Implementation of Partial Differential Equations in Image Compression

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Abstract: **In this paper, the compression algorithm that is Bilinear Interpolation is used to compress the medical images. The partial Differential equations are used to improve the blurring in the images compressed by the compression algorithm. Partial Differential Equation mode for image compression has been discussed. The comparison of the images compressed with the Bilinear Interpolation and that of partial differential equations has shown in this paper. The experimental results shows that the proposed technique yields better performance by yielding good compression ratio and PSNR which is much desired in medical images.**

Keywords- Image Compression, Bilinear Interpolation, Partial Differential Equations, Peak Signal To Noise Ratio, Mean Square Error.

I. INTRODUCTION

Medical images are having large size and it becomes difficult to transmit them over the network as it is. So, there is a need to compress the images before transmitting them to the receiver end. The compression is the technique of reducing the size of the image so that it becomes easy to transmit them over the network. The reduction of the size is the removal of the redundancies in the image like that of data redundancy, Interpixel redundancy etc. By removing these types of redundancies, the size of the image gets reduced. Other benefits of image compression are that it also reduces the probability of transmission errors as there are fewer bits to transmit over the network. It not only reduces storage requirements but also overall execution time. It also provides a level of security against illicit monitoring. The compression of the image is done by Bilinear Interpolation method for image compression. Bilinear Interpolation method gives better results than that of other interpolation methods. The other part of the paper is the improvement of the Bilinear Interpolation. By compressing the images with this algorithm, there comes some blurring into the images. So, Partial Differential Equations are used to improve this that is to remove the blurring from the images.

II. BILINEAR INTERPOLATION

The Second interpolation method is the Bilinear Interpolation method[2]. Other Interpolation methods are Nearest Neighbour Algorithm and Bi- cubic Interpolation method. Bilinear Interpolation is an extension of Linear Interpolation for interpolating functions of two variables on a regular grid. The Bilinear interpolation method is based upon four nearest neighbouring pixels. Let us assume, if we want to find out the value of the unknown function f at any point P = (x , y). The values of the function f at the four points $Q_{11}(x_1, y_1)$, $Q_{12}(x_1, y_2)$, $Q_{21}(x_2, y_1)$, $Q_{22}(x_2, y_2)$. The idea is to firstly perform the linear interpolation in the x- direction and then in the y- axis. The result of the Bilinear Interpolation is independent of other interpolation methods. If the linear interpolation is done in the y- axis first and then in the x- axis, the result would be the same.

Do the linear interpolation in x- axis, we get,

$$f(R1) \approx \frac{x2-x}{x2-x1} f(Q11) + \frac{x-x1}{x2-x1} f(Q21)$$

$$where \;\; R1 = (x, y1) \qquad \ldots (1)$$

$$f(R2) \cong \frac{x2-x}{x2-x1} f(Q_{12}) + \frac{x-x1}{x2-x1} f(Q22)$$

$$where \;\; R2 = (x, y2) \qquad \ldots (2)$$

Now Proceed in y- axis,

$$f(P) \approx \frac{y2-y}{y2-y1} f(R1) + \frac{y-y1}{y2-y1} f(R2) \qquad ..(3)$$

This gives the estimation of f(x , y)

$$f(x,y) \cong \frac{f(Q11)}{(x2-x1)(y2-y1)}(x2-x)(y2-y) + \frac{f(Q21)}{(x2-x1)(y2-y1)}(x-x1)(y2-y) + \frac{f(Q12)}{(x2-x1)(y2-y1)}(x2-x)(y-y1) + \frac{f(Q22)}{(x2-x1)(y2-y1)}(x-x1)(y-y1) \quad \ldots (4)$$

