Implementation of Opposition based Biogeography-based Optimization for Optimal Digital IIR Filter Design

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Abstract— **The present work proposes a revised solution methodology for the design of digital infinite impulse response (IIR) filters using the biogeography-based optimization (BBO) algorithm. BBO is a population based stochastic optimization algorithm which is inspired by the mathematics of biogeography. So far BBO has been effectively applied to variety of application areas. BBO searches the solution space through basic migration and mutation operators in order to find the global optimum solution. The conventional BBO algorithm owns good local exploitation capability but is deficient in global exploration. Moreover, it experiences premature convergence and simply falls into the local minima. To remedy this defect and to improve the performance of BBO an improved variant, called the opposition based biogeography based optimization (OBBO) is proposed in this paper. In OBBO, for improving the population diversity, the polyphyletic migration operator is incorporated in place of the basic migration operator and for the purpose of starting with a better solution set, opposition-based learning strategy is included. Then, the OBBO algorithm is applied for the IIR filter designing problem. The multivariable optimization is taken as the design criterion to obtain the robust and stable digital IIR filter considering the minimization of the magnitude approximation error and ripple magnitudes of both the pass-band and the stop-band while satisfying the stability constraints that are imposed throughout the design process. Further, constraints are considered as additional objective function to be maximized using membership function of constraint violation. OBBO algorithm is effectively applied for designing the low-pass (LP) and high-pass (HP) filters. To demonstrate the effectiveness of OBBO for designing the digital IIR filters, the results obtained are compared with some well established algorithms and it is observed that the employed algorithm produces superior or atleast comparable results and can be also be applied for the design of higher order filters.**

Keywords—Digital IIR filters, biogeography-based optimization, opposition based learning, multiparameter optimization, filter design.

I. INTRODUCTION

The digital IIR filters are the most important part of application areas like the digital signal processing, digital image processing and digital video processing. Because of the numerous advantages over the analog filters, the digital filters are in demand in almost all the applications related to the field of science and technology. Digital IIR filters often provide a much better performance, improved selectivity and less computational cost than their equivalent FIR filters. Moreover, the IIR filters usually have much sharper roll-offs in their frequency responses than the FIR filters of equal complexity. Compared with an FIR filter design problem, an IIR filter design problem is more challenging. The design task of IIR digital filters is to approximate a given ideal frequency response by a stable IIR digital filter under some design criterion. If both magnitude and phase/group delay responses are considered, an IIR digital filter design problem is essentially a non-convex optimization problem due to the presence of the denominator of the transfer function [1].

The designing of the digital IIR filters is considered as a very important subject in the field of signal processing and a large amount of work has been carried out in this field. The digital IIR filter designing essentially follows two approaches namely, the transformation approach and the optimization approach. The former method firstly takes into account the designing of an analog filter for a given set of prescribed specifications and then it is transformed into the digital IIR filter [2]. But this approach usually returns a single solution in most of the cases and requires too much of pre-knowledge. Because of the nonlinear and multimodal error surface of the digital IIR filters, the conventional gradient-based algorithm simply got trapped at local minima [3]. To overcome this problem design techniques that can efficiently achieve the global minima in the multimodal error surface are needed. Under the optimization approach, in the past years, various methods have been proposed [4-22]. These methods struggle to obtain optimal filter design models and use the magnitude approximation error, and ripple magnitudes of both pass-band and stop-band as the performance measures for the optimal and stable digital IIR filters designs.

In the past years, many evolutionary heuristic search optimization algorithms have been applied to design and optimize the digital IIR filters. The important digital IIR filter design approaches include the genetic algorithms (GAs) [4-6], hierarchical genetic algorithm (HGA) [7], genetic algorithm improved using the hybrid taguchi method (HTGA) [8], taguchi immune algorithm (TIA) [9], particle swarm optimization [10-13], predator pray optimization (PPO) algorithm [14], real structured genetic (RSGA) algorithm [15], simulated annealing (SA) [16] differential evolution (DE) algorithm [17, 18], artificial immune algorithm [19], ant colony optimization [20] and artificial bee colony optimization [21]. In continuation to these works, algorithms like hybrid differential evolution with exploratory search [22], real coded genetic algorithms (RCGA) [23], gravitational search algorithm [24], integrated cat swarm and differential evolution (CSO-DE) [25], hybrid [26] and heuristic search method (HSM) [27] have been implemented for designing of optimal digital IIR filters. Many of the evolutionary optimization algorithms mentioned above exhibit problems such as local search stagnation, premature convergence, control parameters tuning and in many cases arrives at the same solution repeatedly [15, 28]. Therefore, it is required to put efforts in improving the existing evolutionary heuristic optimization techniques or for the development of new technique to overcome the various problems that exist in most of the algorithms used for designing the optimal digital filters.

In this paper, the BBO algorithm is introduced to design optimal digital IIR filters. BBO is inspired by the science of biogeography and operates by probabilistically sharing information between the individuals in the population of species representing the candidate solutions in the search space. Until now, BBO has shown quite good performance on the various well known benchmark functions [29] and has been applied to numerous real world applications [30-35]. The convergence speed of BBO is quite good but similar to other evolutionary algorithms it easily got stuck in the local minima and suffers from premature convergence.

This occurs because the exploration as well as the exploitation processes is equally essential and too much stress on exploitation leads to a pure local search, whereas too much stress on exploration leads to a pure random search. To remedy these limitations, many variants of BBO have been proposed [36-40] and applied to various problems. In the present work, instead of the basic migration operator, the polyphyletic migration operator is used in the BBO algorithm to increase its population diversity [41]. With the intention of starting with better solutions, the oppositional learning strategy is also included. In the design process, multivariable optimization is applied as a design measure which undertakes the design of optimal stable digital IIR filter at the same time satisfying the different performance prerequisites like minimizing the magnitude approximation error and ripple magnitude of the pass-band and the stop-band. The designing of LP and HP filters is independently carried by the proposed algorithm and for performance estimation the results are compared with some existing filter design techniques. The experimental results and comparisons demonstrate that BBO is distinctly suitable for designing optimal digital IIR filters.

