

# Image Denoising Techniques in Spatial Domain and Wavelet Domain

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**Abstract - This paper proposes performance comparison of different image denoising algorithms in spatial domain and wavelet domain. This paper provide a comprehensive evaluation of PSNR and MSE and comparison results are based on different types of noise such as Salt and Pepper noise and Gaussian noise. Spatial domain employs a low pass filtering on pixels by considering that noise occupies the higher region of the frequency spectrum. In wavelet domain, Orthogonal Wavelet is used for image decomposition. Peak signal to Noise ratio (PSNR) and Mean Square Error (MSE) are the two factors for measuring the quality of different denoising techniques.**

## 1. INTRODUCTION

A Digital Image is generally encoded as a matrix of gray level or intensity level. Each pixel value of the image is a result of a light intensity, falling on the sensors of camera. Images are often corrupted with noise, arise due to imperfect instrument behaviour, low lighting conditions. Each element in the imaging chain such as lenses, film, digitizer, etc. contributes to the degradation. The objective of the denoising algorithms is to remove noise while retaining as much as possible the important image features. In this paper there are two basic approaches of image denoising such as spatial filtering methods [7] and Wavelet domain filtering methods [5]. In spatial filtering method, we work directly on the pixel value by assuming that noise occupies the higher region of the frequency spectrum. Spatial filters can be further classified into linear and non-linear filters. Mean filter [6] is an example of linear filter. But it has a drawback of blurring at the edges. Non – linear method of spatial filtering is Median filter [6]. Denoising algorithms in the wavelet domain consist of three steps, first Calculate the Discrete Wavelet Transform of the noisy image, second Threshold the wavelet coefficients and last compute the Inverse Discrete Wavelet Transform to get the denoised image. These existing methods are applied on the image which is corrupted by the salt and pepper noise and Gaussian noise. Performance of denoising algorithms is measured using quantitative performance measures such as PSNR, MSE as well as in terms of visual quality of the images.

The MSE is the difference between the input image and the estimated image. Let input image is  $f(x, y)$  of size  $m \times n$  and

$\hat{f}(x, y)$  is estimated image after denoising then MSE can be defined as-

$$MSE = \frac{1}{m \times n} \sum_{x=1}^m \sum_{y=1}^n [f(x, y) - \hat{f}(x, y)]^2$$

PSNR measures the quality of reconstruction with respect to original image. A higher PSNR would normally indicate that the reconstruction is of higher quality. PSNR is usually expressed in terms of the logarithmic decibel scale (dB).

$$PSNR = 10 \log_{10} \left( \frac{255^2}{MSE} \right)$$

## 2. DENOISING METHODS

### 2.1. Spatial Domain

Mean filter, Median filter come under Spatial Domain denoising methods [3]. The concept behind image denoising using these filters is convolution and moving window principle.

**Linear Filtering (Mean Filter)** The Mean filter use a simple sliding window that replaces the center value of the window with the average of all the neighbouring pixel values including itself. By doing this, it replaces pixels, which are unrepresentative of their surroundings. The sliding window is a square of size  $m \times m$ ,  $m$  is odd number. If the coefficients of the mask sum up to one, then the average brightness of the image is not changed. If the coefficients sum to zero, the average brightness is lost, and it returns a dark image. The Mean or Average filter is also called a Linear filter, works on the shift-multiply-sum principle.

**Non-Linear Filtering (Median Filter)** A Median filter belongs to the class of nonlinear filters. The Median filter also follows the moving window principle similar to the Mean filter. In Median filter, all the pixel value of the window is arranged into ascending order or in decending order. The Median of the pixel values of the window is computed, and the center pixel of the window is replaced with the computed median.

2.2 Wavelet Domain

A wavelet system is a set of building blocks to construct or represents a signal or function. It is a two dimensional expansion set, usually a basis, for some class one or higher dimensional signals.

**Wavelet Decomposition** Wavelets measure functional variations - intensity or gray-level variations for images along different directions. The 2D Scaling and Wavelet functions are expressed as:

$$\phi_{j,m,n}(x,y) = 2^{j/2} \phi(2^j x - m, 2^j y - n)$$

$$\psi^i_{j,m,n}(x,y) = 2^{j/2} \psi^i(2^j x - m, 2^j y - n),$$

$$i = \{H, V, D\}$$

Where index i identifies the directional wavelets. By the discrete wavelet transform of the function f(x, y) of size M x N, Approximation & Details wavelets coefficients are calculated by the following equations:

$$W_\phi(j,m,n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) \phi_{j,m,n}(x,y)$$

$$W_\psi^i(j,m,n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) \psi^i_{j,m,n}(x,y)$$

$$i = \{H, V, D\}$$

Here H represents the Horizontal components, V the vertical components and D the Diagonal components.  $\psi^H(x,y)$  measures variations along columns (like horizontal edges),  $\psi^V(x,y)$  corresponds to variations along rows (like vertical edges), and  $\psi^D(x,y)$  corresponds to variations along diagonals.

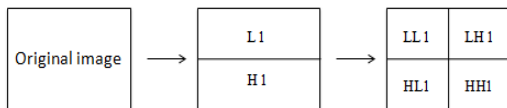


Figure 1: Two dimensional wavelet transform

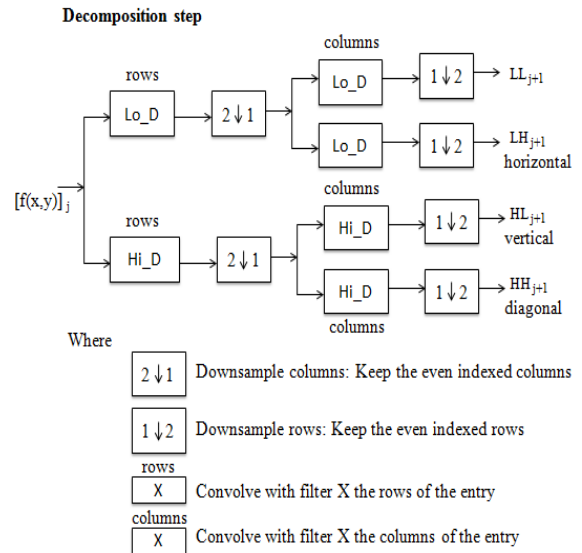


Figure 2: Decomposition of Image into Wavelet Coefficients

LL sub band is also called the Approximation sub band. All three sub bands HL, LH and HH are called the Detail sub bands, because they add the high-frequency detail to the approximation image. For calculating the filter coefficients corresponding to a Wavelet, there are some conditions which are:

Condition. (a) Unit area under scaling function:

$$\phi(x) = \sqrt{2} \sum_{k=0}^{N-1} a_k \phi(2x - k)$$

If we take  $\int \phi(x) = 1$

then a relation about choosing filter coefficients  $a_m$  i.e.

$$\sum_{k=0}^{N-1} a_k = \sqrt{2}$$

Condition. (b) Orthogonality of translates of scaling functions:-

The translates of Scaling function must be orthogonal i.e.

$$\int \phi(x) \phi(x - k) dx = \delta_{0,k}$$

If  $k=0$ , we get the **Square Normalization Condition** i.e.

$$\sum_{k=0}^{N-1} a^2_k = \sqrt{2}$$

For  $k \neq 0$ , we get a **Double Shift Orthogonality Condition** i.e.

$$\sum_{l=0}^{N-1} a_l a_{l-2k} = 0 \quad \text{for all } k \neq 0$$

**Condition. (c) Orthogonality of Scaling & Wavelet functions**

$$\phi(x) = \sqrt{2} \sum_{k=0}^{N-1} a_k \phi(2x - k)$$

$$\psi(x) = \sqrt{2} \sum_{k=0}^{N-1} b_k \phi(2x - k)$$

for Orthogonality of  $\phi(x)$  &  $\psi(x)$ , a relation b/w coefficients of high pass filter & low pass filter which is

$$b_k = (-1)^{N-k-1} a_{N-k-1}$$

### Wavelet domain denoising technique

**(a) VisuShrink:** VisuShrink was introduced by Donoho [1]. It is a Non-adaptive method. It uses a threshold value  $T$  that is proportional to the standard deviation of the noise. It is also referred to as universal threshold because all the wavelets coefficients are thresholded by using this threshold and is defined as:

$$T = \sigma \sqrt{2 \log M}$$

$\sigma^2$  is the noise variance present in the noisy image and  $M$  represents the image size or number of pixels. An estimate of the noise level  $\sigma$  was defined based on the Median Absolute Deviation given by:

$$\hat{\sigma} = \frac{\text{Median}(|Y_{ij}|)}{0.6745},$$

**(b) BayesShrink:** BayesShrink was proposed by Chang, Yu and Vetterli [2]. It uses soft thresholding and is subband-dependent, which means that thresholding is done at each band of resolution in the wavelet decomposition. It is smoothness adaptive. In particular, it is assumed that, for the various subbands and decomposition levels, the wavelet coefficients of the original image follow approximately a Generalized Gaussian Distribution (GGD) [3].

The Bayes threshold  $T$ , is defined as:

$$T = \frac{\hat{\sigma}^2}{\hat{\sigma}_x}$$

Where  $\hat{\sigma}^2$  is the noise variance and  $\hat{\sigma}_x^2$  is the signal variance without noise. The noise variance  $\hat{\sigma}^2$  is estimated from the subband  $HH_1$  in by the median estimator. From the definition of additive noise we have:

$$Y(x, y) = X(x, y) + n(x, y)$$

Since the noise and the signal are independent of each other, it can be stated that:

$$\hat{\sigma}_y^2 = \hat{\sigma}_x^2 + \hat{\sigma}^2$$

The variance of noisy image  $\hat{\sigma}_y^2$  can be determined by averaging the squared value of wavelet coefficients & shown below by equation:

$$\hat{\sigma}_y^2 = \frac{1}{n^2} \sum_{i,j=1}^n Y_{ij}^2$$

The variance of the signal,  $\hat{\sigma}_x^2$  is computed as:

$$\hat{\sigma}_x^2 = \sqrt{\max(\hat{\sigma}_y^2 - \hat{\sigma}^2, 0)}$$

With  $\hat{\sigma}^2$  and  $\hat{\sigma}_x^2$  the Bayes threshold is computed. Using this threshold, the wavelet coefficients are thresholded at each band. An estimate of the noise level  $\sigma$  was defined based on the Median Absolute Deviation given by

$$\hat{\sigma} = \frac{\text{Median}(|Y_{ij}|)}{0.6745},$$

$$Y_{ij} \in \text{subband}(HH_1)$$

### “3. Simulation Results”

#### (1) Qualitative View of Denoising Techniques

**(a) For the Removal of Gaussian Noise:** We are comparing all these de-noising techniques over Lena image size 512\*512 which is corrupted by Gaussian noise of mean 0 and standard deviation ( $\sigma$ ) 20.



Original Image of Lena



Image corrupted by Gaussian noise (mean = 0,  $\sigma = 20$ )



Denoised image using BayesShrink Method

Figure 3: Qualitative View of Denoising Techniques for Gaussian Noise



Denoised image using Mean filter

**(b) For the Removal of Salt & Pepper Noise** We are comparing all these Denoising techniques over Lena image of size 512\*512 which is corrupted by Salt & Pepper Noise of Noise Density= .02 .



Image corrupted by Salt & Pepper Noise of Noise Density = .02



Denoised image using Median filter



Denoised image using BayesShrink Method



Denoised image using Universal Threshold Method (VisuShrink)



Denoised image using Mean filter



Denoised image using Median filter

Figure 4: Qualitative View of Denoising Techniques for Salt and Paper Noise

**(2) Quantitative Comparative Study of Denoising Techniques** We are comparing the various denoising techniques in terms of two parameters which are Peak signal to noise ratio (in db) & Mean square error.

#### (a) Performance for the Removal of Gaussian Noise

Performance of Denoising Methods for Removal of Gaussian Noise (Mean=0,  $\sigma=20$ )

Domain	Methods	Noise Removal	PSN R (db)	MSE
Spatial Domain	Mean Filtering	Gaussian Noise	22.14 12	397.1 512
	Median Filtering	Gaussian Noise	22.05 32	405.2 872
Wavelet Domain	Universal Threshold Method	Gaussian Noise	22.82 27	339.4 770
	BayesShrink Method	Gaussian Noise	23.02 99	323.6 612

Table 1.

#### (b) Performance for the Removal of Salt & Pepper Noise

Performance of Denoising Methods for Removal of Salt & Pepper Noise (Noise Density=.02)

Domain	Methods	Noise Removal	PSN R (db)	MSE
Spatial Domain	Mean Filtering	Salt & Pepper Noise	21.01 34	514.9 225
	Median Filtering	Salt & Pepper Noise	21.25 28	487.3 047
Wavelet Domain	Universal Threshold	Salt & Pepper Noise	20.00 23	538.8 653
	BayesShrink	Salt & Pepper Noise	20.83 64	525.9 225

Table 2.

**Conclusion:** We have compared two techniques of image denoising in spatial domain and two techniques in Wavelet domain. The simulation result shows that for Salt & Pepper noise Median filter is appropriate. Median filter is also robust as compared to mean filter because median filter does not create new unrealistic pixel value when filter straddles an edge.

In the case where an image is corrupted with Gaussian Noise, the wavelet domain denoising has proved to be nearly optimal. From Table 1 and 2, it can be seen that the Wavelet domain techniques outperforms spatial domain techniques in this case. The smoothness of recovered image is better in VisuShrink. This method gives threshold, which depend on the number of pixels. If the number of pixels are large, we get bigger and bigger threshold, which tends to oversmoothen the image. But VisuShrink does not reduce the Mean Squared Error. From Table 1, it can be seen BayesShrink method gives better performance as compared to VisuShrink for the removal of Gaussian Noise.

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