

Image Segmentation using Kernel Metric and Modified Weighted Fuzzy Factor

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Abstract—In this paper, a modified Kernel Weighted Fuzzy Local Information C-means clustering (MKWFLICM) algorithm for image segmentation is proposed. The proposed method is a modification of Kernel Weighted Fuzzy Local Information C-means clustering (KWFLICM) algorithm. In the proposed method the trade-off weighted fuzzy factor in KWFLICM algorithm is modified by replacing the local coefficient of variation with a distance measure. The proposed algorithm is tested by applying on a synthetic image corrupted by salt & pepper noise, speckle noise and additive white Gaussian noise (AWGN). Performance of MKWFLICM algorithm is evaluated using the parameters, Segmentation Matching Factor (SMF), Segmentation Accuracy (SA) and Normalized Mean Square Error (NMSE). Results shows that the proposed algorithm is fast and efficient compared to KWFLICM.

Keywords—Fuzzy clustering; gray-level constraint; spatial constraint; image segmentation; kernel metric; modified weighted fuzzy factor.

I. INTRODUCTION

Image segmentation is the process of extracting foreground from background of an image. Fuzzy c-means (FCM) clustering algorithm is one of the most widely used fuzzy clustering algorithms for image segmentation. This method is developed by Dunn [1] in 1973 and improved by Bezdek [2] in 1981. Conventional FCM works well on most of the noise free images but it cannot accurately segment images corrupted by noise and outliers. Results of FCM are non-robust because of ignoring spatial contextual information in image and use of non-robust Euclidean distance. Biju V.G. and Mythili P. [3] proposed an improved FCM algorithm based on genetic algorithm for image segmentation. To deal with the problem of ignoring spatial contextual information, many improved FCM algorithms have been proposed by modifying the original FCM objective function by incorporating local spatial information.

M.Ahmed et al. [4] formulated FCM_S algorithm by modifying the objective function of the standard FCM algorithm. Although the spatial contextual information can increase sensitivity to noise to some extent, still it lacks enough robustness to noise and outliers and is not suitable for revealing non Euclidean structure of input data due to the use of Euclidean distance. Also the spatial neighbourhood term calculated in each iteration is time consuming.

D.Zhang and S.Chen [5], proposed a kernel based fuzzy clustering algorithm (KFCM) which introduces a kernel

induced distance measure into the objective function of FCM to replace the conventional measures. A spatial penalty term considers the effect of neighbouring pixels on the central pixel. But the calculation of penalty term in each iteration is very time consuming. Reference [5] also proposed two variants of FCM_S, FCM_S1 and FCM_S2 which uses a mean filtered and median filtered image to increase robustness of FCM to noise by directly modifying the objective function. FCM_S1 and FCM_S2 are proposed to simplify the computation of parameters and then extended them to corresponding kernalized versions KFCM_S1 and KFCM_S2.

L. Szilagyi et al. [6] proposed Enhanced Fuzzy C-means Clustering (En_FCM) algorithm to speed up the clustering process for grey level images. Image segmentation is performed on a linearly weighted sum image. By introducing a new factor γ the amount of required calculation is considerably reduced. Thus the computational time of En_FCM is very small. W.Cai.S et al. [7] proposed Fast Generalized Fuzzy C-means Clustering (FGFCM) algorithm. FG_FCM combines both spatial and gray-level information to form a non-linearly weighted sum image and clustering is performed. A new factor local similarity measure is used to guarantee both noise immunity and detail preserving for image. Its computational time is also very small. En_FCM and FG_FCM need some parameters whose selection is to be made by either trial and error or by experience. Also these algorithms do not directly apply on the original image.

S. Krinidis and V. Chatzis [8], proposed Fuzzy Local information C-means Clustering (FLICM) algorithm which incorporates the local spatial information and gray-level information in a novel fuzzy way. The major characteristic of FLICM is the use of a fuzzy local similarity measure which guarantees noise insensitiveness and image detail preservation. This fuzzy factor replaces the parameters used in above algorithms. M.Gong et al. [9], proposed Reformulated Fuzzy Local information C-means Clustering (RFLICM) algorithm in which local coefficient of variation is adopted to replace the spatial distance. This algorithm introduces the reformulated factor as a local similarity measure to make a trade-off between image detail and robustness to noise. But it is unreasonable to ignore the effect of spatial distance constraint on the relationship between central pixel and neighboring pixels, when the size of window is enlarged. Also the damping extent of neighbors can't be accurately calculated when there is same gray-level distribution and different spatial constraint.

