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Higher Dimensional Bianchi Type-I Cosmological Model in Lyra's Geometry

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Abstract **In the present study, we explore a five-dimensional Bianchi type-I cosmological model incorporating both dark matter and holographic dark energy within the framework of Lyra's geometry. By applying suitable transformations under appropriate assumptions, we derive a deterministic solution describing the evolution of the universe. Various physical and kinematical properties of the derived cosmological model have been examined in detail. Furthermore, the expansion dynamics of the universe are analyzed, and it is noted that the model exhibits a singularity at the initial epoch, i.e., when cosmic time is zero. Additionally, key parameters of cosmology such as redshift, $Om(z)$ diagnostic are studied along with thermodynamics of the universe.**

Keywords **Five-dimensional Bianchi type-I, Lyra's geometry, Dark matter, Holographic Dark Energy.**

I. INTRODUCTION

Higher Dimensional Cosmologywas introduced by Theodor Kaluza [1], in an attempt to unifygravitywith electromagnetism. Later Oskar Klein [2] extended this theory by proposing the idea of compactification of extra dimension. The extra spatial dimension gets curled up in a circle of a very small radius; so that a particle moving along that axis returns to its initial position after traveling only a short distance. This curled extra spatial dimension is a compact set, and this phenomenon is referred to as compactification. The solution of Einstein's Field Equations in higher-dimensional space-times is believed to be of physical relevance possibly at extremely early stages of the universe before the universe underwent compactification. It is argued that the extra dimensions are observable at the present time.

Einstein geometrizes gravitation in his theory of general relativity which is treated as a basis for a model of the universe. After that, many cosmologists and astrophysicists attempted to study gravitation in different contexts. For the illustration of both gravitation and electromagnetism, Weyl [3] tried to formulate a novel gauge theory containing a metric tensor. But, due to the non-integrability condition, it does not get importance in the cosmological society. In order to eliminate the non-integrability condition inWeyl's geometry, Lyra [4] introduced a gauge function into the structureless manifold and recommended an adjustment to it.Einstein's modified field equation in normal gauge for Lyra's manifold obtained by Sen [5] is given by

$$R_{ij}-\frac{1}{2}g_{ij}R+\frac{3}{2}\phi_i\phi_j-\frac{3}{4}\phi_\nu\phi^\nu=-T_{ij} \qquad (1)$$

Where, R_{ij} is Ricci tensor, g_{ij} is metric tensor, R is Ricci curvature, ϕ_i is displacement vector, T_{ij} is Energy Momentum Tensor.

The proposed work contains a study of a five-dimensional Bianchi Type-I universe filled with Dark Matter and Holographic Dark Energy (HDE) in the framework of Lyra's manifold.

The energy-momentum tensor for Dark Matter and HDE are respectively given by, (Singh *et. al.*[6])

$$T_{ij}=\rho_m u_i u_j \qquad (2)$$

$$\bar{T}_{ij}=(\rho_\Lambda+p_\Lambda)u_i u_j+g_{ij}p_\Lambda \qquad (3)$$

The field equation in equation (1) for Energy Momentum Tensor of Dark Matter and Dark Energy becomes,

$$R_{ij}-\frac{1}{2}g_{ij}R+\frac{3}{2}\phi_i\phi_j-\frac{3}{4}\phi_\nu\phi^\nu=-\left(T_{ij}+\bar{T}_{ij}\right) \qquad (4)$$

In recent years many researchers have studied higher-dimensional cosmology. P.K. Sahoo *et. al.* [7] analysed Higher-dimensional Bianchi type-III universe with strange quark matter attached to string cloud in general relativity. A. Pradhan *et. al.* [8] studied A new class of holographic dark energy models in LRS Bianchi Type-I. D.D pawar *et.al.* [9] examined Magnetized Dark Energy Cosmological Modes with Time dependent Cosmological Term in Lyra geometry. G.U. Khapekar *et.al.*[10] studied Characteristic values of Six-Dimensional Symetric Tensor for generalized Peres space-time. J. Daimary *et. al.* [11] have studied five-dimensional Bianchi type-I string cosmological model with electromagnetic field. M.R. Mollah *et. al.* [12] proposed a work On Bianchi type III Cosmological Model with Quadratic

