Image Compression Using Wavelet Transform And Differential Pulse Code Modulation Technique

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Abstract
The objective of image compression is to reduce irrelevance and redundancy of the image data in order to be able to store or transmit data in an efficient form. Lot of work has been done on image compression and still a lot of scope is wide open. This work evaluates the performance of image compression algorithm based on wavelet transform and Differential Pulse Code Modulation (DPCM). In the first step wavelet transform is used and in the second step the image is processed through the DPCM. The simulation results show a improvement in the performance compared to the wavelet transform technique. The comparison was also made by choosing various combinations of band and measuring the compression ratio and PSNR.

Keywords: Image Compression, DPCM, Wavelet, Matlab, Haar

Introduction
Digital video compression techniques have played an important role in the world of telecommunication and multimedia systems where bandwidth is still a valuable commodity. Hence, video compression techniques are of prime importance for reducing the amount of information needed for picture sequence without losing much of its quality. All the image compression algorithms are mainly focused in removing the statistical redundancy. Over last few decades, researchers have proposed many competing techniques such as prediction coders, transform coders, vector quantizers, trellis-coded quantizers and fractal image representation. Every algorithm has its own advantages and disadvantages. The main objective of this paper is to perform image compression by cascading wavelet transform and DPCM.

Digital images are characterized by multiple parameters. The first feature of a digital image is its color mode. A digital image can have one of three modes: binary, grayscale or color. A binary (bilevel) image is an image in which only two possible values for each pixel. A grayscale image means that its each pixel can contain only a tint of gray color. A digital image is a set of pixels. Each pixel has a value that defines color of the pixel. All the pixels are composed into one array. There are several different ways in which image files can be compressed.

Lossless Image Compression
When hearing that image data are reduced, one could expect that automatically also the image quality will be reduced. A loss of information is, however, totally avoided in lossless compression, where image data are reduced while image information is totally preserved. It uses the predictive encoding which uses the gray level of each pixel to predict the gray value of its right neighbor. Only the small deviation from this
prediction is stored. This is a first step of lossless data reduction. Its effect is to change the statistics of the image signal drastically. Statistical encoding is another important approach to lossless data reduction. Statistical encoding can be especially successful if the gray level statistics of the images has already been changed by predictive coding. The overall result is redundancy reduction, that is reduction of the repetition of the same bit patterns in the data. Of course, when reading the reduced image data, these processes can be performed in reverse order without any error and thus the original image is recovered. Lossless compression is therefore also called reversible compression.

Lossy Image Compression
Lossy data compression has of course a strong negative connotation and sometimes it is doubted quite emotionally that it is at all applicable in medical imaging[3]. In transform encoding one performs for each image a mathematical transformation that is similar to the Fourier transform thus separating image information on gradual spatial variation of brightness (regions of essentially constant brightness) from information with faster variation of brightness at edges of the image (compare: the grouping by the editor of news according to the classes of contents). In the next step, the information on slower changes is transmitted essentially lossless (compare: careful reading of highly relevant pages in the newspaper), but information on faster local changes is communicated with lower accuracy (compare: looking only at the large headings on the less relevant pages). In image data reduction, this second step is called quantization. Since this quantization step cannot be reversed when decompressing the data, the overall compression is ‘lossy’ or ‘irreversible’

Wavelet Transform
The Wavelet Transform provides a time-frequency representation of the signal. It was developed to overcome the short coming of the Short Time Fourier Transform (STFT), which can also be used to analyze non-stationary signals. While STFT gives a constant resolution at all frequencies, the Wavelet Transform uses multi-resolution technique by which different frequencies are analyzed with different resolutions.

Wavelet transform represents an image as a sum of wavelet functions with different locations and scales[4]. Any decomposition of an image into wavelets involves a pair of waveforms: one to represent the high frequencies corresponding to the detailed parts of an image (wavelet function) and one for the low frequencies or smooth parts of an image (scaling function).[1] The result of the wavelet transform is a set of wavelet coefficients, which measure the contribution of the wavelets at these locations and scales.