More recently M.Gong et al. [10], proposed a variant of FLICM algorithm (KWFLICM), which incorporates a trade-off

weighted fuzzy factor and kernel metric which are parameter free. The trade-off weighted fuzzy factor depends on the spatial distance of all neighboring pixels and their gray-level difference simultaneously. Kernel metric uses Gaussian Radial basis function kernel. The kernel parameter is determined by using a fast bandwidth selection rule based on the distance variance of all data points. But in KWFLICM the fuzzy factor computed in each iteration step is time consuming.

In this paper, a trade-off weighted fuzzy factor is designed whose computation cost is minimum. This weighted fuzzy factor gives more accurate segmentation result compared to KWFLICM. The rest of this paper is organized as follows. In the second section, details of the proposed algorithm is described. In Section III, Results and discussion is presented. Conclusions are drawn in section IV.

II. METHODOLOGY

MKWFLICM algorithm is a variant of KWFLICM algorithm. The trade-off weighted fuzzy factor in KWFLICM algorithm is modified by replacing the local coefficient of variation with a distance measure. The computation of this modified weighted fuzzy factor is easier than that in KWFLICM, and it makes the algorithm efficient.

A. General Framework of MKWFLICM Algorithm

Let I be an image having N pixels. Pixels in the image is denoted as x_i ($i=1, \dots, N$). This image is to be segmented into c clusters. The objective function of MKWFLICM is defined as follows

$$J_m = \sum_{i=1}^N \sum_{k=1}^c u_{ki}^m (1 - K(x_i, v_k)) + G'_k \quad (1)$$

Where u_{ki} represents the membership matrix, $1 - K(x_i, v_k)$ represents a non-Euclidean distance measure based on kernel method, m is the fuzzyfication parameter, v_k is the cluster prototype and G'_k is the reformulated fuzzy factor. The reformulated fuzzy factor is written as follows

$$G'_{ki} = \sum_{i=1}^N \sum_{k=1}^c u_{ki}^m \sum_{j \in N_i} w_{ij} (1 - u_{ki})^m (1 - K(x_j, v_k)) \quad (2)$$

Where N_i stands for the set of neighbors in a window around x_i , w_{ij} is the modified weighted fuzzy factor of pixel in a local window around x_i , $(1 - u_{ki})^m$ is a penalty which can accelerate the iterative convergence to some extent. The membership matrix must satisfy the following equation.

$$u_{ki} \in [0, 1] \left| \sum_{k=1}^c u_{ki} = 1 \text{ and } 0 < \sum_{i=1}^N u_{ki} < N, \forall k \right. \quad (3)$$

B. Calculating The Modified Weighted Fuzzy Factor

The weighted fuzzy factor is calculated based on the local spatial constraint and gray-level constraint. The spatial constraint gives the damping extent of the neighboring pixels with the spatial distance from the central pixel. The spatial constraint makes the influence of the pixels within the local window to change flexibly according to their distance from the central pixel. So more local information is used in the algorithm. The spatial constraint is defined as follows

$$w_{sc} = \frac{1}{(d_{ij} + 1)} \quad (4)$$

Where d_{ij} is the spatial Euclidean distance between the central pixel i and neighboring pixels j in the local window N_i .

To reflect the relationship between central pixel and neighboring pixels the intensity distance is also considered. KWFLICM is computationally time consuming because of the gray-level constraint computed in each iteration of the algorithm. So in the proposed algorithm the gray-level constraint is calculated as described below.

$$w_{id} = \frac{1}{M} \sum_{k=1}^M I_{N_i}(k) I_{N_j}(k), \quad I_{N_j}(k) \neq 0 \quad (5)$$

Where I_{N_i} and I_{N_j} are the intensity vectors of two same sized square image patches N_i and N_j . M denotes the number of pixels in the image patches. The definition of gray-level constraint allows to use more local information. Here natural logarithm function is used to map this distance into the intensity distance factor, which is defined as

$$w_{gc} = |1 - \log(w_{id})| \quad (6)$$

Here the constant one guarantees w_{gc} to be non-negative. Therefore the weighted fuzzy factor is written as

$$w_{ij} = w_{sc} \cdot w_{gc} \quad (7)$$

C. Calculating The Distance Based On Kernel Metric

Kernel method aims at transforming the complex non-linear problems in original low dimensional feature space to the problems which can be easily solved in the transformed space. Commonly used kernel method is Gaussian Radial Basis Function kernel (GRBF). The kernel distance is defined as

$$K(x_i, v_k) = \exp \left(-\frac{\|x_i - v_k\|^2}{\sigma} \right) \quad (8)$$

Where parameter σ is the bandwidth. It is calculated by using a fast bandwidth selection rule based on the distance variance of all pixels in the image, defined as follows.

Given an image X , where x_i denotes the pixels in the image, \bar{x} is the mean of the image, d_i represents the distance from each pixel x_i to \bar{x} and \bar{d} is the mean of the distances. Then bandwidth σ is calculated as follows.

$$\bar{x} = \frac{\sum_{i=1}^N x_i}{N} \quad (9)$$

$$d_i = \|x_i - \bar{x}\|^2 \quad (10)$$

$$\bar{d} = \frac{\sum_{i=1}^N d_i}{N} \quad (11)$$

Then bandwidth is given as

$$\sigma = \left(\frac{1}{N-1} \sum_{i=1}^N (d_i - \bar{d})^2 \right)^{\frac{1}{2}} \quad (12)$$

So the parameter σ is determined by the distance variance of all the data points. Then the distance based on kernel method is

$$D_{ik}^2 = 1 - \exp \left(- \frac{\|x_i - v_k\|^2}{\sigma} \right) \quad (13)$$

D. Proposed Algorithm

The updating formulas for minimizing J_m , with respect to u_{ki} and v_k is given as follows

$$u_{ki} = \frac{1}{\sum_{l=1}^c \frac{1 - K(x_i, v_l) + \sum_{\substack{j \in N_i \\ j \neq i}} w_{ij} (1 - u_{kj})^m (1 - K(x_j, v_l))}{1 - K(x_i, v_l) + \sum_{\substack{j \in N_i \\ j \neq i}} w_{ij} (1 - u_{kj})^m (1 - K(x_j, v_l))}} \quad (14)$$

$$v_k = \frac{\sum_{i=1}^N (u_{ki}^m K(x_i, v_k) x_i)}{\sum_{i=1}^N (u_{ki}^m K(x_i, v_k))} \quad (15)$$

So the proposed algorithm is as follows.

Step 1: Set the number c of the cluster prototypes, fuzzyfication parameter m , and window size N_i and the stopping condition ϵ .

Step 2: Initialize randomly the fuzzy cluster prototypes.

Step 3: Set the loop counter $b = 0$.

Step 4: Calculate the modified weighted fuzzy factor and the kernel distance as described in sections B and C.

Step 5: Update the partition matrix using equation (14)

Step 6: Update the cluster prototypes using equation (15)

Step 7: If $\max |v_{new} - v_{old}| < \epsilon$ then stop, otherwise, set $b = b + 1$ and go to step 4.

When the algorithm has converged a defuzzification process takes place in order to convert the fuzzy partition matrix to a crisp partition. Generally maximum membership procedure is adopted. This procedure assigns a pixel i to the class C_k with the highest membership.

$$C_k = \arg \{ \max \{ u_{ki} \} \}, \quad (k = 1, 2, \dots, c) \quad (16)$$

III. RESULTS AND DISCUSSIONS

The efficiency of the proposed algorithm (MKWFLICM) is tested and compared with KWFLICM algorithm. The segmentation result of MKWFLICM algorithm is evaluated using the parameters, Segmentation matching factor (SMF), Segmentation accuracy (SA) and Normalized mean square error (NMSE). The mathematical expression of Segmentation matching factor is as follows

$$SMF = \sum_{i=1}^c \frac{A_i \cap A_{ref}}{A_i \cup A_{ref}} \quad (17)$$

Where c is the number of clusters, A_i represents the set of pixels belonging to the i^{th} class found by the algorithm, while A_{ref} represents the sets of pixels belonging to the i^{th} class in the reference segmented image.

Segmentation accuracy is defined as the sum of correctly classified pixels divided by the sum of the total number of pixels.

$$SA = \frac{\sum_{i=1}^c A_i \cap C_i}{\sum_{j=1}^c C_j} \quad (18)$$

Where A_i represents the sets of pixels belonging to the i^{th} class found by the algorithm, while C_i represents the set of pixels belonging to the i^{th} class in the reference segmented image.

Normalized mean square error (NMSE) is an estimator of the overall deviation between predicted and measured vales. NMSE cost vary between 0 and 1. Zero value shows perfect segmentation. NMSE is given by

$$NMSE = \frac{\sqrt{\frac{1}{MN} \sum_i \sum_j (x_{ij} - \bar{x}_{ij})^2}}{\frac{1}{MN} \sum_i \sum_j (x_{ij})} \quad (19)$$

Where x_{ij} is the reference image and \bar{x}_{ij} is the segmented image, images are of size $M \times N$.

For evaluating the performance of the proposed algorithm, a synthetic image of size 80x80, having intensity values 30 and 90 is generated. Then different levels of Salt & Pepper, Additive White Gaussian and Speckle noise are added to the image. For each level of noise, average of five values of segmentation matching factor and segmentation accuracy is shown in Table I, II and III. Segmentation result on the synthetic image corrupted by Salt & Pepper noise is shown in Fig.1. Fig. 1(a) and 1(b) shows original image and reference image respectively. Image corrupted by 0.15 noise density is shown in Fig.1(c). Segmentation result of KWFLICM and MKWFLICM algorithm are shown in Fig. 1(d) and 1(e) respectively.

Table I gives the Segmentation matching factor and Segmentation accuracy of KWFLICM and MKWFLICM algorithms on the synthetic image corrupted by Salt & Pepper noise. The noise densities applied varies from 0.05 to 0.25. From the result, the proposed algorithm gives an average of 4.002% improvement in SMF and 2.002% improvement in SA.

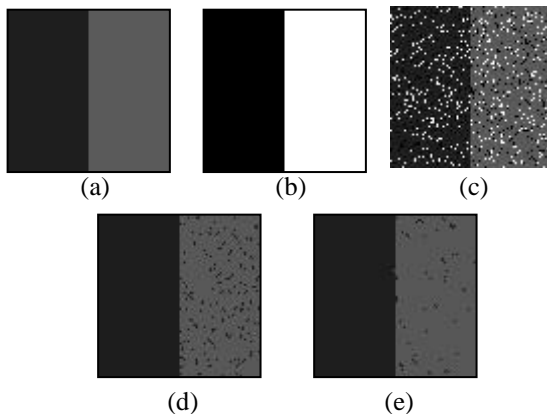


Fig. 1. Segmentation results on the synthetic corrupted by Salt & Pepper noise (d=0.15). (a) Original image. (b) Reference image. (c) Noisy image. (d) KWFLICM result. (e) MKWFLICM result

TABLE I. SEGMENTATION MATCHING FACTOR (%) AND SEGMENTATION ACCURACY (%) FOR THE SYNTHETIC IMAGE WITH DIFFERENT LEVELS OF SALT & PEPPER NOISE.

Noise density (d)	SMF		SA	
	KWFLICM	MKWFLICM	KWFLICM	MKWFLICM
0.05	98.188	99.394	99.094	99.697
0.10	96.088	98.651	98.044	99.325
0.15	94.185	98.001	97.091	99.000
0.20	91.493	97.027	95.744	98.513
0.25	88.744	95.637	94.366	97.816

Numerical values obtained on applying different levels of speckle noise are shown in Table II. MKWFLICM algorithm has an average of 6.09% improvement in SMF and 3.04% in SA.

TABLE II. SEGMENTATION MATCHING FACTOR (%) AND SEGMENTATION ACCURACY (%) FOR THE SYNTHETIC IMAGE WITH DIFFERENT LEVELS OF SPECKLE NOISE.

variance (v)	SMF		SA	
	KWFLICM	MKWFLICM	KWFLICM	MKWFLICM
0.05	98.006	98.825	99.003	99.413
0.10	87.231	92.194	93.616	96.097
0.15	77.856	85.619	88.928	92.809
0.20	72.169	79.975	86.084	89.988
0.25	67.556	76.701	83.778	88.350

Table III shows the results obtained on applying Additive White Gaussian noise. Noise levels applied varies from 1 dB to 10 dB. MKWFLICM has an average improvement of 9.17% in SMF and 5.40% in SA.

TABLE III. SEGMENTATION MATCHING FACTOR (%) AND SEGMENTATION ACCURACY (%) FOR THE SYNTHETIC IMAGE WITH DIFFERENT LEVELS OF GAUSSIAN NOISE

SNR (dB)	SMF		SA	
	KWFLICM	MKWFLICM	KWFLICM	MKWFLICM
1	64.720	76.070	79.275	86.484
3	72.661	84.611	84.313	91.650
5	81.104	92.273	89.559	95.975
7	88.348	96.450	93.791	98.188
10	96.038	99.334	97.978	99.666

Fig. 2. shows the segmentation results of the synthetic image corrupted by speckle noise (v=0.05). Fig. 2(a) and 2(b) shows the original and reference image respectively. Noisy image is shown in Fig. 2(c). Segmentation results of KWFLICM and MKWFLICM are shown in Fig. 2(d) and 2(e) respectively.

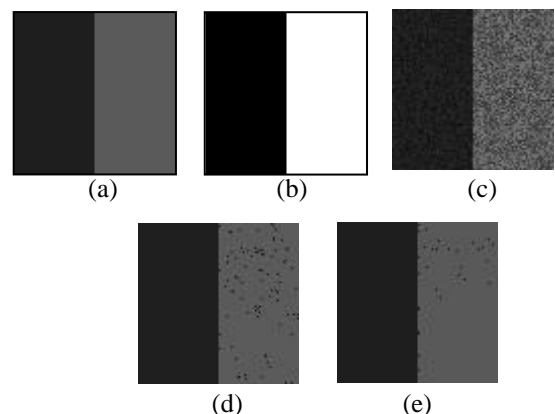


Fig 2. Segmentation result on synthetic image corrupted by Speckle noise (v=0.05). (a) Original image. (b) Reference image. (c) Noisy image. (d) KWFLICM result. (e) MKWFLICM result.

The computation time of the two algorithms for the synthetic image with different types of noises is compared in Table IV. 2.16 GHz, Pentium N3520 processor and 2GB

RAM is used. From the table it can be seen that the computation time for the proposed algorithm is smaller than that of the KWFLICM algorithm.

TABLE IV. COMPUTATION TIME (S) FOR THE ALGORITHMS ON THE SYNTHETIC IMAGE CORRUPTED BY DIFFERENT NOISES

Noise	KWFLICM	MKWFLICM
<i>Salt & Pepper</i> ($d = 10$)	78.99	57.02
<i>Speckle</i> ($v = 10$)	328.89	146.88
<i>Gaussian</i> ($SNR = 10$)	188.09	77.34

Table V, VI and VII gives the normalized mean square error of the two algorithms on the synthetic image corrupted by Salt & Pepper, Gaussian and Speckle noises respectively. For each level of noise average of five values of Normalized mean square error is shown in the Tables.

Table V shows the Normalized mean square error obtained on applying Salt & Pepper noise. From the Table, the average error of KWFLICM is greater than that of MKWFLICM by 0.0639. Comparison of Normalized mean square error obtained on applying different levels of Speckle noise is shown in Table VI. Average error of KWFLICM is 0.0715 greater than that of MKWFLICM.

TABLE V. NORMALISED MEAN SQUARE ERROR OF THE SYNTHETIC IMAGE CORRUPTED WITH SALT & PEPPER NOISE.

Noise density (d)	0.05	0.10	0.15	0.20	0.25
<i>KWFLICM</i>	0.0947	0.1436	0.1784	0.2173	0.2559
<i>MKWFLICM</i>	0.0551	0.0850	0.1084	0.1389	0.1831

TABLE VI. NORMALISED MEAN SQUARE ERROR FOR THE SYNTHETIC IMAGE CORRUPTED WITH SPECKLE NOISE.

Variance (v)	0.05	0.10	0.15	0.20	0.25
<i>KWFLICM</i>	0.0943	0.2679	0.3487	0.3968	0.4290
<i>MKWFLICM</i>	0.0769	0.1964	0.2601	0.3088	0.3370

Table VII shows NMSE value obtained on applying Additive White Gaussian noise. For AWGN noise, average error of MKWFLICM is 0.1168 lesser than that of KWFLICM.

TABLE VII. NORMALISED MEAN SQUARE ERROR OF THE SYNTHETIC IMAGE CORRUPTED WITH ADDITIVE WHITE GAUSSIAN NOISE.

SNR (dB)	1	3	5	7	10
<i>KWFLICM</i>	0.4981	0.4164	0.3299	0.2496	0.1422
<i>MKWFLICM</i>	0.3686	0.2864	0.2023	0.1351	0.0597

IV. CONCLUSION

KWFLICM and MKWFLICM algorithms are applied to noisy synthetic image. The results shows that the proposed algorithm is able to attain a maximum segmentation accuracy

of 99.69% for an image corrupted with 0.05 noise density Salt & Pepper noise. MKWFLICM algorithm has better segmentation accuracy than KWFLICM. Also the time taken for the proposed algorithm for segmentation is lesser than that of the KWFLICM algorithm. From Table IV it is clear that MKWFLICM is a fast and efficient algorithm for segmentation. Table V, VI and VII shows that the error rate of MKWFLICM is smaller than KWFLICM.

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