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EoS in Lyra Geometry. M.K. Singh Ranawat *et. al.* [13] analyzed Five-Dimensional Bianchi Type I String Cosmological Model in General Relativity. J.K. Singh *et. al.* [14] studied The Bianchi type-V Dark Energy Cosmology in Self Interacting Brans Dicke Theory of Gravity. M.V. Santi *et. al.* [15] studied Bianchi type-III holographic dark energy model with quintessence. M.P.V.V. Bhaskara Rao *et. al.* [16] studied Five-dimensional FRW Modified Holographic Ricci Dark Energy Cosmological Models with Hybrid Expansion Law in a Scalar-Tensor Theory of Gravitation. F. Rahaman *et. al.* [17, 18] proposed work on higher dimensional homogeneous cosmology in Lyra Geometry and Higher-dimensional string theory in Lyra geometry. J. Baro *et. al.* [19] studied Mathematical analysis on anisotropic Bianchi type-III inflationary string cosmological models in Lyra geometry, K.P. Singh *et. al.* [20] has studied higher dimensional Bianchi type-III string cosmological models in Lyra geometry. G. Mohanty [21]*et. al.* examined Higher-dimensional String Cosmological model with bulk viscous fluid in Lyra geometry. D.R.K. Reddy [22] studied Five-Dimensional spherically symmetric perfect fluid cosmological model in the Lyra manifold.K. S. Adhav [23] has studied LRS Bianchi type-I anisotropic dark energy in Lyra geometry.

II. METRIC AND FIELD EQUATIONS

Five-dimensional Bianchi type-I space-time is given as,

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 + D^2dm^2 \tag{5}$$

Where, A, B, C, D are functions of time (t) only.

Displacement vector defined as $\phi_i = (\beta(t), 0, 0, 0, 0)$

β is the gauge function.

In a co-moving coordinate system, the modified field equation (4) for five-dimensional Bianchi type-I space-timein (5) with equations (2) and (3) are

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}\dot{D}}{AD} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{B}\dot{D}}{BD} + \frac{\dot{C}\dot{D}}{CD} - \frac{3}{4}\beta^2 = \rho_m + \rho_\Lambda \tag{6}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\ddot{D}}{D} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{B}\dot{D}}{BD} + \frac{\dot{C}\dot{D}}{CD} + \frac{3}{4}\beta^2 = -p_\Lambda \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\ddot{D}}{D} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}\dot{D}}{AD} + \frac{\dot{C}\dot{D}}{CD} + \frac{3}{4}\beta^2 = -p_\Lambda \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{D}}{D} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{D}}{AD} + \frac{\dot{B}\dot{D}}{BD} + \frac{3}{4}\beta^2 = -p_\Lambda \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} + \frac{3}{4}\beta^2 = -p_\Lambda \tag{10}$$

Here overhead dots denote the ordinary differentiation concerning time t .

By the use of the energy conservation equation $T^{j}_{i;j} + \overline{T}^{j}_{i;j} = 0$, the right-hand side of equation (4) leads to,

$$\begin{aligned} &\frac{3}{2}\phi_i\left(\phi^{j}_{,j} + \phi^k\,\Gamma^{j}_{kj}\right) + \frac{3}{2}\phi^j\left(\phi_{i,j} - \phi_k\,\Gamma^{k}_{ij}\right) \\ &-\frac{3}{4}\phi_\nu\left(\phi^{\nu}_{,j} + \phi^k\,\Gamma^{\nu}_{kj}\right) - \frac{3}{4}\phi^\nu\left(\phi_{n,j} - \phi_k\,\Gamma^{k}_{\nu j}\right) = 0 \end{aligned} \tag{11}$$

Above equation (11) satisfies identically for $i = 1, 2, 3, 4$ and for $i = 0$, we get

$$\frac{3}{2}\beta\dot{\beta} + \frac{3}{2}\beta^2\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{\dot{D}}{D}\right) = 0 \tag{12}$$

III. COSMOLOGICAL SOLUTION

Field equations (6) to (10) is a system of five highly non-linear differential equations in eight unknown functions hence to derive the exact solution of field equations, we use the following transformations,

$$\begin{aligned} &A = \exp(a),\quad B = \exp(b),\quad C = \exp(c), \\ &D = \exp(d)\ \text{ and }\ dt = ABCD\,dT \end{aligned} \tag{13}$$

Where, a, b, c, d are functions of T only.