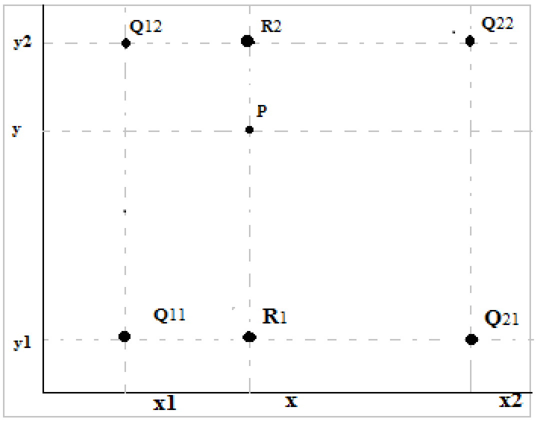

Fig 1. Bilinear Interpolation

It is not compulsory that in which direction do the linear interpolation first. The result is same in both the cases.
Bilinear interpolation considers the closest 2x2 neighborhood of known pixel values surrounding the unknown pixel's computed location. It then takes a weighted average of these 4 pixels to arrive at its final, interpolated value. The weight on each of the 4 pixel values is based on the computed pixel's distance in 2d space from each of the known points.

III THE PDE BASED IMAGE COMPRESSION ALGORITHM

The PDE model we shall introduce is based on a noise removal algorithm proposed in [4]. Authors proposed a fourth-order PDE to image de noising, which is to recover an image u from a noisy observation u0. This model is referred as the LLT model. For noise removal, one needs to solve the following minimization problem:

$$\min_v E(v), where\ E(v) = \int_\Omega \left(v_{xx}^2 + v_{xy}^2 + v_{yx}^2 + v_{yy}^2\right)^{\frac{1}{2}} dxdy + \frac{\lambda}{2}\int_\Omega \left|v - v_0\right|^2 dxdy \quad ...(5)$$

Assume the noise level σ2 = R |u−u0|2dxdy is approximately known, then one can use a Lagrangian multiplier to solved the above constrained minimization problem. The resulted equation is a fourth order nonlinear partial differential equation. High- order PDEs are known to recover smoother surfaces better. The nonlinear PDE resulted from LLT can also preserve jump rather well. In this work, we try to use this idea for image compression. The Partial Differential Equations are used for the removal of noise, so we are using it to remove the artefact from the image that gets compressed with bilinear interpolation method. The various artefacts are there that comes into an image after compression. These are blurring, edge halos etc.

IV PERFORMANCE CRITERIA

The performance of the algorithm can be obtained by comparing them on the basis of PSNR, MSE and that of the Compression Ratio (CR). The PSNR is the Peak Signal to Noise Ratio that gets improved in the case of Partial Differential Equation. The MSE is the Mean Square Error that gets reduced in case of Partial differential Equations.

A. Compression Ratio

Compression Ratio (C.R) is the ratio between the size of the original image and the size of the compressed image.

$$C.R = \frac{n_1}{n_2} \quad ...(6)$$

B. Distortion Measure

Distortion Measure is measured by the Mean Square Error (MSE) .

$$MSE = \frac{1}{M * N}\sum_{x=1}^{M}\sum_{y=1}^{N}[f(x,y) - f'(x,y)]^2 \quad ...(7)$$

C. Peak signal to Noise Ratio

PSNR is used as an approximation to human perception of reconstruction quality. PSNR is accepted as a widely used quality measurement in the field of image compression.

$$PSNR = 10 log_{10}\frac{(MAX)^2}{MSE} \quad ...(8)$$

Where MSE stands for Mean Square Error and it is the power of corrupting noise that affects the fidelity of its representation and MAX is the maximum possible power of the signal. Most of MAX can be taken as 255*255. The signal in our case is the original image and the noise is the error introduced by compression. Image having high PSNR value is having high reconstruction quality.