The remainder of the paper is structured as follows: Section 2 describes the formulation of the problem statement for digital IIR filter design. The details and underlying mechanism of the BBO and OBBO algorithms are described in section 3. Section 4 explains in detail the steps of the OBBO algorithm for designing the digital IIR filters. The performance of the proposed method has been evaluated and the results obtained are compared with the design results given by some well established optimization algorithms like HGA [7], HTGA [8], TIA [9], HSM [27], RCGA [23], hybrid method [26], PPO [14], hybrid DE [22], and CSO-DE [25] in section 5. Finally, section 6 contains the concluding remarks and scope for future work.

II. DIGITAL IIR FILTER DESIGN PROBLEM

The design of digital IIR filter is usually realized by the following difference equation [3]:

$$y(n)=\sum_{i=0}^{M}x_i u(n-i)-\sum_{k=1}^{N}x_{N+k}\,y(n-k) \qquad (1)$$

where, M and N are the number of x_i and $xN+k$ filter coefficients, respectively, such that $N \ge M$. $u(n)$ and $y(n)$ are its input and output, respectively. An equivalent transfer function of digital IIR filter is expressed as follows:

$$H(z)=\frac{\sum_{i=0}^{M}x_i z^{-i}}{1+\sum_{k=1}^{N}x_{M+k} z^{-k}} \qquad (2)$$

For designing of digital IIR filter the values of the filter coefficients x_i and $xM+k$, which produce the desired response, are needed to be found out. In general the digital IIR filter is realized by cascading different first-order and second-order sections together. The transfer function of the cascaded digital IIR filter is denoted by $H(w, X)$, where X indicates the filter coefficients. The magnitude of $H(w, X)$ is denoted by $|H(w, X|$ and the basic structure of $H(w, X)$ can be stated as [2]:

$$H(w,X)=x_1\prod_{i=1}^{M}\left(\frac{1+x_i e^{-jw}}{1+x_{2i+1}e^{-jw}}\right)\times\prod_{k=1}^{N}\left(\frac{1+x_l e^{-jw}+x_{l+1}e^{-2jw}}{1+x_{l+3}e^{-jw}+x_{l+4}e^{-2jw}}\right) \qquad (3)$$

where, vector $X=[x_1 x_2 \ldots\ldots x_D]^T$ denotes the filter coefficients of dimension D×1, such that, $D=2M+4N+1$ and $l=2M+4(k-1)+2$.

In the IIR filter design process, the coefficients are optimized to minimize the approximation error function for the magnitude. The magnitude response is specified at K discrete and equally spaced frequency points in the pass-band as well as the stop-band. The absolute error is denoted by $e(X)$ and is stated below:

$$e(X)=\sum_{k=0}^{K}\left|H_d(w_k)-|H(w_k,X)|\right| \qquad (4)$$

where, $H_d(w_k)$ is the desired magnitude response of IIR filter and is given as:

$$H_d(w_k)=\begin{cases}1 & for \quad w_k \in passband\\ 0 & for \quad w_k \in stopband\end{cases} \qquad (5)$$

The ripple magnitudes of pass-band and stop-band are denoted by $\delta_p(x)$ and $\delta_s(x)$, respectively and are given as:

$$\delta_p(X)=\max_{w_k}\{|H(w_k,X)|\}-\min_{w_k}\{|H(w_k,X)|\};\ w_k \in passband \qquad (6)$$

$$\delta_s(X)=\max_{w_k}\{|H(w_k,X)|\};\ w_k \in stopband \qquad (7)$$

The design of stable digital IIR filter normally requires the inclusion of stability constraints. Therefore, the stability constraints found by using the Jury method [34] on the coefficients of the digital IIR filter stated in (9.1) - (9.5), are included in the optimization process. The multivariable constrained optimization problem is then stated as:

Minimize $e(X)$ (8)

Subject to the stability constraints:

$$1 + x_{2k+1} \geq 0 \quad (k = 1,2,\ldots,M) \tag{9.1}$$

$$1 - x_{2k+1} \geq 0 \quad (k = 1,2,\ldots,M) \tag{9.2}$$

$$1 - x_{l+3} \geq 0 \quad (l = 2M + 4(k-1) + 2, k - 1,2,\ldots N) \tag{9.3}$$

$$1 + x_{l+2} + x_{l+3} \geq 0 \quad (l = 2M + 4(k-1) + 2, k - 1,2,\ldots,N) \tag{9.4}$$

$$1 - x_{l+2} + x_{l+3} \geq 0 \quad (l = 2M + 4(k-1) + 2, k - 1,2,\ldots M) \tag{9.5}$$

A. Membership Function of Magnitude Response Error

The digital IIR filter designing mainly aims to minimize the magnitude response of the defined frequency band in which the frequency is either allowed to pass or attenuated. Owing to imprecise nature of designer's judgment, it is presumed that designer may have fuzzy goals for each objective functions. The fuzzy sets are defined by membership function. The membership function of magnitude response error for the pass-band frequency is given in Fig. 1 and is mathematically expressed as:

$$\mu_{pj}\left(H\left(e^{jw}\right)\right) = \begin{cases} 1 & ;\delta_p^{\,L} \leq \left|H\left(e^{jw}\right)\right| \leq \delta_p^{\,U} \\ \dfrac{\left|H\left(e^{jw}\right)\right| - \delta_p^{\,\min}}{\delta_p^{\,L} - \delta_p^{\,\min}} & ;\delta_p^{\,\min} < \left|H\left(e^{jw}\right)\right| < \delta_p^{\,L} \\ \dfrac{\delta_p^{\,\max} - \left|H\left(e^{jw}\right)\right|}{\delta_p^{\,\max} - \delta_p^{\,U}} & ;\delta_p^{\,U} < \left|H\left(e^{jw}\right)\right| < \delta_p^{\,\max} \; (j = 1,2,\ldots,A_n) \\ 0 & ;\delta_p^{\,\min} \leq \left|H\left(e^{jw}\right)\right| \leq \delta_p^{\,\max} \end{cases} \tag{10}$$

where, ω represents the pass-band frequency. $\delta_p^{\,L} = (1 - \delta_p)$ and $\delta_p^{\,U} = (1 + \delta_p)$ are tolerable limits acceptable for the magnitude in the pass-band. $\delta_p^{\,\min}$ and $\delta_p^{\,\max}$ are the minimum and maximum magnitude values for which the membership function is defined in the pass-band.

In the same way, the membership function of magnitude response error for the stop-band frequency is given in Fig. 2 and is mathematically expressed as:

$$\mu_{si}\left(H\left(e^{jw}\right)\right) = \begin{cases} 1 & ;\left|H\left(e^{jw}\right)\right| \leq \delta_s \\ \dfrac{\delta_s^{\,\max} - \left|H\left(e^{jw}\right)\right|}{\delta_s^{\,\max} - \delta_s} & ;\delta_s \leq \left|H\left(e^{jw}\right)\right| \leq \delta_s^{\,\max} \; (i = 1,2,\ldots,B_n) \\ 0 & ;\left|H\left(e^{jw}\right)\right| \geq \delta_s^{\,\max} \end{cases} \tag{11}$$

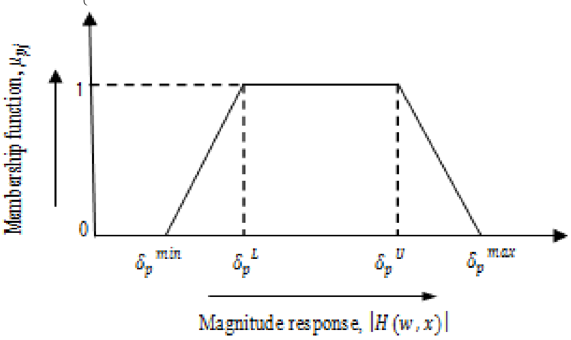


Fig. 1 Membership function for magnitude response in pass-band

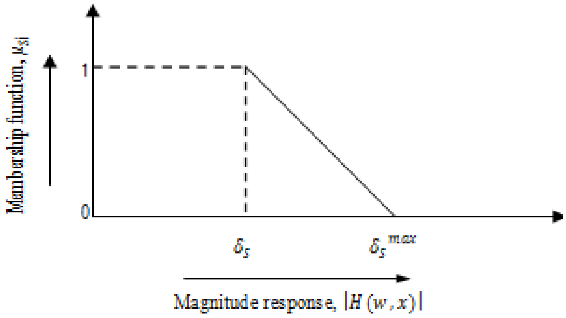


Fig. 2 Membership function for magnitude response in stop-band

where, ω represents the stop-band frequency. δ_s is the acceptable limit of the magnitude in the stop-band and $\delta_s^{\,\max}$ is the maximum magnitude value for which the membership function is defined in the stop-band.

Aggregating the membership functions, the objective is formulated for the magnitude response error in the pass-band and the stop-band and is mathematically expressed as:

$$f_1 = \frac{1}{A_n}\sum_{j=1}^{A_n}\mu_{pj}\left(H\left(e^{jw}\right)\right) + \frac{1}{B_n}\sum_{i=1}^{B_n}\mu_{si}\left(H\left(e^{jw}\right)\right) \tag{12}$$

where, A_n and B_n represents the sampling frequencies in the pass-band and stop-band, respectively.

B. Constraint Handling

To design the digital IIR filter, the stability constraints in (9.1) – (9.5), which are obtained by using the jury method on the coefficients of the digital IIR filter, are included in the optimization process. The cumulative membership function of constraints is considered as another objective to be maximized and is defined below.

$$f_2 = \frac{1}{2}\left\{\frac{1}{2}\left[\sum_{j=1}^{2}\frac{1}{M}\left(\sum_{i=1}^{M}\mu_{c_{ji}}\right)\right] + \frac{1}{3}\left[\sum_{j=3}^{5}\frac{1}{N}\left(\sum_{k=1}^{N}\mu_{c_{jk}}\right)\right]\right\} \tag{13}$$

where, the membership function for the first constraint is defined as:

$$\mu_{c_{1i}} = \begin{cases} 1 & ;1 + x_{1i+1} \geq 0; i = 1,2,\ldots,M \\ 0 & ;1 + x_{1i+1} < 0; i = 1,2,\ldots,M \end{cases} \tag{14}$$

And the membership function for the forth constraint is defined.

$$\mu_{c_{4k}} = \begin{cases} 1 & ;1 + x_{l+2} + x_{l+3} \geq 0; k = 1,2,\ldots N, l = 2M + 4(k-1) + 2 \\ 0 & ;otherwise, k = 1,2,\ldots N, l = 2M + 4(k-1) + 2 \end{cases} \tag{15}$$

Similarly, the membership functions for second, third and fifth constraint are defined.

The optimization problem is then redefined as below, whereby the cumulative membership function of constraints is considered as another objective to be maximized.

$$\text{Maximize } F = [f_1, f_2]^T \tag{16}$$

III. BIOGERGRAPHY BASED OPTIMIZATION

The BBO algorithm is a population based global optimization algorithm. It is based on the mathematical models of the natural phenomenon of biogeography, which deals with the distribution of species over time and space [44]. In this algorithm, each individual which corresponds to a candidate solution is called a "habitat" which is geographically isolated from other habitats. Each habitat has a performance index called the habitat suitability index (HSI) for measuring its goodness or suitability for living. BBO imitates the immigration (colonization) and emigration (extinction) of species between habitats in a multidimensional space. Habitats that are well suited for biological species are said to have high HSI and habitats that are less suited have a low HSI. HSI of all the habitats depends upon number of features like rainfall, environment temperature, diversity of vegetation, topography etc. All of these features that portray the habitability of the habitat are called the suitability index variables (SIVs) and are composed of D-dimensional real vector. The movement of species between the habitats is governed by two important parameters called the immigration rate (λ) and the emigration rate (μ) [45]. The rate at which the new species arrive in a habitat is known as the immigration rate and the rate at which the old species become extinct from the habitat is known as the emigration rate. These rates are functions of the number of species in the habitat. Habitats with smaller populations are more susceptible to extinction (i.e. the immigration rate is high). But as more species inhabit the habitat, the immigration rate reduces and the emigration rate increases. The greater the total number of species in the habitat, which corresponds to a high HSI, the better the solution it contains. In BBO, good solutions (i.e. habitats with many species) share their features with poor solutions (i.e. habitats with few species), and poor solutions accept a lot of new features from good solutions. The immigration and emigration rates when there are K species in the habitat can be calculated using the sinusoidal migration model [46, 47] and are given as follows:

$$\lambda_k = \frac{I}{2}\left(\cos\left(\frac{\pi K_k}{K_{\max}}\right)+1\right), \quad k = 1,2,...,NP \tag{17}$$

$$\mu_k = \frac{E}{2}\left(-\cos\left(\frac{\pi K_k}{K_{\max}}\right)+1\right), \quad k = 1,2,...,NP \tag{18}$$

where, I is the maximum possible immigration rate, which occurs when there are zero species on the island and E is the maximum possible emigration rate, which occurs when the island contains the maximum number of species, $K_{\max}$ that it can support. Fig. 3 shows the relationships between the fitness of habitats (a function of the number of species), emigration rate and immigration rate. The equilibrium point K_0 is reached at the point of intersection of immigration curve and the emigration curve. The operation of BBO algorithm is managed by two basic operators i.e., migration and mutation operators. The migration operation helps to share the information among the habitats and the mutation operation is used to increase the population diversity. These concepts are used in BBO to find good solution for a given optimization problem.

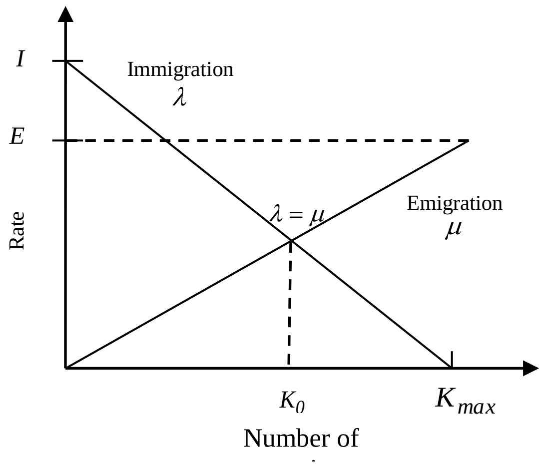


Fig. 3 Species model of a single habitat [48]

A. Population Initilaization

For applying the BBO algorithm to solve the optimization problems, the primary step is to decide the number of species that are to be used in the algorithm. Each species in the habitat has position made up of D-dimensions and a fitness value according to the fitness function. The position of specie represents the solution set and the fitness value represents the accommodation of the specie to the fitness function. For a D-dimensional optimization problem, a habitat is an $1 \times D$ array. The population consists of NP parameters vector $X_k, k = 1,2,..., NP$. Within the solution search space, the initial population containing the candidate solutions is initialized as follows:

$$X_{kd}^{\ t} = X_d^{\min} + R\left(X_d^{\max} - X_d^{\min}\right) \quad (k = 1,2,..., NP; d = 1,2,..., D) \tag{19}$$

where, $X_{kd}^{\ t}$ represents the position of the k^{th} specie in d^{th} dimension. $X_d^{\max}$ and $X_d^{\min}$ are the lower and the upper bounds of the k^{th} specie and R represents a uniform random number between 0 and 1. To get the feasible solutions, the initially generated population must satisfy the equality and inequality constraints. Therefore, the random perturbation method is applied to check if there occur any violations of the stability constraints.

B. Opposition Based Learning

The BBO optimization methods start with some initial random solutions that are improved by moving towards optimal solution. The computation time, among others, is related to the distance of these initial guesses from the optimal solution. It can be improved by the chance of starting with a better solution by simultaneously checking the opposite solution in the search space. The guess or its opposite guess has been chosen as an initial solution. A guess is farther from the solution than its opposite guess with 50% probability [49]. Therefore, starting with better guesses adjudged by its objective function has the potential to accelerate convergence. The same approach can be applied not only to initial solutions but also continuously to each solution in the current population, during the run.

$$X^{t}_{k+NP,d} = X_d^{\,lower} + X_d^{\,upper} - X_{kd}^{\ t} \quad (k = 1,2,..., NP; d = 1,2,..., D) \tag{20}$$

where, X_d^{lower} and X_d^{upper} are lower and upper limits of filter coefficients and are expressed as:

$$X_d^{lower} = \begin{cases} X_d^{\min} & ;t=1 \\ \min\{X_{kd}; k=1,2,...,NP\} & ;t>1 \end{cases} \quad (21)$$

$$X_d^{upper} = \begin{cases} X_d^{\max} & ;t=1 \\ \max\{X_{kd}; k=1,2,...,NP\} & ;t>1 \end{cases} \quad (22)$$

C. Fitness Evaluation

To solve the digital IIR filter design problem by employing the multi-objective optimization, the objective function is changed to the following generalized form:

$$F = \max\{\min\{f_{k1}, f_{k2}\}; k=1,2,\cdots,NP\} \quad (23)$$

D. Migration

In BBO, the migration operation which deals with the immigration and emigration rates, facilitates the sharing of information among the different habitats in the solution search space and modifies the selected habitat's SIVs and can be expressed as:

$$X_{kd} = X_{ed} \quad (24)$$

where, X_{kd} and X_{ed} represent the immigration and the emigration habitat, respectively.

This basic migration operation exhibit simple exploitation ability by allowing the sharing of information between the habitats and the new solutions are produced simply by copying the features from a constant pool. This results in a lack of the exploration ability of OBBO to produce new solutions from new areas in the search space. To overcome this drawback and to balance the exploitation and exploration features, an improved migration operator called the polyphyletic migration, is used in OBBO [41]. The basic information sharing between habitat and habitat's SIVs modification operation can be explained by the following expression:

$$v_{kd} = X_{ed} + \phi_{kd}(X_{ed} - X_{rd}) \quad ; k \neq e \neq r \in \{1,2,..., NP\} \quad (25)$$

where, v_{kd} is the candidate solution and X_e represent the emigration habitat. X_r is a random habitat that satisfies $r \neq k \neq e$ and ϕ_{kd} is a uniform random number distributed between the range (-1,1). The polyphyletic migration operation is illustrated in Algorithm 1.