The transformed field equations are given by,

$$\begin{aligned} &a'b' + a'c' + a'd' + b'c' + b'd' + c'd' - \\ &\frac{3}{4}\beta^2\exp(2(a+b+c+d)) = (\rho_m + \rho_\Lambda)\exp(2(a+b+c+d)) \end{aligned} \tag{14}$$

$$\begin{aligned} &b'' + c'' + d'' - a'b' - a'c' - a'd' - b'c' - b'd' - c'd' + \\ &\frac{3}{4}\beta^2\exp(2(a+b+c+d)) = -p_\Lambda\exp(2(a+b+c+d)) \end{aligned} \tag{15}$$

$$\begin{aligned} &a'' + c'' + d'' - a'b' - a'c' - a'd' - b'c' - b'd' - c'd' + \\ &\frac{3}{4}\beta^2\exp(2(a+b+c+d)) = -p_\Lambda\exp(2(a+b+c+d)) \end{aligned} \tag{16}$$

$$\begin{aligned} &a'' + b'' + d'' - a'b' - a'c' - a'd' - b'c' - b'd' - c'd' + \\ &\frac{3}{4}\beta^2\exp(2(a+b+c+d)) = -p_\Lambda\exp(2(a+b+c+d)) \end{aligned} \tag{17}$$

$$\begin{aligned} &a'' + b'' + c'' - a'b' - a'c' - a'd' - b'c' - b'd' - c'd' + \\ &\frac{3}{4}\beta^2\exp(2(a+b+c+d)) = -p_\Lambda\exp(2(a+b+c+d)) \end{aligned} \tag{18}$$

Here, a dash denotes the differentiation with respects to T .

Equating equations (15) and (16), we get

$$b = a + C_1 T + C_2 \quad (19)$$

Equating equations (16) and (17), we get

$$c = a + (C_1 + C_3)T + (C_2 + C_4) \quad (20)$$

Similarly, equating equations (16) and (18), we get

$$d = a + (C_1 + C_5)T + (C_2 + C_6) \quad (21)$$

Using equations (19), (20), and (21) in the condition $a' + b' + c' + d' = 0$, (Ranawat et. al. [8]) we obtain

$$a = C_7 T + C_8 \quad (22)$$

Where,

$C_7 = -\frac{3C_1 + C_3 + C_7}{4}$ and C_8 - constant of integration

Using the value of a from equation (22), in (19), (20), and (21) we get

$$b = (C_1 + C_7)T + (C_2 + C_8) \quad (23)$$

$$c = (C_1 + C_3 + C_7)T + (C_2 + C_4 + C_8) \quad (24)$$

$$d = (C_1 + C_5 + C_7)T + (C_2 + C_6 + C_8) \quad (25)$$

Hence, we obtain

$$A = \exp(a) = \exp(C_7 T + C_8) \quad (26)$$

$$B = \exp(b) = \exp((C_1 + C_7)T + (C_2 + C_8)) \quad (27)$$

$$C = \exp(c) = \exp((C_1 + C_3 + C_7)T + (C_2 + C_4 + C_8)) \quad (28)$$

$$D = \exp(d) = \exp((C_1 + C_5 + C_7)T + (C_2 + C_6 + C_8)) \quad (29)$$

Now from equation (12) displacement vector β can be obtained as

$$\beta = C_9 \exp(-(C_{10}T + C_{11})) \quad (30)$$

Where,

C_9 - constant of integration,

$C_{10} = 3C_1 + C_3 + C_5 + 4C_7,\ C_{11} = 3C_2 + C_4 + C_6 + 4C_8$

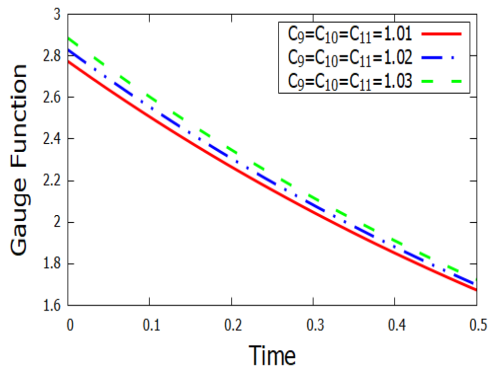

Figure 1: Plot of Gauge function vs. Time for $C_{10} = C_{11} = C_{12} = 1.01, 1.02, 1.03$

Figure 1 shows the behavior of gauge function with change in cosmic time for $C_{10} = C_{11} = C_{12} = 1.01, 1.02, 1.03$. It is observed that the gauge function is a decreasing function of the cosmic time. At an early stage, the gauge function β is infinite and decreases with the evolution of the universe and vanishes as time tends to infinity. Thus, the model has singularity when the cosmic time is zero.

Space-time in equation (5) reduces to

$$ds^2 = -\exp(2(C_{10}T + C_{11}))dT^2 + \exp(2(C_7 T + C_8))dx^2$$
$$+ \exp(2(C_1 + C_7)T + 2(C_2 + C_8))dy^2$$
$$+ \exp(2(C_1 + C_3 + C_7)T + 2(C_2 + C_4 + C_8))dz^2$$
$$+ \exp(2(C_1 + C_5 + C_7)T + 2(C_2 + C_6 + C_8))dm^2 \quad (31)$$

IV. PHYSICAL AND KINEMATICAL PROPERTIES

Physical properties of the model are studied including Energy density and pressure of Dark matter and HDE.