The integral wavelet transform is the integral transform defined as

$$\text{[W}_a f](a,b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} \phi \left( \frac{x-b}{a} \right) f(x) dx$$

The wavelet coefficients $c_{jk}$ are then given by

$$c_{jk} = [W_a f](2^{-j}k2^{-j})$$

Here, $a=2^{-j}$ is called the binary dilation or dyadic dilation, and $b = k2^{-j}$ is the binary or dyadic position. Following is the working of wavelet transform in brief. First a wavelet transform is applied. This produces as many coefficients as there are pixels in the image (i.e., there is no compression yet since it is only a transform). These coefficients can then be compressed more easily because the information is statistically concentrated in just a few coefficients. This principle is called transform coding. After that, the coefficients are quantized and the quantized values are entropy encoded and/or run length encoded.

Common applications of wavelet transforms include speech and audio processing, image and video processing, biomedical imaging, and 1-D and 2-D applications in communications and geophysics.

Wavelet transform are classified into 2 types

Continuous Wavelet Transform (CWT)
Discrete Wavelet Transform (DWT)

Continuous wavelet Transform
Continuous wavelet transform (CWT) of 1D signal is defined as

$$\text{(W}_a f)(b) = \int f(x) \psi_{a,b}(x) dx$$

The $\psi_{a,b}$ is computed from the mother wavelet by translation and dilation

$$\psi_{a,b}(x) = \frac{1}{\sqrt{a}} \phi \left( \frac{x-b}{a} \right)$$

Discrete wavelet Transform
The Discrete Wavelet Transform (DWT), which is based on sub-band coding is found to yield a fast computation of Wavelet Transform. It is easy to implement and reduces the computation time and resources required

In CWT, the signals are analyzed using a set of basis functions which relate to each other by simple scaling and translation. In the case of DWT, a time-scale representation of the digital signal is obtained using
digital filtering techniques. The signal to be analyzed is passed through filters with different cutoff frequencies at different scales.
DWT separates the high and low-frequency portions of a signal through the use of filters
One level of transform: Signal is passed through G & H filters. Down sample by a factor of two. Multiple levels (scales) are made by repeating the filtering and decimation process on lowpass outputs

Fig. 1 Workflow of Discrete Wavelet Transform

**Haar Wavelet Transform**

In mathematics, the Haar wavelet is a certain sequence of rescaled "square-shaped" functions which together form a wavelet family or basis. Wavelet analysis is similar to Fourier analysis in that it allows a target function over an interval to be represented in terms of an orthonormal function basis. The Haar sequence is now recognised as the first known wavelet basis and extensively used as a teaching example in the theory of wavelets.

In functional analysis, the Haar systems denotes the set of Haar wavelets
\[ t \rightarrow \varphi_{n,k}(t) = \varphi(2^n t - k); n \in \mathbb{N}, 0 \leq k < 2^n \]

Following is the algorithm that is followed for haar wavelet.

1. Find the average of each pair of samples
2. Find the difference between the average and sample
3. Fill the first half with averages
4. Fill the second half with differences
5. Repeat the process on the first half

**Step 1:**

\[
\begin{bmatrix}
3 & 5 & 4 & 8 & 13 & 7 & 5 & 3 \\
\end{bmatrix}
\]

Averaging

\[
\begin{bmatrix}
4 & 6 & 10 & 4 & -1 & -2 & 3 & 1 \\
\end{bmatrix}
\]

Differencing

**Step 2:**

\[
\begin{bmatrix}
4 & 6 & 10 & 4 & -1 & -2 & 3 & 1 \\
\end{bmatrix}
\]

Averaging

\[
\begin{bmatrix}
5 & 7 & -1 & 3 & -1 & -2 & 3 & 1 \\
\end{bmatrix}
\]

Differencing

\[
\begin{bmatrix}
(4 + 6)/2 = 5 \\
-5 = -1 \\
\end{bmatrix}
\]

**Step 3:**

\[
\begin{bmatrix}
5 & 7 & -1 & 3 & -1 & -2 & 3 & 1 \\
\end{bmatrix}
\]

