Impedance Characteristics H-plane Tee Junction using L band Wave Guide

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Abstract - In all radar and communication applications antennas play an important role, for transmitting and receiving purposes. For certain special applications, it is essential to radiate with desired polarization. Wave guide junction radiators are preferred for this purpose. In H-Plane Tee junctions, the Tee arm is commonly coupled to the main wave guide by a longitudinal slot. The analysis of such structures is reported in the literature. However, the coupling can be done by inclined slot in the narrow wall of main wave guide. This structure acts as a radiator to produce vertically polarized waves with polarization limits. The knowledge of admittance characteristics of this new coupling system provides additional design parameters for the array designer.

In the present work, the analysis is made to obtain variation of conductance, susceptance, coupling and VSWR as a function of frequency after determining the resonant slot length of L band H-plane Tee junction wave guide. The results are numerically computed for varied slot width and slot inclinations. The concepts of self-reaction and discontinuity in modal currents of the main guide as well as Tee arm are used in the analysis. The data presented are extremely useful for the design of small and large arrays of L band H-plane Tee junction radiators, which are more suitable in navigations, GSM mobile phones, and in military applications. They are also used to measure the soil moisture of rain in forests.

Key words: Admittance, wave guide junction, shunt Tee, H-plane Tee

1. INTRODUCTION:

Basically the H-Plane Tee junction is a three port device. The main guide containing two ports and the coupled arm contains third port. The main wave guide is in shunt with the coupled arm. In power division applications Shunt Tees are usually preferred, to divide the power equally into two main ports when fed through shunt port. In the present work H-Plane Tee junctions are used as radiators with vertical polarization. For this purpose, the power is fed at the input port of main guide with the corresponding output port matched terminated. The power is radiated through the coupled arm. The Tee arm is coupled to the main guide usually by a longitudinal slot. However, the coupling can be made by inclined slot in the narrow wall of main guide. This structure is also useful to produce vertically polarized waves. For the array designer additional design parameter will be provided by this coupling system i.e. waveguide dimensions and slot dimensions. Literature on Longitudinal slot coupled Shunt Tee wave guides is available, but no one reported on inclined slot coupled wave guide Shunt Tee. The rectangular waveguides are used due to their compact size and space considerations. Radiation pattern will be distorted in case of open ended slot arrays because of mutual coupling exists between the slots. In array applications, cross polarized components can be suppress by Slot coupled Shunt Tees which in turn reduces mutual coupling between slots.

The analysis of different slots is presented by many researchers [1-4]. Results on studies of impedance characteristics of slots are reported. Raju and Das have reported how To obtain a desired radiation pattern for a wave guide array by suppressing cross polarization [5] and to reduce mutual coupling between the slots[6]. Pandharipande et al [7] derived an expression for the equivalent network of long axial slot in the case of H-plane T junction coupled through longitudinal slot in the narrow wall of primary wave guide. Oliner[8] presented impedance properties of different types of slots using equivalent circuit and variational method. The results include with thickness and without thickness. Marcuvitz[9] has developed concept that Discontinuities in Waveguides walls produce fields. Discontinuity Electric and Magnetic Fields equivalent represents Discontinuity in modal Currents. Hsu. [10] obtained some admittance properties of the inclined slots in the narrow wall and investigated on the possible resonant length. Raju [11] has reported on variation of resonant length as a function of slot width and Admittance of inclined Slots in narrow wall of rectangular waveguide that are sufficiently wide as a function of frequency. Very useful investigations on slot coupled waveguide junctions and slot radiators carried out by Watson [12]. The coupled slots are either in the narrow wall or broad wall of a rectangular waveguide. Das [13] derived an equivalent circuit for waveguide T-junction using variational technique considering the slot thickness. Raju[14] and Das [15]have obtained admittance characteristics and resonant length of inclined slots in the narrow wall of a rectangular waveguide by using self-reaction and discontinuity in modal current approach. The variation of resonant length as a function of inclination of the slot is given using variational analysis as well as method of moments. Cheng-Geng jan [16] has reported the analysis of side wall inclined slots using method of moment technique.
2. ANALYSIS FOR ADMITTANCE CHARACTERISTICS:

It is well known that a vertical slot in narrow wall of rectangular waveguide does not radiate. The electric field in such a slot is horizontally directed. But in applications where vertically polarized fields are required from inclined slots, it is possible to obtain them by coupling the slot into shunt Tee arm forming a Shunt Tee. In the present paper, the admittance characteristics of inclined slot in narrow wall of L band Shunt Tee is determined from self-reaction and discontinuity in modal current [8]. The analysis consists of two parts: first part consists of evaluation of self-reaction for the feed guide. This in turn consists of evaluation of self-reaction of horizontal and vertical components of the magnetic current. The second part consists of evaluation of self-reaction for the Tee arm.

In the present work, the analysis is carried out to obtain variation of slot conductance and susceptance as a function of resonant slot length. The result is numerically obtained for varied slot widths and slot inclination. Consider a L band waveguide shunt Tee coupled through an inclined slot of length 2L and width 2w, on the narrow wall as shown in Fig.1.

The analysis for admittance characteristics is obtained using self-reaction and discontinuity in modal current. The admittance characteristics in the coupled waveguide radiator are evaluated using TE and TM mode field concepts. In the present work the equivalent network parameter is obtained [14]. It is assumed that slot is inclined at an angle θ from the vertical axis and coupling takes place through inclined slot in narrow wall of the primary feed waveguide.

As shown in fig (1) a and b are narrow wall and broad wall dimensions of primary and secondary rectangular waveguide. An inclined slot in the narrow wall of coupled junction of two different standard waveguides with slot length 2L and width 2W .θ is the angle of inclination of slot from vertical axis. The slots admittance characteristics are analyzed using self-reaction and discontinuity in modal current. Using TE and TM mode field concepts, slot radiators are analyzed.

3. FORMULATION:

3.1 Self-reaction equations in H plane Tee junction coupled through inclined slot:

The Electric field in aperture plane of slot is replaced by an equivalent magnetic current. The total self-reaction <a,a>Τ of this magnetic current, with magnetic Fields produced by This Magnetic currents. The admittance seen by primary guide can be expressed as

\[ Y_T = -\frac{(l_1 l_3)}{<a,a>Τ} \]  

where l_2 is discontinuity in modal current. – (1)

Expression for self-reaction is given by [3]

\[ <a,a>Τ = -\int \bar{H}_S \bar{M}_S dv \]  

where \( \bar{H}_S \) is magnetic field and \( \bar{M}_S \) is magnetic current. V is the coupled volume.

The equivalent network parameter is given by [9] the expression of the form [5]. In present work Self-reaction \( <a,a>Τ \) is determined separately for the two guides. The self-reaction \( (a,a)_h \) in primary guide is longitudinal component of magnetic current, the self-reaction \( (a,a)_v \) in primary guide is transverse component of magnetic current, the self-reaction \( (a,a)_s \) in secondary guide, obtained from the modal expansion of the magnetic field in the coupled guide, is given by [14]. The shunt impedance loading on the primary guide due to the slot coupled shunt Tee can be expressed as the total self-reaction is equal to the sum of self-reactance\( (a,a)_h, (a,a)_v \) and \( (a,a)_s \). Hence, the equivalent network parameter will be

\[ <a,a>Τ = (a,a)_h + (a,a)_v + (a,a)_s \]

The expression for shunt impedance loading on the primary guide due to slot coupled matched terminated Tee arm will be

\[ Z_T = -\frac{(a,a)_h}{l_1 l_2} - \frac{(a,a)_v}{l_1 l_2} - \frac{(a,a)_s}{l_1 l_2} \]  

\[ Z_T = Z_1 + Z_2 + Z_3 \]  

3.1.1 Self-reaction due to longitudinal component of magnetic current in primary wave guide \( (a,a)_h \):

The Electric field \( \bar{E}_S \) in aperture plane of slot of fig 1 is related to equivalent magnetic Current \( \bar{M}_S \) by the relation

\[ \bar{M}_S = \bar{E}_S X \bar{a}_n \]  

\[ \bar{a}_n \] (This work is licensed under a Creative Commons Attribution 4.0 International License.)
where $\hat{a}_n$ is unit vector normal to the aperture plane.