Energy Density of Dark matter

$$\rho_m = C_{12} \exp(-(C_{10}T + C_{11})) \quad (32)$$

Where, C_{12} is resolved constants.

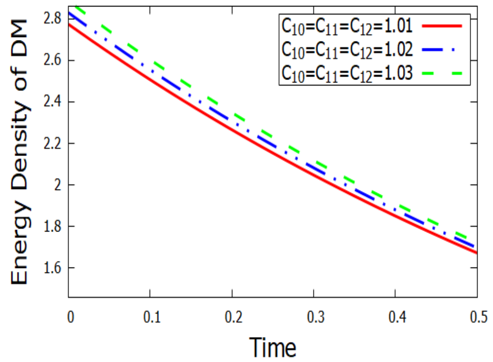

Figure 2: Plot of Energy density of Dark matter vs. Time for $C_{10} = C_{11} = C_{12} = 1.01, 1.02, 1.03$

Figure 2 gives the plot of Energy density of Dark matter versus Cosmic time for the constants $C_{10} = C_{11} = C_{12} = 1.01, 1.02, 1.03$.

It can be seen from figure 2 that, at an early stage of evolution of the universe, energy density of dark matter dominates and at late times it approaches zero.

Pressure of HDE

$$p_\Lambda = -\frac{3}{4} C_9{}^2 \exp(-2(C_{10}T + C_{11})) + C_{13} \quad (33)$$

Where, C_{13} is resolved constants.

Figure 3 shows the plot of pressure of Holographic Dark Energy (HDE) versus cosmic time (T) for constants $C_9 = C_{10} = C_{11} = 0.01, 1.02, 1.03,\ C_{13} = 0$.

As it is indicated in figure 3, the negative pressure of HDE increases with increase in time. The negative pressure indicates the accelerated phase of expansion of universe.

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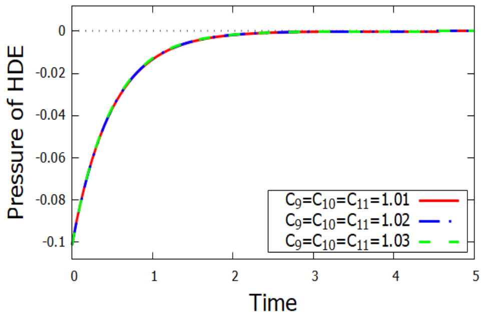

Figure 3: Plot of Pressure of HDE vs. Time for $C_9 = C_{10} = C_{11} = 0.01, 1.02, 1.03,\ C_{13} = 0$

Energy density of HDE

$$\rho_\Lambda = C_{15} \exp\left(-2(C_{10}T + C_{11})\right) - C_{12} \exp\left(-(C_{10}T + C_{11})\right) \quad (34)$$

Here $C_{15} = C_{14} - \frac{3}{4} C_9{}^2$

Where, C_{14} is resolved constants.

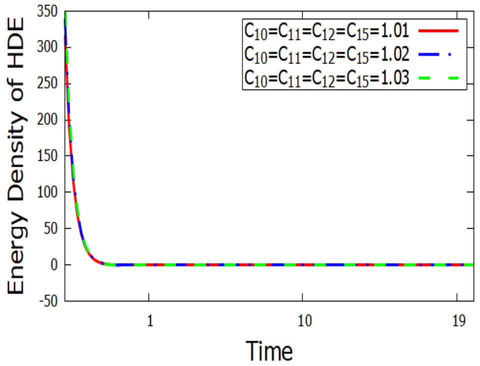

Figure 4: Plot of Energy density of HDE vs. Time for $C_{10} = C_{11} = C_{12} = C_{15} = 1.01, 1.02, 1.03$

Equation (34) gives the energy density of HDE and figure 4 shows the plot of energy density of HDE versus cosmic time. It is seen from the figure 4 that energy density of HDE is decreasing function of cosmic time.

EoS Parameter of Holographic Dark Energy

$$\omega_\Lambda = \frac{p_\Lambda}{\rho_\Lambda}$$

$$= \frac{-\frac{3}{4} C_9{}^2 \exp\left(-2(C_{10}T + C_{11})\right) + C_{13}}{C_{15} \exp\left(-2(C_{10}T + C_{11})\right) - C_{12} \exp\left(-(C_{10}T + C_{11})\right)} \quad (35)$$

Figure 5 gives the graph of EoS parameter of HDE as a function of time for s $C_9 = C_{10} = C_{11} = C_{12} = C_{15} = 1.01, 1.02, 1.03$ and $C_{13} = 0$. It is observed from the graph that EoS parameter decreases as time increases.