Algorithm 1: Polyphyletic migration operation

```
For a target habitat X_k
for k=1 to NP do
   if rand(0,1) < λ_k then
      Select habitat X_e with respect to the immigration rate, μ_e
      if rand(0,1) < μ_e then
         Randomly select r ≠ k ≠ e
         v_kd = X_ed + φ_kd (X_ed − X_rd)
   else
         Randomly select s ≠ k
         v_kd = X_sd
   end if
else
   v_kd = X_kd
end if
end for
```

The polyphyletic migration operator has three advantages. First, it directly copy information from habitat and uses the fairly good habitat as the base to produce a symmetrical perturbation. So, it fetches more new information from the unexploited feasible space. Second, it ensures the emigration of new features from other habitats. The third advantage is that the emigration habitat principally focuses on the exploitation while the other two habitats which are randomly selected from the current population mainly emphasize the exploration. These three types of habitats help to attain a good balance between exploitation and exploration processes.

E. Mutation

In BBO, the mutation operator is used to increase the population diversity, which helps to lessen the chances of trapping in the local minima [36]. The immigration and emigration processes can be represented mathematically by a probabilistic model. Consider the probability, P_k that a habitat contains K species. P_k changes from time t to time $t+\Delta t$ as follows:

$$P_k(t+\Delta t) = P_k(t)(1-\lambda_k \Delta t - \mu_k \Delta t) + P_{k-1}\lambda_{k-1}\Delta t + P_{k+1}\mu_{k+1}\Delta t \quad (26)$$

where, λ_k and μ_k are the immigration and emigration rates when there are K species in the habitat. This equation holds because in order to have K species at time $t+\Delta t$, one of the following conditions must hold:

a) There were *K* species at time *t*, and no immigration or emigration occurred between *t* and $t+\Delta t$.

b) There were $(K-1)$ species at time *t*, and single specie has immigrated.

c) There were $(K+1)$ species at time *t*, and single specie has emigrated.

If time Δt is small enough so that the probability of more than one immigration or emigration can be ignored, then taking the limits of the Eqn. (24) as $\Delta t \to 0$ gives the following equation:

$$\dot{P}_k = \begin{cases} -(\lambda_k + \mu_k)P_k + \mu_{k+1}P_{k+1}, & K = 0 \\ -(\lambda_k + \mu_k)P_k + \lambda_{k-1}P_{ks-1} + \mu_{k+1}P_{k+1}, & 1 \le K \le K_{\max} - 1 \\ -(\lambda_k + \mu_k)P_k + \mu_{k-1}P_{k-1}, & K = K_{\max} \end{cases} \quad (27)$$

where, λ_{k-1} is the immigration rate for one less than the species count of habitat i and μ_{k+1} is the emigration rate for one more than the species count of habitat k and are mathematically expressed as:

$$\lambda_{k-1} = \lambda * \left(1 - \frac{(spc-1)}{NP}\right) \quad (28)$$

$$\mu_{k+1} = \mu\left(\frac{(spc+1)}{NP}\right) \quad (29)$$

where, spc represents the species count of the habitat k and NP represents the maximum number of species in the habitat.

The probability is then updated as:

$$P_k = P_k + \dot{P}_k \times \Delta t \quad (30)$$

$$P_k = \frac{P_k}{\sum_{k=0}^{NP} P_k} \quad (31)$$

In BBO, the mutation rate, m_{rate} of a SIV in the habitat X_k is selected to be replaced by a randomly generated SIV according to a probability of existence P_k. The mutation rate, m_{rate} can be expressed as follows:

$$m_{rate} = P_{mute}\left(1 - \frac{P_k}{P_{\max}}\right) \quad (32)$$

where, P_{mute} is a user defined parameter called the initial mutation probability and $P_{\max} = \max\{P_k\}$. The mutation operator is described in Algorithm 2.

F. Stopping Criterion

OBBO is population based optimization technique and its stopping criterion is same as all other population based algorithms. Usually the maximum number of iterations (generations) is used as the stopping criteria. After completion of the generations the diversity of the solution is verified. If the obtained solution is found to be reliable, then the program is terminated otherwise the generation number is incremented as $(g = g + 1)$ and the whole procedure is repeated until an optimal solution to the problem is obtained.

Algorithm 2: Mutation operation

for $k=1$ to NP **do**
  Compute the probability P_k and time derivative of P_k i.e. $\dot{P}_k$ using Eqn. (26) and Eqn. (27), respectively.
  Update the probability using Eqn. (30) and Eqn. (31)
  Compute the mutation rate, m_{rate} using Eqn. (32).
  Select X_{kd} with respect to P_k
  if $rand(0,1) > m_{rate}$ **then**
    Replace X_{kd} with a random SIV
  end if
end for

IV. IMPLEMENTATION OF OBBO FOR DIGITAL IIR FILTER DESIGN

For the designing of the digital IIR filters the OBBO algorithm is implemented with the incorporation of the polyphyletic migration operator instead of the basic migration operator together with the mutation operator. The inclusion of the polyphyletic migration operator not only increases the population diversity but also increases the exploitation and the exploration abilities of OBBO. Here, OBBO tries to have an optimal IIR filter structure while satisfying the stability constraints that are imposed during the designing and considers the minimization of magnitude approximation error and ripple magnitudes of both the pass-band and the stop-band. The implementation of OBBO algorithm for digital IIR filter design is explained step by step as follows:

Algorithm 3: The main procedure of OBBO algorithm

Initialize the BBO parameters viz. Population size (NP), Maximum immigration rate (I), Maximum emigration rate (E), Mutation probability (P_{mute}), Maximum number of iterations ($MGEN$)
Initialize the population or the random set of habitats, X_{kd} using Eqn. (19). Each habitat corresponds to a candidate solution to the optimization problem.
Evaluate the fitness i.e. the HSI for each habitat.
Apply opposition and again evaluate the fitness.
Sort the population from best to worst and keep the best habitats.
Initialize the generation counter, $g = 1$ and improve=1
while the stopping criteria is not met **do**
  Sort the entire population from best to worst.
  Map the HSI to the number of species.
    Calculate the immigration rate, λ and emigration rate, μ for each specie count using Eqn. (17) and Eqn. (18), respectively.
    Compute the rate of change of probabilities using Eqn. (32).
    Compute the probabilities for each species count and use λ and μ to modify the habitats.
    For migration operation apply the Algorithm 1 on the population.
    Evaluate the fitness of newly obtained migrated population i.e. the HSI for each habitat.
    Perform the Algorithm 2 on the population.
    Sort them in ascending order and keep the best habitats.
  Ensure the population does not have duplicates.
  Procure the global best habitat (solution).
  if (global best is not improved) **then**
    Improve=improve+1
  else
    Improve=1
  if mod(improve, trial) **then**
    Apply opposition and select best NP habitats
  endif
  $g = g + 1$
end do

A. Low Order Digital IIR Filter Design

For the designing of cascaded digital IIR filters and evaluation of filter coefficients, the OBBO algorithm has been employed. The designing of LP and HP filters have been undertaken and 200 equally spaced points are set within the frequency domain [0, π]. The objective of the optimization problem is to minimize the magnitude approximation error and ripple magnitudes of both the pass-band and the stop-band, subject to the stability constraints given by (9.1) - (9.5) under the prescribed design conditions stated in Table I. So, (16) is maximized that maximizes the membership function of magnitude error and satisfaction level of constraints. For the purpose of comparison the lowest order of the IIR filter is set exactly the same as given in [7] i.e. 3 for both LP and HP filters.

The control parameters settings for OBBO are listed in Table II. The final filter models obtained for the LP and HP filters are given in (33)-(36), respectively. The magnitude responses and pole-zero plots for the LP and HP filters are represented in Fig. 4 and Fig. 5, respectively. The best results obtained by implementing OBBO for the filters are summarized in Table III and Table IV, where the comparison of the obtained results is carried out with the design results given by other methods like HGA [7], HTGA [8], TIA [9], HSM [27], RCGA [23], hybrid method [26], PPO [14], hybrid DE [22] and CSO-DE [25]. From Table III and Table IV, it can be concluded that for all the filter types, OBBO algorithm outperform all other algorithms in terms of providing a lower magnitude approximation error except in the case of BP filter, where the CSO-DE method listed in [24] provides a further lower value of the magnitude approximation error. And in terms of pass-band performance and stop-band performance also, OBBO is capable of providing smaller values of the ripple magnitudes and surpasses all other algorithms for all the cases except in the case of HP filter, where the heuristic method [27], shows a better pass-band performance.

TABLE I. PRESCRIBED DESIGN CONDITIONS FOR LP AND HP FILTERS

Filter Type	Maximum value of $\lvert H(w_i, x)\rvert$	Pass band	Stop band
LP	1	$0 \le w \le 0.2\pi$	$0.3\pi \le w \le \pi$
HP	1	$0.8\pi \le w \le \pi$	$0 \le w \le 0.7\pi$

TABLE II. VALUES OF CONTROL PARAMETERS FOR LP AND HP FILTERS

Parameters	Notation	LP	HP
Population size	NP	100	100
Maximum number of iterations	$MGEN$	500	500
Maximum immigration rate	I	1.0	1.0
Maximum emigration rate	E	1.0	1.0
Step size	Δt	1.0	1.0
Top best habitat	$KEEP$	4	3
Mutation probability	P_{mute}	0.01	0.01
Lower bound of immigration probability	λ^{lower}	0.0	0.0
Upper bound of immigration probability	λ^{upper}	1.0	1.0

TABLE III. DESIGN RESULTS FOR LP FILTER

Method	Magnitude error	Pass-band performance	Stop-band performance
HGA [7]	4.3395	$0.8870 \le \lvert H(e^{iw})\rvert \le 1.0090$ (0.1139)	$\lvert H(e^{iw})\rvert \le 0.1802$ (0.1802)
HTGA [8]	4.2511	$0.9004 \le \lvert H(e^{iw})\rvert \le 1.0000$ (0.0996)	$\lvert H(e^{iw})\rvert \le 0.1247$ (0.1247)
TIA [9]	4.2162	$0.9012 \le \lvert H(e^{iw})\rvert \le 1.0000$ (0.0988)	$\lvert H(e^{iw})\rvert \le 0.1243$ (0.1243)
HSM [27]	4.1145	$0.9246 \le \lvert H(e^{iw})\rvert \le 1.0110$ (0.0864)	$\lvert H(e^{iw})\rvert \le 0.1238$ (0.1238)
RCGA [23]	4.0095	$0.9335 \le \lvert H(e^{iw})\rvert \le 1.0160$ (0.0825)	$\lvert H(e^{iw})\rvert \le 0.1510$ (0.1510)
HYBRID [26]	3.7903	$0.9283 \le \lvert H(e^{iw})\rvert \le 1.0260$ (0.0977)	$\lvert H(e^{iw})\rvert \le 0.1405$ (0.1405)
PPO [14]	3.6611	$0.9178 \le \lvert H(e^{iw})\rvert \le 1.0000$ (0.0822)	$\lvert H(e^{iw})\rvert \le 0.1611$ (0.1611)
Hybrid DE [22]	3.5014	$0.8838 \le \lvert H(e^{iw})\rvert \le 1.0190$ (0.1352)	$\lvert H(e^{iw})\rvert \le 0.1505$ (0.1505)
CSO-DE [25]	3.4678	$0.8455 \le \lvert H(e^{iw})\rvert \le 1.0400$ (0.1945)	$\lvert H(e^{iw})\rvert \le 0.1129$ (0.1129)
OBBO	**3.4343**	$\mathbf{0.9511 \le \lvert H(e^{iw})\rvert \le 1.0328}$ **(0.0817)**	$\mathbf{\lvert H(e^{iw})\rvert \le 0.1118}$ **(0.1118)**

TABLE IV. DESIGN RESULTS FOR HP FILTER

Method	Magnitude error	Pass-band performance	Stop-band performance
HGA [7]	14.5078	$0.9224 \le \lvert H(e^{iw})\rvert \le 1.0030$ (0.0806)	$\lvert H(e^{iw})\rvert \le 0.1819$ (0.1189)
HTGA [8]	4.8372	$0.9460 \le \lvert H(e^{iw})\rvert \le 1.0000$ (0.0540)	$\lvert H(e^{iw})\rvert \le 0.1457$ (0.1457)
TIA [9]	4.7144	$0.9467 \le \lvert H(e^{iw})\rvert \le 1.0000$ (0.0533)	$\lvert H(e^{iw})\rvert \le 0.1457$ (0.1457)
HSM [27]	4.6635	$0.9584 \le \lvert H(e^{iw})\rvert \le 1.0080$ (0.0496)	$\lvert H(e^{iw})\rvert \le 0.1477$ (0.1477)
RCGA [23]	4.5296	$0.9677 \le \lvert H(e^{iw})\rvert \le 1.0186$ (0.0509)	$\lvert H(e^{iw})\rvert \le 0.1540$ (0.1540)
HYBRID [26]	3.9724	$0.9625 \le \lvert H(e^{iw})\rvert \le 1.0265$ (0.0640)	$\lvert H(e^{iw})\rvert \le 0.1536$ (0.1536)
PPO [14]	3.9332	$0.9401 \le \lvert H(e^{iw})\rvert \le 1.0010$ (0.0609)	$\lvert H(e^{iw})\rvert \le 0.1692$ (0.1692)
Hybrid DE [22]	2.8960	$0.8955 \le \lvert H(e^{iw})\rvert \le 1.0140$ (0.1185)	$\lvert H(e^{iw})\rvert \le 0.1100$ (0.1100)
CSO-DE [25]	2.7119	$0.9396 \le \lvert H(e^{iw})\rvert \le 1.0090$ (0.0694)	$\lvert H(e^{iw})\rvert \le 0.1567$ (0.1567)
OBBO	**2.6762**	$\mathbf{0.9640 \le \lvert H(e^{iw})\rvert \le 1.0145}$ **(0.0505)**	$\mathbf{\lvert H(e^{iw})\rvert \le 0.0923}$ **(0.0923)**

$$H_{LP(z)} = 0.0319\frac{(z+0.7405)(z^2-0.3104z+0.9546)}{(z-0.6991)(z^2-1.4639z+0.7572)} \quad (33)$$

$$H_{HP(z)} = 0.0313\frac{(z-1.2999)(z^2+0.5992z+0.9603)}{(z+0.6715)(z^2+1.4570z+0.7505)} \quad (34)$$

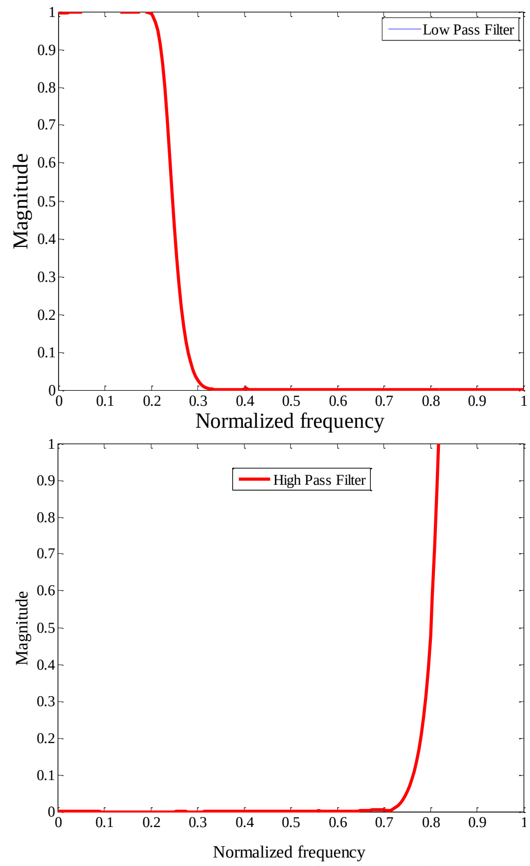
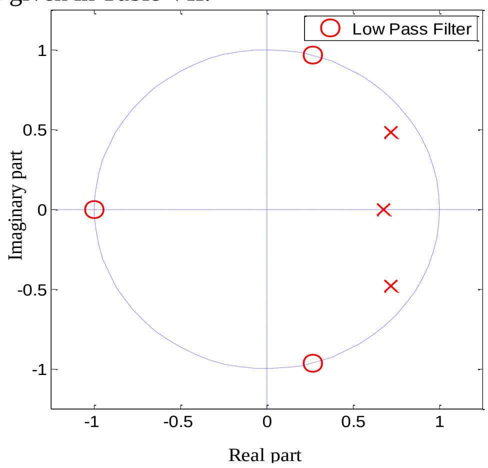


Fig. 4 Magnitude responses of LP and HP filters

B. *Higher order digital IIR filter design*

The developed OBBO algorithm has also been successfully implemented for the higher order digital IIR filter design problem. Similar to the lower order design, 200 equally spaced points are set within the frequency domain [0, π]. The order of the digital IIR filter is given as M+2N; where M and N denotes the number of filter coefficients. Here, the objective of the optimization problem is to maximize the absolute error of magnitude response of the filter subject to the stability constraints given by (9.1) - (9.5) under the prescribed design conditions given in Table I. For the LP filter, the maximum number of iterations is kept as 500 and the population size of 150 habitats is considered with the maximum immigration and maximum emigration rate of 1.0 each. The value of top 5 habitats is kept during the optimization process and the mutation probability and step size are set as 0.01 and 1.0, respectively. The proposed algorithm shows the capability to design a stable LP filter with values of M and N equal to 3 and 4, respectively i.e. the order of the filter is 11. This designed filter with an order of 11 showed better magnitude approximation error over all other orders. The magnitude approximation error and the pass-band and stop-band ripple values for the higher order LP filter are summed up in Table V and the values of the best optimized numerator and denominator coefficients are shown in Table VI.

A similar approach has been followed for designing the higher order digital IIR HP filter. The maximum value of order for the digital IIR HP filter for which the implemented algorithm shows competitive results is given in Table V. The magnitude approximation error and the pass-band and stop-band ripple values for these filters are also summed up in Table V and the best optimized coefficient values for the HP filter are given in Table VII.