The field distribution in the slot is assumed to be of form given by [6]

$$E_S = \hat{a}_x E_m \sin(k(L - |z'|)) \quad (5)$$

for $$\frac{a}{2} - W \leq |x'| \leq \frac{a}{2} + W$$ and $$-L \leq |z'| \leq L$$

where $E_m$ is maximum Electric field, $\hat{a}_x$ is unit vector along $x$ direction and $K=2\pi/\lambda$. $\lambda$ is wave length. $a$ and $b$ are narrow wall and broad wall dimensions of feed guide.

$$L_t = L \cos \theta, W_t = W \cos \theta$$ with respect to $x$-component of magnetic current.

Corresponding magnetic current is $\overline{M}_S$

$$\overline{M}_S = E_S \times \overline{a}_n \quad (8)$$

The magnetic current is along $x$-direction in present case

$$\overline{M}_S = \overline{a}_n \times \overline{a}_x E_m \sin(k(L - |x'|))$$

for $$-W_t \leq |x'| \leq W_t$$ and $$\frac{a}{2} - L_t \leq |x'| \leq \frac{a}{2} + L_t$$

By using self-reaction expressions given by [3]

$$<a,a> = - \iint \overline{H} \cdot \overline{M} \, dv$$

As the magnetic current distributed over the surface, the volume integral reduces to surface integral

$$<a,a> = - \int \overline{H}_S \cdot \overline{M}_S \, ds \quad \ldots \ldots \ldots (9)$$

By integrating and simplifying the expression for Self-reaction given by [9]

$$(a,a)_S = \frac{\lambda L_t}{24kW} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{E_m E_n \sin(n \pi L)}{\alpha} \int \frac{w}{2n} \frac{\sin(kz \sin \theta)}{\sin(kz \sin \theta)} \left(2 \cos(kz \sin \theta) - \cos(kz \sin \theta) - \frac{k^2}{k^2 - \frac{n^2}{2}} \right) \left(2 \cos(kz \sin \theta) - \cos(kz \sin \theta) - \frac{k^2}{k^2 - \frac{n^2}{2}} \right) \left(\cos \alpha - \frac{\cos^2 \alpha}{a} \right)$$

It should be noted that the integral $- \int \overline{H}_S \cdot \overline{M}_S \, ds$ is performed at $y=b$ plane. Because the magnetic current $M_S$ is the surface current and extended in $x$ direction from $\frac{a}{2} - L_t$ to $\frac{a}{2} + L_t$ and in $z$ direction $-W_t$ to $W_t$. Self-reaction of magnetic current along to $x$-component of magnetic current can be obtained by replacing $L_t$ by $L \cos \theta$, $W_t$ by $W \cos \theta$ and $E_n$ by $V \cos \theta$.

With modification Self-reaction due to transverse component of magnetic current in primary wave guide

$$<a,a> = - \int \overline{H}_S \cdot \overline{M}_S \, ds \quad \ldots \ldots \ldots (11)$$
\[
(a, a)_s = \frac{jv_c k_o \beta_{ab} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \frac{Z_{01}}{Y_{mn}} \cos^2 \theta \sin^2 k_z \int \frac{1}{\beta_{ab} \beta_{mn}} \cos \left( \frac{2\pi \sin \alpha}{\lambda} \right) - \cos (k \sin \theta) \right]^2 \left[ 2 \cos \theta \cos \left( \frac{y w a}{\tan \theta} \right) - \frac{1}{\tan \theta} \right] \right] - (12)
\]

3.1.3 Self-reaction in coupled/ secondary wave guide \((a, a)_s\):

For the coordinates shown in fig.2 the variables are related as

\[x = x' + \left( \frac{a}{2} \right) \quad \text{and} \quad z = z' + \left( \frac{b}{2} \right);\]

\[k = 2\pi/\lambda. \quad -------- (13)\]

From formulation given by [3] and using the relations above (13) the normalized vectors for electric \((E_m^e)\) and magnetic \((E_m^m)\) are found. The electric and magnetic voltages are given

\[V_m^e = \int_{-w}^{w} \int_{-L}^{L} \bar{E}_m \bar{E}_m^e \, dx' \, dz' \]

\[V_m^m = \int_{-w}^{w} \int_{-L}^{L} \bar{E}_m \bar{E}_m^m \, dx' \, dz' \quad -------- (14)\]

where \(E_m\) is maximum Electric field, and \(K = 2\pi/\lambda \cdot \lambda\) is wave length. \(a\) and \(b\) are narrow wall and broad wall dimensions of feed and coupled guide. From the knowledge of [6] the expressions for modal voltages are obtained.

The field distribution in the aperture plane of slot is assumed having length \(2L\) and width \(2w\) given by

\[\bar{E}_S = \bar{a}_x E_m \sin k (L_t - |z'|) \quad -------- (15)\]

for \(-L \leq |z'| \leq L\) and \(-W \leq |x'| \leq W\)

where \(E_m\) is maximum Electric field, \(\bar{a}_x\) is unit vector along \(x\) direction and

\[K = 2\pi/\lambda \cdot \lambda\] is wave length. \(2L\) is length of slot and \(2W\) is width of slot and \(V = 2WE_m\)

The transvers component of magnetic field in \(y = 0\) plane of guide 2 is of the form

\[\bar{H}_S = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \left[ (Y_0)^m_{mn} + (Y_0)^m_{mn} \right] \quad -------- (16)\]

Here \((Y_0)^m_{mn}\), and \((Y_0)^m_{mn}\) are characteristic admittance of TE and TM modes. \(h^p_{mn}\) and \(h^m_{mn}\) are modal vector functions for transvers component of magnetic field.

Since magnetic current is in ground plane \(y = 0\), The total magnetic current considering its image in the ground plane is given by

\[\bar{M}_S = 2 \bar{E}_y X \bar{a}_y \]

The electrical field distribution in the aperture plane of slot can represent by an equivalent magnetic current. The self-reaction \((a, a)_s\) of the magnetic current. \(M_S\) in coupled guide given by

\[(a, a)_s = - \int \bar{H}_S \bar{M}_S \, dx' \, dz' \quad -------- (17)\]

The self-reaction \((a, a)_s\) is reduced to

\[\langle a, a \rangle_S = 2 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (Y_0)^m_{mn} \left( (Y_0)^m_{mn} \right)^2 + 2 \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} (Y_0)^m_{mn} \left( (Y_0)^m_{mn} \right)^2 - (18)\]

where \((Y_0)^m_{mn}\) and \((Y_0)^m_{mn}\) are characteristic admittance and \(Y_{mn}\) is admittance.

3.2 Expression for discontinuity in modal current:

The expression for discontinuity in modal current \(I_3\) is given by [8] and it is expressed as

\[I_3 = jY_{01} \int \bar{a}_x \times E_x (h_{01}\sin \beta_{01} \gamma + jh_{01} \cos \beta_{01} \gamma) \, ds \quad -------- (19)\]

Here \(h_{01}\) and \(h_{01}\) are transverse and longitudinal modal vector functions respectively. \(Y_{01}\) is characteristic wave admittance and \(\beta_{01}\) is propagation constant.