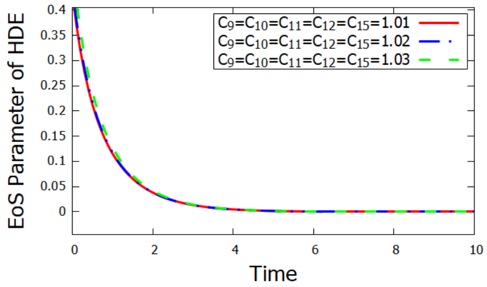

Figure 5: Plot of EoS parameter of HDE vs. Time for $C_9 = C_{10} = C_{11} = C_{12} = C_{15} = 1.01, 1.02, 1.03$ and $C_{13} = 0$

Coincidence Parameter

$$\bar{r} = \frac{\rho_m}{\rho_\Lambda}$$

$$= \frac{C_{12} \exp\left(-(C_{10}T + C_{11})\right)}{C_{15} \exp\left(-2(C_{10}T + C_{11})\right) - C_{12} \exp\left(-(C_{10}T + C_{11})\right)} \quad (36)$$

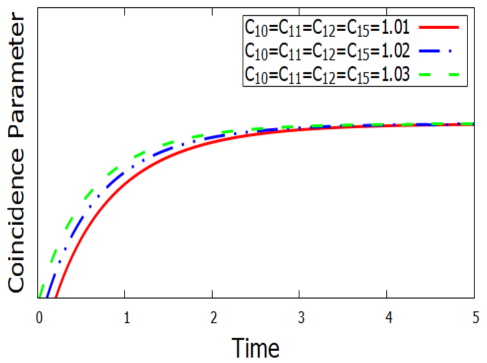

Figure 6: Plot of Coincidence Parameter vs. Time for $C_{10} = C_{11} = C_{12} = C_{15} = 1.01, 1.02, 1.03$

Figure 6 gives the behavior of the Coincidence parameter $\bar{r} = \frac{\rho_m}{\rho_\Lambda}$ against time. It shows that Coincidence parameter changes at initial stage of evolution of universe. But after some time, it converges to some constant value and does not vary throughout the evolution. Hence, it can be stated that there is proper kind of interaction between energy densities of the Dark Matter and Holographic Dark Energy since their ratio attains constant value during evolution.

Spatial Volume $V = \exp\left(\frac{C_{10}T + C_{11}}{2}\right) \quad (37)$

Equation (37) represents the spatial volume of the model, which is also depicted in figure 7. From figure 7, it is observed that initiallythe spatial volume of the model is constant and expands exponentially. Volume tends to infinity as time

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tends to infinity. That means the universe must be expanding.

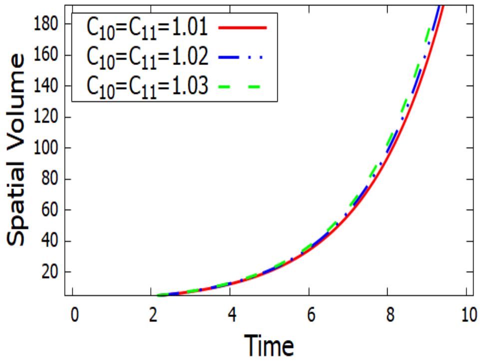

Figure 7: Plot of Spatial volume vs. Time for $C_{10} = C_{11} = 1.01, 1.02, 1.03$

Average Scale Factor $a = \exp\left(\dfrac{C_{10}T + C_{11}}{8}\right)$ (38)

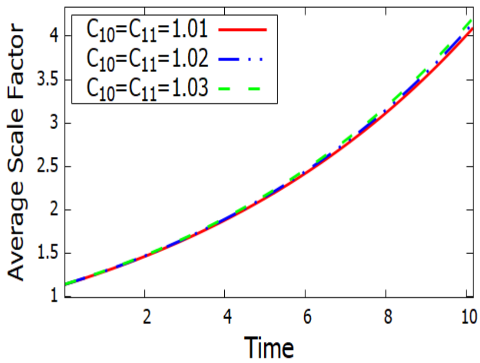

Figure 8: Plot of Average Scale Factor vs. Time for $C_{10} = C_{11} = 1.01, 1.02, 1.03$

Equation (38) gives the Average Scale factor. Figure 8 shows the plot of average scale factor with respects to cosmic time. It is observed from the figure that average scale factor is increasing function of time.