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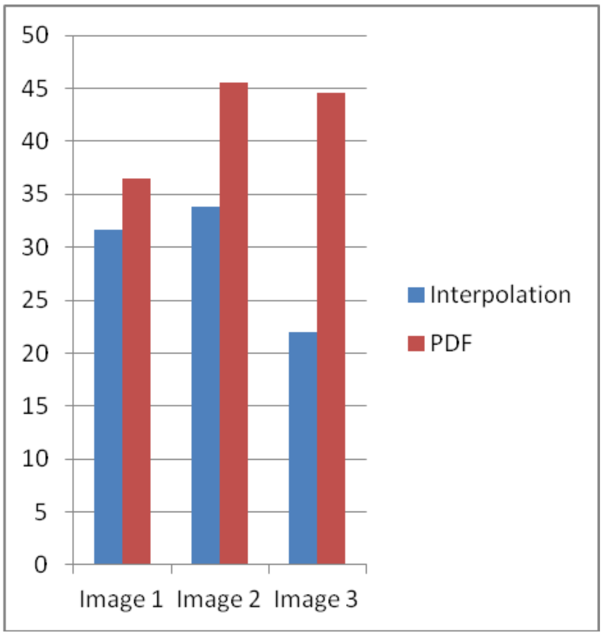


Fig 2. Comparison on the basis of PSNR

V IMPLEMENTATION RESULTS

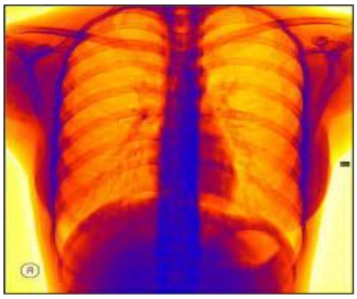
The effectiveness of our algorithm can be shown below by comparing the results of our algorithm with that of Interpolation. As shown in various images, there is some blurring artefact in the images compressed by the Interpolation algorithm. The both algorithms are applied on various medical images like that of MRI, CT Scan etc. The comparison on the basis of PSNR (Peak- Signal -To- Noise- Ratio), MSE (Mean Square Error) and that of size is described in the table below.

TABLE I COMPARISON OF TWO ALGORITHMS

Image	Method	PSNR	MSE	Size
1	Interpolation	31.64	95459	11 KB
	PDE	36.49	10142	10 KB
2	Interpolation	33.88	2408	15 KB
	PDE	45.53	18263	12 KB
3	Interpolation	21.94	7247	11 KB
	PDE	44.55	2761	11 KB

a) Original Image (16 KB)

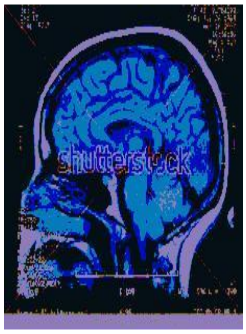
b) Image compressed by Interpolation



(c) Image compressed by Partial Differential Equations

Fig 3. Image 1

a) Original Image(15 KB)

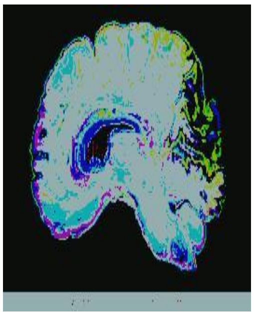


b) Compressed by Interpolation

c) Image Compressed By Partial differential Equations

Fig 4. Image 2

a) Original Image(14 KB)



b) Image compressed by interpolation

c) Image compressed by Partial Differential Equations

Fig 5. Image 3

V CONCLUSION AND FUTURE WORK

In this paper, we proposed an image compression method based on nonlinear PDEs. The quality differs in the images compressed by both the algorithms. All the images taken as an example shows that our algorithm gives better results and removes the artefact at a certain level. The images indicate that our algorithm gives efficient result for both grey- scale as well as coloured images. After compressing the images with both the algorithms and comparing the results on the basis of Compression Ratio, PSNR and MSE, the next work for the future is to provide security to the resultant image at the time of transmission. When the compressed image is transmitting to the receiver end, it has to be very secure. So, providing security is our next work.

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