These are given by

\[h_{01} = \left( \frac{2}{ab} \right) \sin \left( \frac{\pi}{b} \bar{a}_x \right) \quad -------- (20)\]

\[h_{01} = -j \left( \frac{2}{ab} \right) \sin \left( \frac{\pi}{b} \bar{a}_x \right) \quad -------- (21)\]

For a slot on the narrow wall the expression \(h_{201}\) is turns to

\[h_{201} = \left( \frac{2}{ab} \right) \sin \left( \frac{\pi}{b} \bar{a}_x \right) \quad -------- (23)\]
Since the slot is located at y=b plane, \( h_{01} = 0 \) and \( h_{02} \) takes the form of (21). Using the equations (20),(22) and evaluating the integral in the equation (19)

The expression for discontinuity in modal current \([9]\) reduces to the form

\[
I_x = 2j\gamma \sqrt{\frac{2}{\mu_0 \varepsilon_0}} \frac{2\pi}{b_i \beta_i} \frac{1}{\beta_i - kT} \left( \cos \beta_i \frac{L}{2} - \cos \left( \frac{kT}{2} \right) \right) \frac{\sin \beta_i \frac{w}{2}}{\beta_i \frac{w}{2}}
\]

\[\text{---- (19)}\]

Here \( Y_{01} = \frac{\beta_{01}}{a_0 \mu_{01}} \) and \( \beta_{01} = \sqrt{k^2 - \left( \frac{\pi}{b_i} \right)^2} \);

\[
V = 2E_0 m W
\]

3.3. Expression for admittance loading:

The normalized shunt admittance is related to normalized impedance by the relation and can be calculated from the knowledge of self-reaction and discontinuity in modal current

\[
Y_x = g_s + jb_b = \frac{1}{\varepsilon_r} \frac{1}{r + jx} \quad \text{---- (20)}
\]

where \( g_s \) the normalized conductance and \( b_b \) is the normalized susceptance

3.4 Expression for Coupling and VSWR:

It has been possible to represent the radiation of present interest by the equivalent circuit which consists of admittance parameters. The transmission matrix of the shunt admittance parameters \([5]\) given by

\[
\begin{bmatrix}
  c_t^+ \\
  c_t^-
\end{bmatrix} = \begin{bmatrix}
  1 + Y/2 & Y/2 \\
  -Y/2 & 1 - Y/2
\end{bmatrix} \begin{bmatrix}
  c_t^+ \\
  c_t^-
\end{bmatrix}
\]

When port2 of guide1s terminated with matched load \( c_t^2 = 0 \)

The reflection coefficient seen by port1 is given by

\[
\rho = \frac{1 - Y_{L_N}}{1 + Y_{L_N}} \quad \text{where} \quad Y_{L_N} = 1 + Y_T
\]

Using power balanced condition the radiated power coupled to free space is given by

\[
C = 4g_s^2 / [(2 + g_s)^2 + b_b^2] \quad \text{---- (21)}
\]

The VSWR in terms of reflection coefficient is given by \([9]\)

\[
\text{VSWR} = \frac{1 + |\rho|}{1 - |\rho|} \quad \text{---- (22)}
\]

4. RESULTS:

Using the expressions of normalized admittance presented above, the variations of normalized conductance, normalized susceptance with the length of the slot is numerically computed at the central frequency of L-band wave guide. For the slot inclination of \( \theta = 30^\circ, 35^\circ, 40^\circ, 45^\circ, 50^\circ \) the resonant lengths of the slot \( 2L = 10.6cm, 11.0cm, 11.4cm, 11.8cm, 12.0cm \) are obtained respectively. The variation of conductance, susceptance, coupling and VSWR as a function of frequency for slot widths of \( 2W = 0.0cm, 0.1cm, 0.15cm, 0.2cm, 0.25, 0.3cm \) are presented in fig.2, fig.3, fig.4, fig.5, fig.6 and fig.7 respectively.

From the results the variation of normalized admittance with slot width for fixed resonant length are presented in fig.(8-9). The resonant length is obtained from variation of normalized admittance with slot length for Centre frequency of L-band. The results are presented in appendix-I
Fig. 2. Variation in conductance, susceptance, coupling and VSWR for a=16.5 cm, b=8.2 cm, slot width W=0.05 and with slot inclination θ=30°, 35°, 40°, 45°, 50°.

Fig. 3. Variation in conductance, susceptance, coupling and VSWR for a=16.5 cm, b=8.2 cm, slot width W=0.1 and with slot inclination θ=30°, 35°, 40°, 45°, 50°.
Fig. 4. Variation in conductance, susceptance, coupling and VSWR for $a=16.5\text{ cm}$, $b=8.2\text{ cm}$, slot width $W=0.15$ and with slot inclination $\theta=30^\circ, 35^\circ, 40^\circ, 45^\circ, 50^\circ$.

Fig. 5. Variation in conductance, susceptance, coupling and VSWR for $a=16.5\text{ cm}$, $b=8.2\text{ cm}$, slot width $W=0.2$ and with slot inclination $\theta=30^\circ, 35^\circ, 40^\circ, 45^\circ, 50^\circ$. 

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Fig. 6. Variation in conductance, susceptance, coupling and VSWR for \(a=16.5\) cm, \(b=8.2\) cm, slot width \(W=0.25\) and with slot inclination \(\theta=30^\circ, 35^\circ, 40^\circ, 45^\circ, 50^\circ\).

Fig. 7. Variation in conductance, susceptance, coupling and VSWR for \(a=16.5\) cm, \(b=8.2\) cm, slot width \(W=0.3\) and with slot inclination \(\theta=30^\circ, 35^\circ, 40^\circ, 45^\circ, 50^\circ\).

Fig. 8. Variation of conductance as a function of slot width at central frequency = 1.3 GHz and slot inclination \(\theta=30^\circ, 35^\circ, 40^\circ, 45^\circ, 50^\circ\) with resonant lengths \(2L=10.6\) cm, 11 cm, 11.4 cm, 11.8 cm and 12 cm respectively.
On the Representation of susceptance has a cross over from positive to negative. variation with frequency i.e. Conductance has a peak and normalized admittance with slot length is similar to that of conductance in all cases is found to appear slightly away from resonant frequency. The shift is to the left of resonant frequency, f, and normalized susceptance is found to have change of sign at f. These observations are irrespective of slot angle and slot width. Coupling is found to vary from -6dB to -16dB and VSWR has a variation between 1 to 1.5. It is interesting to note that normal conductance does not exhibit any peak as a function of slot width. But variation of susceptance is different as a function of slot width. In some cases it has polarity changes and in some cases there no such cross over. From the results presented in appendix-I, the variation of normalized admittance with slot length is similar to that of variation with frequency i.e. Conductance has a peak and susceptance has a cross over from positive to negative.

5. CONCLUSIONS:

It is evident from the results that the maximum conductance in all cases is found to appear slightly away from resonant frequency. The shift is to the left of resonant frequency, f, and normalized susceptance is found to have change of sign at f. These observations are irrespective of slot angle and slot width. Coupling is found to vary from -6dB to -16dB and VSWR has a variation between 1 to 1.5. It is interesting to note that normal conductance does not exhibit any peak as a function of slot width. But variation of susceptance is different as a function of slot width. In some cases it has polarity changes and in some cases there no such cross over. From the results presented in appendix-I, the variation of normalized admittance with slot length is similar to that of variation with frequency i.e. Conductance has a peak and susceptance has a cross over from positive to negative.

REFERENCES:


Appendix-I

Variation of conductance and susceptance as a function of slot length, at f=1.3Ghz. for a= 16.5 cm , b= 8.25 cm and θ= 30° The resonant length obtained is 10.6 cm

Variation of conductance and susceptance as a function of slot length, at f=1.3Ghz. for a= 16.5 cm , b= 8.25 cm and θ= 35° The resonant length obtained is 11 cm
Variation of conductance and susceptance as a function of slot length, at $f=1.3$ GHz for $a=16.5$ cm, $b=8.25$ cm, and $\theta=40^\circ$. The resonant length obtained is 11.4 cm.

Variation of conductance and susceptance as a function of slot length, at $f=1.3$ GHz for $a=16.5$ cm, $b=8.25$ cm, and $\theta=45^\circ$. The resonant length obtained is 11.8 cm.

Variation of conductance and susceptance as a function of slot length, at $f=1.3$ GHz for $a=16.5$ cm, $b=8.25$ cm, and $\theta=50^\circ$. The resonant length obtained is 12.0 cm.