Hubble Parameter $H = \dfrac{C_{10}}{8}\exp\left(-\left(C_{10}T + C_{11}\right)\right)$ (39)

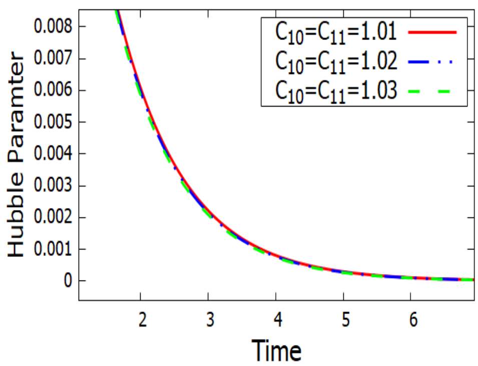

Figure 9: Plot of Hubble Parameter vs. Time for $C_{10} = C_{11} = 1.01, 1.02, 1.03$

Hubble parameter is calculated as in equation (39) and a graph of the Hubble parameter is shown in figure 9. The graph shows that the Hubble parameter decreases with time, it suggests that the expansion of the universe is accelerating.

Expansion Scalar $\theta = \dfrac{3C_{10}}{8}\exp\left(-\left(C_{10}T + C_{11}\right)\right)$ (40)

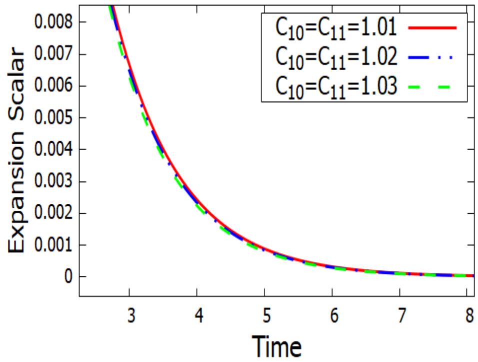

Figure 10: Plot of Expansion Scalar vs. Time for $C_{10} = C_{11} = 1.01, 1.02, 1.03$

Equation (40) gives the expression for Expansion Scalar and figure 10 gives the graph of Expansion Scalar versus time. It is observed from the graph that the Expansion Scalar reduces as time increases and vanishes as time tends to infinity.

Anisotropic Parameter $A_m = \dfrac{16C_{16}}{{C_{10}}^2}$ (41)

The anisotropic parameter of the cosmological model given in equation (41) is obtained to be constant $(C_{10} \neq 0)$. It is observed that, the anisotropy parameter is constant and different from zero for $C_{16} \neq 0$ and vanishes at $C_{16} = 0$. That means the universe is anisotropic throughout the evolution except for $C_{16} = 0$.

Shear Scalar $\sigma^2 = \dfrac{C_{16}}{2}\exp\left(-2\left(C_{10}T + C_{11}\right)\right)$ (42)

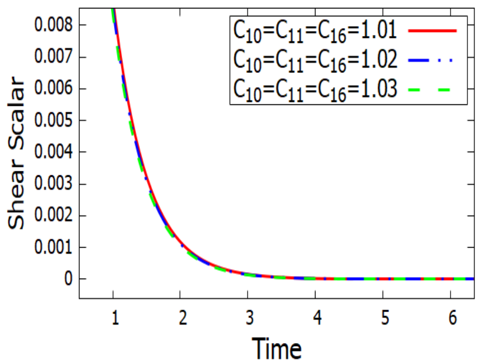

Figure 11: Plot of Shear Scalar vs. Time for $C_{10} = C_{11} = C_{16} = 1.01, 1.02, 1.03$

Shear Scalar is obtained as expressed in equation (42) which is plotted against time in figure 11 and it is observed from the graph that the shear scalar decreases as time increases and vanishes as time tends to infinity.
Directional Hubble parameters are obtained as

$$H_1 = C_7 \exp\left(-\left(C_{10}T + C_{11}\right)\right)$$
$$H_2 = \left(C_1 + C_7\right)\exp\left(-\left(C_{10}T + C_{11}\right)\right)$$
$$H_3 = \left(C_1 + C_3 + C_7\right)\exp\left(-\left(C_{10}T + C_{11}\right)\right)$$
$$H_4 = \left(C_1 + C_5 + C_7\right)\exp\left(-\left(C_{10}T + C_{11}\right)\right)$$

Red shift-
For red shift, average scale factor a is related to a_0 by the relation,

$$1 + z = \frac{a_0}{a}$$

Where, subscript 0 denotes the present phase and a_0 is the present scale factor
Hence, we get,

$$\text{Red shift } z = -1 + \exp\left(\frac{C_{10}\left(T_0 - T\right)}{8}\right) \qquad (43)$$

Here, T_0 is the age of universe at present time.

$$T = \frac{8}{C_{10}}\left(1+z\right) - \frac{C_{11}}{C_{10}} \qquad (44)$$

$\boldsymbol{Om(z)}$ diagnostic- In the context of the $Om(z)$ diagnostic, the Hubble parameter serves as the foundational element. This diagnostic is independent of the model and helps to distinguish between different dark energy models, such as the ΛCDM model and its alternatives. In terms of observations and cosmological parameters, the $Om(z)$ diagnostic is then defined as,

$$Om(z) = \frac{\dfrac{H_z^{\,2}}{H_o^{\,2}} - 1}{\left(1+z\right)^3 - 1}$$

$$Om(z) = \frac{\left(\dfrac{C_{10}}{8}\left(1+z\right)^{-8}\right)^2 - H_o^{\,2}}{\left[\left(1+z\right)^3 - 1\right]H_o^{\,2}} \qquad (45)$$

From equation (45), it can be observed that $Om(z)$ increase as z decrease, indicating that dark energy is dynamically increasing and is playing a more significant role in the recent expansion of the universe.

Energy Conditions- The examination of energy conditions plays a crucial role in analyzing the behavior of both null and time-like geodesic congruence. These conditions are also fundamental in understanding the dynamics of strong gravitational fields. The commonly used or standard energy conditions are outlined as follows:

Null Energy Condition (NEC)-
$p_\Lambda + \rho_\Lambda \geq 0$
Weak Energy Condition (WEC)-
$\rho_\Lambda \geq 0,\ p_\Lambda + \rho_\Lambda \geq 0$
Strong Energy Condition (SEC)-
$p_\Lambda + \rho_\Lambda \geq 0,\ \rho_\Lambda + 3p_\Lambda \geq 0$
Dominant Energy Condition (DEC)-
$\rho_\Lambda \geq \left|p_\Lambda\right|$

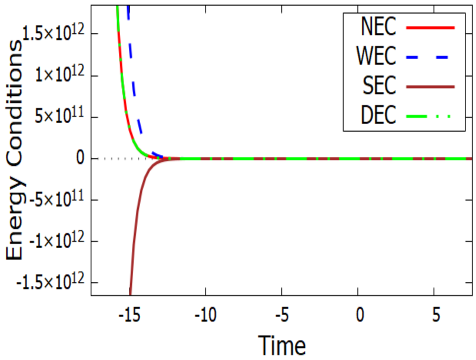

Figure 12: Plot of Energy conditions for HDE vs. Time

From figure 12, it is observed that the obtained cosmological model satisfies all the energy conditions.

V. THERMODYNAMICAL BEHAVIOUR AND ENTROPY OF THE UNIVERSE

The total entropy associated with the universe, including both the interior and the horizon boundary, increases monotonically with time [24]. By applying the first and second laws of thermodynamics to a system enclosed by a horizon with volume V, we get,

$$\tau dS = d\left(\rho V\right) + pdV \qquad (46)$$

Where, $\rho = \rho_m + \rho_\Lambda$, τ and S are the temperature and entropy respectively [24]. Again, above equation can be written as,

$$\tau dS = d\left[\left(p+\rho\right)V\right] + pdV$$

Since dark matter and dark energy are often represented as perfect fluids, this assumption helps simplify the analysis of their behaviour in cosmological models. The perfect fluid in thermodynamics can be defined as,

$$dp = \left(\frac{p+\rho}{\tau}\right)d\tau$$

Hence, from above we get,

$$dS = \frac{1}{\tau} d[(p+\rho)V] - (p+\rho)V \frac{d\tau}{\tau}$$

This leads to,

$$dS = d\left[\frac{(p+\rho)V}{\tau}\right]$$

On integrating above, we get

$$S = \frac{(p+\rho)V}{\tau}$$

The thermodynamics of the Universe, particularly its entropy, does not depend on the properties of individual fluid components. Instead, it is determined by the total matter density and the isotropic pressure of the fluid. Let us denote the entropy density by $\bar{S}$ so that

$$\bar{S} = \frac{S}{V} = \frac{p+\rho}{\tau} = \frac{(1+\gamma)\rho}{\tau} \quad (47)$$

By expressing the entropy density in terms of temperature, the first law of thermodynamics can be reformulated as follows:

$$d(\rho V) + \gamma\rho dV = (1+\gamma)\tau d\left(\frac{\rho V}{\tau}\right) \quad (48)$$

This yields the expressions for the temperature and entropy density as follows,

$$\tau = \rho^{\frac{\gamma}{1+\gamma}}, \; \bar{S} = (1+\gamma)\rho^{\frac{1}{1+\gamma}} \quad (49)$$

Thus, from equations (32), (34), the Hawking temperature τ and Entropy density $\bar{S}$ can be written as,

$$\tau = [C_{15}\exp(-2(C_{10}T + C_{11}))]^{\gamma/1+\gamma} \quad (50)$$

$$\bar{S} = (1+\gamma)[C_{15}\exp(-2(C_{10}T + C_{11}))]^{1/1+\gamma} \quad (51)$$

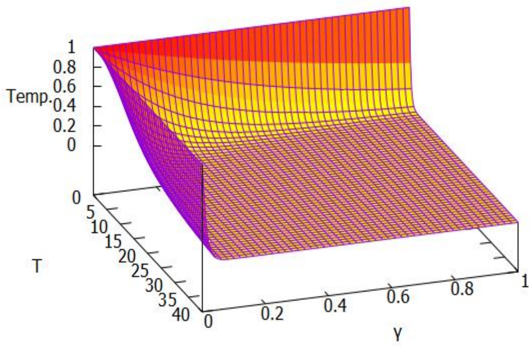

Figure 13: Plot of Thermodynamic Temperature vs. Time for $0 < \gamma < 1$

In the derived model, the thermodynamic temperature is found to be a decreasing function of time (Figure 13). It starts with an infinite value at the initial stage of the Universe and gradually approaches a constant value as the Universe expands. This behavior supports the second law of thermodynamics.

Furthermore, Equation (51) describes the entropy density of the proposed dark energy model. The entropy density is positive and decreases over time, mirroring the behavior of the energy density (as shown in Figure 14. The total entropy of the Universe increases continuously throughout cosmic evolution since $\frac{\dot{S}}{S} > 0$.

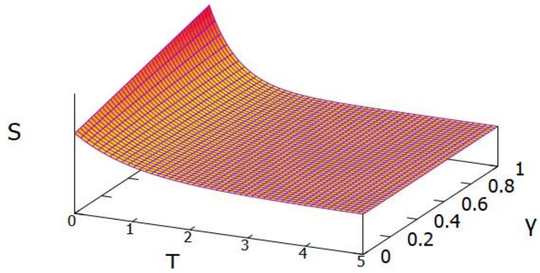

Figure 14: Plot of Entropy vs. Time for $0 < \gamma < 1$

VI. CONCLUSION

In this paper, we have obtained a cosmological model by solving field equations using a transformation. Along with the gauge function, Energy density and pressure of both Dark matter and HDE are calculated. Gauge function is found to be a decreasing function of time. At an early stage, the gauge function β is infinite and decreases with the evolution of the universe and vanishes as time tends to infinity. Hence, the model has singularity when the cosmic time is zero. It is observed that Energy density of DM is decreasing function of time. At an early stage of evolution of the universe, energy density of DM was dominating and it vanishes as time tends to infinity. Negative pressure of HDE is obtained to be increasing as cosmic time increases which indicates accelerated expansion of the universe. Some Physical and Kinematical properties of the obtained model are calculated and discussed. Plot of EoS parameter is observed to be decreasing function of cosmic time. Also, Coincidence parameter of HDE is obtained which shows that there is proper kind of interaction between energy densities of the Dark Matter and Holographic Dark Energy since their ratio approaches to a constant value during evolution. It is found that the universe must be expanding and this expansion of the universe must be accelerating since spatial volume increases exponentially as time increases and Hubble parameter decreases with increase in time. This idea stems from the observation that all galaxies seem to be receding from each other at an accelerating pace. Other properties like Average Scale Factor, Expansion Scalar, Anisotropic Parameter, Shear Scalar, Red shift are studied. It is observed from the value of Anisotropic parameter that the universe is anisotropic throughout the evolution except for $C_{16} = 0$. Also, energy conditions for HDE are studied and plotted against cosmic time. It is concluded that each energy condition is satisfied.

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Study $Om(z)$ diagnostic shows that, $Om(z)$ increases as redshift (z) decreases, suggesting that dark energy is evolving dynamically and becoming increasingly dominant in driving the universe's accelerated expansion at late times. The thermodynamic behavior of the derived dark energy model aligns with the second law of thermodynamics. The temperature decreases over time, approaching a constant value, while the entropy density remains positive and decreases similarly to the energy density. Despite this, the total entropy increases continuously, confirming the model's physical consistency and thermodynamic validity.

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