Averaging

\[
\begin{bmatrix}
6 & -1 & 3 & -1 & -2 & 3 & 1 \\
\end{bmatrix}
\]

Row average

\[
(5 + 7)/2 = 6 \\
5 - 6 = -1 \\
\]

**2-D DWT**

The 2D Discrete Wavelet Transform (DWT) is an important function in many multimedia applications, such as JPEG2000 and MPEG-4 standards, digital watermarking, and content-based multimedia information retrieval systems. The 2D DWT is computationally intensive than other functions, for instance, in the JPEG2000 standard.

- Step 1: replace each row with its 1-D DWT.
- Step 2: Replace each column with its 1-D DWT
- Step 3: Repeat steps 1 & 2 on the lowest subband for the next scale.
- Step 4: Repeat step 3 until as many scales as desired

![2-D DWT Diagram](image-url)
Differential Pulse Code Modulation

Differential pulse-code modulation (DPCM) is a signal encoder that uses the baseline of pulse-code modulation (PCM) but adds some functionalities based on the prediction of the samples of the signal. The input can be an analog signal or a digital signal.

Proposed Method

![Proposed Method Diagram]

**Fig. 3** Block diagram of the proposed approach

**Fig. 4** Workflow of the proposed approach
<table>
<thead>
<tr>
<th>Band1</th>
<th>Band2</th>
<th>Band3</th>
<th>Band4</th>
<th>Band5</th>
<th>Band6</th>
<th>Band7</th>
</tr>
</thead>
</table>

**Fig. 5 Band labeling after wavelet transform**

Figure 3 shows the block diagram of the complete proposed system. The image is first passed through the 2-D wavelet transform. Figure 5 shows the output of the wavelet transform. These bands are made to pass through the DPCM encoder and the encoded data is transmitted. Figure 4 represents the workflow of the proposed approach. Exactly the reverse process if followed at the receiver input.

**Experimental Results and Discussion**

This section presents the experimental evaluation of the proposed algorithm. The experiments were conducted based on the images ‘lena.bmp’, ‘barbara.png’ of size 512x512 with 28 gray levels. The images were decompressed using haar wavelet transform.
### Results for ‘lena.bmp’

<table>
<thead>
<tr>
<th>Bands Chosen</th>
<th>MSE</th>
<th>PSNR</th>
<th>Bytes Stored</th>
<th>Compression Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,2,3,4,5,6</td>
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<td>48.0722</td>
<td>196608</td>
<td>0.25</td>
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<tr>
<td>1,2,3,4,5</td>
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<tr>
<td>1,2,3,5,6</td>
<td>9.6639</td>
<td>44.3339</td>
<td>180224</td>
<td>0.3125</td>
</tr>
<tr>
<td>1,2,3,5</td>
<td>26.7626</td>
<td>39.9101</td>
<td>114688</td>
<td>0.5625</td>
</tr>
<tr>
<td>2,3,4,5,6</td>
<td>7.37E+03</td>
<td>15.5106</td>
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<td>0.3125</td>
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<tr>
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<td>0.375</td>
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</table>

### Results for ‘barbara.png’

<table>
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<th>Bands Chosen</th>
<th>MSE</th>
<th>PSNR</th>
<th>Bytes Stored</th>
<th>Compression Ratio</th>
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<td>12.0208</td>
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<td>1.65E+04</td>
<td>12.0124</td>
<td>163840</td>
<td>0.375</td>
</tr>
</tbody>
</table>
Conclusion:
Image compression was achieved using the Haar wavelet transform and DPCM method. The experiments were performed by selecting various bands received after wavelet transform and passing them through DPCM. After Matlab simulation it was found that omitting band 6 gives us a compression of 25% but the mean square error is only 4.08. Comparing this result with the other results we find that as we go on increasing the compression ratio the mean square error increases. The results with the best band selection are summarized in the above tables. Depending upon our requirement, one can decide whether to go for better accuracy or better compression.

References: