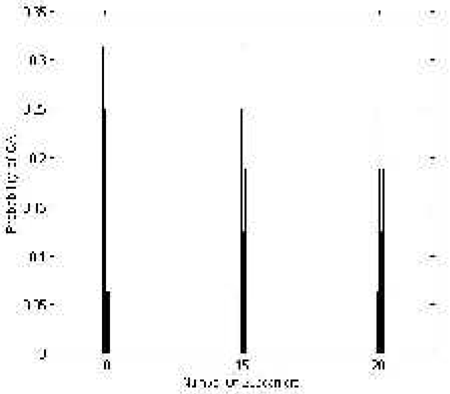

Fig. 9. Joint Multi path RW-KF signal.

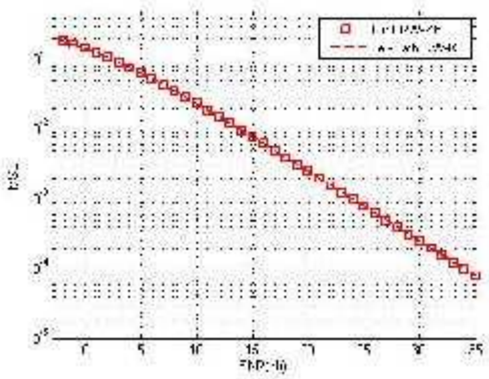

Fig. 10. MSE Of Joint Multi Path RW-KF And Per Path RW-KF.

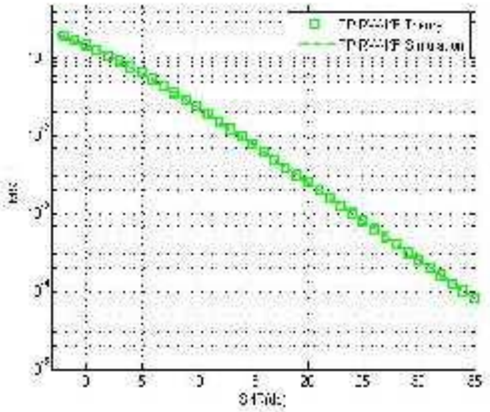

Fig. 11. MSE Of Per Path KF Versus SNR.

Fig. 7 shows the RLS filtered signal which can minimize the error signal in OFDM symbol. Fig. 8 and Fig. 9 shows the output of single-path RW-KF and Joint multi-path RW-KF respectively. This joint multi-path CA is comparable with the input signal CA to analyze the Multi-path channel condition. Fig. 10 presents MSE of Joint Multi Path RW-KF and Per Path RW-KF. Fig. 11 shows MSE of Per Path KF versus SNR.

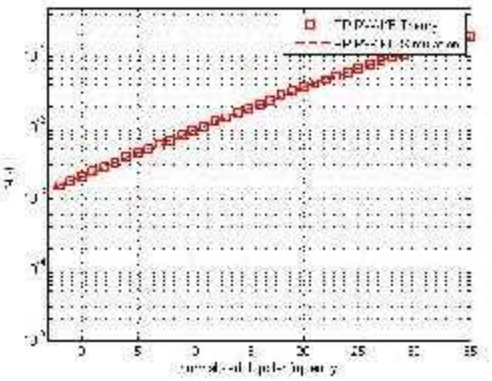

Fig. 12. MSE versus f_dT

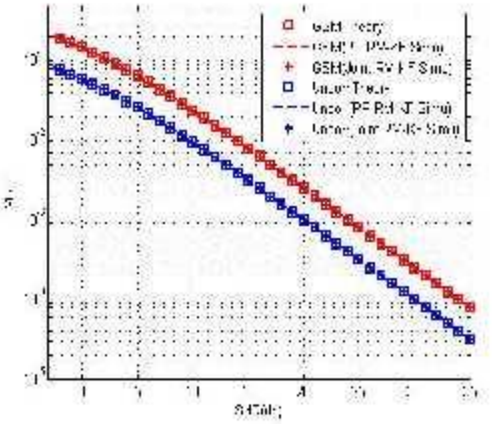

Fig. 13. MSE Comparison Between Per Path and Joint Multi Path RW-KF

Fig. 12 shows the MSE versus f_dT and Fig. 13 shows MSE comparison between Per Path and Joint Multi Path RW-KF.

V. CONCLUSION

This paper proposed the integrated RW-KF for channel condition estimation. Our solution is a two-step solution: first, an error signal and CA for each channel path is calculated with the LS criterion. Secondly, based on this error signal, a KF is applied to each path independently. This per path KF solution explores the time-domain correlation of the channel, while the LS step exploits the frequency-domain correlation of the channel. We have shown how to apply the previous results we obtained for a single-path single-carrier to the multi-path multi-carrier context. Lower dimensional RW-KF that estimates the complex amplitude of each path separately. Furthermore, we have demonstrated that our per-path KF solution can be interpreted as a simplified version of the more complex joint multi-path KF. This has been done through the Woodbury's identities. Hence, this new algorithm consists of a superposition of independent single-path single-carrier KFs, which were allows us to adapt the optimization to the actual multi-path multi-carrier scenario, to provide channel condition estimation analytic formulas for the mean-square error performance and the optimal tuning of the proposed estimator directly as a function of the physical parameters of the channel (Doppler frequency, signal-to-noise-ratio). The simulation results show that the performance of this low-dimensional RW-KF of Per-path solution and Joint Multi Path RW-KF solution to the OFDM symbol which is used to estimate the channel condition. And also the simulation results show that the MSE performance of this low-dimensional Per-Path solution is comparable to that of the joint multi-path KF.

REFERENCES

[1] Y. Zhao and A. Huang, "A novel channel estimationmethod for OFDM mobile communication systems based on pilot signals and transformdomain processing," in*Proc. IEEE 47th Veh. Technol.Conf.*, 1997, vol.3, pp. 2089–2093, vol.3.

[2] M.-H. Hsieh and C.-H. Wei, "Channel estimation for OFDM systems based on comb-type pilot arrangement in frequency selective fading channels," *IEEE Trans. Consum. Electron.*, vol. 44, no. 1, pp. 217–225, 1998.

[3] S. Coleri, M. Ergen, A. Puri, and A. Bahai, "Channel estimation techniques based on pilot arrangement in OFDM systems," *IEEE Trans. Broadcast.*, vol. 48, no. 3, pp. 223–229, 2002.

[4] W. Chen and R. Zhang, "Kalman-filter channel estimator for OFDM systems in time and frequency-selective fading environment," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*,May 2004, vol. 4, pp. iv-377–iv-380, vol.4.

[5] T. Al-Naffouri, "An EM-based forward-backward Kalman filter for the estimation of time-variant channels in OFDM," *IEEE Trans. Signal Process.*, vol. 55, no. 7, pp. 3924–3930, 2007.

[6] E. P. Simon, L. Ros,H. Hijazi, and M. Ghogho, "Joint carrier frequency offset and channel estimation for OFDM systems via the EMalgorithm in the presence of very high mobility," *IEEE Trans. Signal Process.*, vol. 60, no. 2, pp. 754–765, Feb. 2012.

[7] M. Sohail and T. Al-Naffouri, "An EM based frequency domain channel estimation algorithm for multi-access OFDM systems," *Elsevier Signal Process.*, vol. 90, no. 5, pp. 1562–1572, 2010.

[8] R. Gerzaguet, L. Ros, J.-M. Brossier, S. Ghandour-Haidar, and F.Belvèze, "Self-adaptive stochastic Rayleigh flat fading channel estimation," presented at the IEEE Int. Conf. Digit. Signal Process. (DSP), Santorini, Greece, Jul. 2013.

[9] H. Shu, E. P. Simon, and L. Ros, "Third-order Kalman filter: Tuning and steady-state performance," *IEEE Signal Process. Lett.*, vol. 20, no. 11, pp. 1082–1085, 2013.

[10] K. Muralidhar and D. Sreedhar, "Pilot design for vector state scalar observation Kalman channel estimators in doubly-selective MIMO-OFDM systems," *IEEE Wireless Commun. Lett.*, vol. 2, no. 2, pp. 147–150, Apr. 2013.

[11] N. Chen, J. Zhang, and P. Zhang, "Improved channel estimation based on parametric channel approximation modeling for OFDM systems," *IEEE Trans. Broadcast.*, vol. 54, no. 2, pp. 217–225, Jun. 2008.

[12] H. Hijazi and L. Ros, "Bayesian Cramera-Rao bounds for complex gain parameters estimation of slowly varying Rayleigh channel in OFDM systems," *Elsevier Signal Process.*, vol. 89, no. 1, pp. 111–115, 2009.