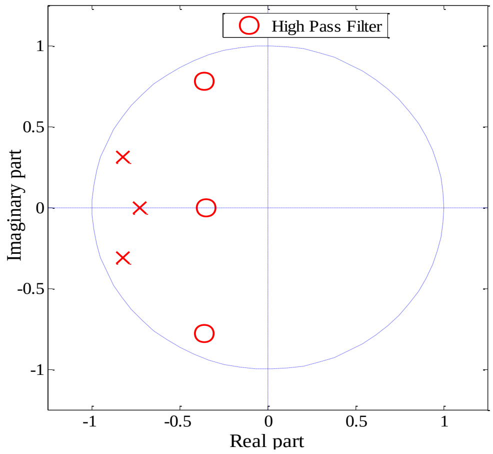


Fig. 5 Pole-zero plots of LP and HP filters

TABLE V. DESIGN RESULTS FOR HIGHER ORDER LP AND HP FILTERS

Filter	Magnitude Error	Pass band ripples	Stop band ripples	Filter order
LP	2.3943	0.9461≤|H(e)|≤ 1.0281 (0.0820)	|H(e)|≤0.0732 (0.0732)	11
HP	2.0301	0.9341≤|H(e)|≤ 1.0087 (0.0746)	|H(e)|≤0.0365 (0.0314)	10

TABLE VI. COEFFICIENTS OF HIGHER ORDER DIGITAL IIR LP FILTER MODEL

i	a_i	b_i	p_i	q_i	r_i	s_i
1	0.1675	-0.3507	-0.7151	0.4411	-0.9245	0.5389
2	0.1368	-0.3456	-0.6154	0.7376	-0.8187	0.4144
3	0.1817	-0.3763	-0.5253	0.6446	-1.0501	0.5276
4			-0.7117	0.5657	-0.7887	0.4098

TABLE VII. COEFFICIENTS OF HIGHER ORDER DIGITAL IIR HP FILTER MODEL

i	a_i	b_i	p_i	q_i	r_i	s_i
1	-0.5253	-0.3763	0.4144	0.6467	0.6467	0.0176
2	0.0176	0.4144	0.0176	0.6446	0.0176	0.0176
3	0.6446	0.6466	0.0176	0.6446	0.0176	0.0176
4	0.0176	-0.3763				

C. Robustness of the designed algorithm

Like other population based stochastic algorithms, in OBBO population is also initialized randomly. So, randomness is an inherent feature of OBBO. Therefore, in order to check the robustness of OBBO algorithm to achieve global optimum design solution for order 3 LP and order 3 HP filter design, 100 independent trial runs have been given with random seed numbers for each case and the variations in the magnitude response has been observed. The maximum value, minimum value, average value and standard deviation in magnitude approximation error are given in Table VIII. From the results, it can be observed that for each case, the value of standard deviation is very small which indicates the robustness of OBBO algorithm.

A similar approach has been followed to check the robustness of OBBO algorithm for the design of higher order digital IIR LP and HP filters and the maximum value, minimum value, average value and standard deviation of the magnitude response error are given in Table IX. The results obtained depict the value of standard deviation is very less in this case too, which proves the robustness of OBBO algorithm again.

TABLE VIII. MAXIMUM, MINIMUM, AVERAGE AND STANDARD DEVIATION OF MAGNITUDE ERROR FOR LOWER ORDER FILTERS

Filter	Maximum magnitude Error	Minimum magnitude Error	Average magnitude Error	Standard Deviation
LP	3.7145	3.4351	3.6201	0.0079
HP	3.0933	2.6795	2.8914	0.0532

TABLE IX. MAXIMUM, MINIMUM, AVERAGE AND STANDARD DEVIATION OF MAGNITUDE ERROR FOR HIGHER ORDER FILTERS

Filter	Maximum magnitude Error	Minimum magnitude Error	Average magnitude Error	Standard Deviation
LP	4.1124	2.3444	3.9762	0.1281
HP	5.4603	2.0386	4.1724	0.1977

D. Statistical Analysis

For further validation of the results obtained by implementing OBBO for IIR filter design, the Wilcoxon's signed rank test has been applied. This test performs a pairwise comparison between all the results obtained by using OBBO algorithm and the results of other existing algorithms. The Wilcoxon's signed rank test (Table X) depicts that OBBO algorithm shows a significant improvement over HGA, HTGA, TIA, HSM, RCGA, hybrid method, PPO, hybrid DE and CSO-DE algorithms with a significant level of α=0.05 and facilitates the designing of not only stable but optimal digital IIR LP and HP filters.

TABLE X. STATISTICAL ANALYSIS RESULTS BASED ON WILCOXON'S SIGNED RANK TEST

Filter	Performance	R^+	R^-	*p*-value
LP	Magnitude approximation error	55	0	0.005062
	Pass-band performance	55	0	0.005062
	Stop-band performance	55	0	0.005062
HP	Magnitude approximation error	55	0	0.005062
	Pass-band performance	53	2	0.009344
	Stop-band performance	55	0	0.005062

V. CONCLUSION

Till-date, a lot of work has been carried out for the designing of digital IIR filters. But, most of the population based optimization algorithms face difficulties like search stagnation, premature or slow convergence etc. and are not capable of providing promising results. This paper implements a revised algorithm i.e. the OBBO algorithm, for the robust and optimal design of digital IIR LP and HP filters. To increase the population diversity and to remedy the problems like premature convergence, trapping in the local minima etc., the polyphyletic migration operator and oppositional learning strategy are incorporated in the basic BBO algorithm. Then, the OBBO algorithm is implemented for optimal digital IIR filter design. The constraints are taken care of by considering them as another objective function to be optimized. The performance estimation of OBBO algorithm is carried out by comparing the obtained results with other well known algorithms. From the results obtained, it is clear that under prescribed design conditions, the OBBO algorithm is very much feasible and outperforms other well known algorithms for the designing of digital IIR filters of low as well as high orders. Further, the OBBO algorithm, allows each filter to be designed independently. Parameter tuning is still a potential area for further research. The OBBO algorithm possesses a quite good exploitation and exploration abilities to search for the optimal solution locally as well as globally.

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