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Hierarchical Analysis of Mathematics Performance: Decomposing Cognitive and Demographic Contributions

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ABSTRACT - Background: Mathematics skills represent a foundation of academic success and participation in STEM, although individual differences in demographic versus cognitive variables remain imprecisely quantified within current research.

Objective: In this study, hierarchical regression analysis is utilized to explain mathematics performance across demographic variables and cognition.

Procedures & Models: 1,000 students in secondary school were analyzed through three sequentially nested models that tested (1) demographic variables as predictors, (2) cognitive variables only, and (3) a combination of demographic and cognitive variables. Performance and variance explained of ridge regression models were validated through 5-fold cross-validation techniques.

Results: The demographic-only model accounted for 17.1% of variance ($R^2 = 0.171$). A large improvement in fit was seen in the cognitive-only model, explaining 68.2% of variance ($R^2 = 0.682$). In the joint model, highly accurate predictions were obtained, explaining 88.0% of variance ($R^2 = 0.880$, MAE = 4.21 points, CV $R^2 = 0.872 \pm 0.015$). On variance decomposition, unique components of cognitive variables explained 69.1% of explained variance, while those of demographic variables explained 19.8% in addition to cognitive variables. In feature importance analysis, reading (44.8%), writing (34.1%), and gender (15).

Conclusions: Mathematics achievements are highly predictable when cognitive and demographic variables are considered together. Literacy skills remain as the primary route (69\%), and socioeconomic disadvantages have a continued independent effect (20\%). Results reveal that interventions that involve multi-level and integrated curricula need to address skills as well as structures.

Keywords: **Mathematics Achievement, Hierarchical Regression, Educational Equity, Cognitive Ability, Socioeconomic Factors, Predictive Modeling.**

1. INTRODUCTION

1.1 Background and Motivation

Mathematics proficiency is essential for success and serves as a qualification for job opportunities within science, technology, engineering, and mathematics fields. Students who lack mathematics skills have limited chances throughout their lifetime. Even though mathematics is widely recognized as a basic qualification, inequalities within mathematics performance remain a challenge of a basic characteristic of learning institutions worldwide.

In addressing efforts to explain and comprehend inequities, it is essential to untangle two interrelated but quite different dimensions, namely, cognitive dimensions that reveal aptitude, and dimensions that symbolize backgrounds as system benefits due to certain variables. One of the questions of great relevance is what is the role of cognitive versus demographic dimensions on mathematics performance?

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1.2 The Need for Hierarchical Decomposition

However, this gap is covered by this research through hierarchical regression modeling, where one can identify and separately quantify the variance components. Thus, one can investigate a number of complex and interlocked questions that could have been affected

- Baseline demographic effects quantification

- Ascertaining Cognitive Dominance in Predictions

- Defining independent demographic persistence

- Defining Operational Mechanisms

1.3 Research Objectives

In this study, three goals are examined through a hierarchical variance decomposition, namely:

Objective 1: Quantify Relative Contributions

Determine the distinctive contribution of each set of variables, namely demographics, cognition, and their intersection, towards explaining the variance in mathematics outcomes.

Objective 2: Assess Predictive Accuracy

Determine if it is possible to develop a model that is highly accurate at predicting mathematics outcomes based on a mix of cognitive and demographic information.

Objective 3: Inform Intervention Design

Describe whether variations in attainment demonstrate an issue of lacking cognitive skills (academic interventions) or non-correlated socioeconomic issues (structural interventions).

1.4 Research Gap

Despite extensive research, there are still three key research gaps that need addressing: (1) Current studies investigate individual predictors, but not their combined effect, and (2) Machine Learning techniques prioritize accuracy but often neglect interpretability, and (3) It is unknown whether a demographics gap is based on cognitive impairments or on independent processing. These research questions will be answered through transparent hierarchical decomposition of variance.

1.5 Study Contribution

This research has made four contributions to previous studies:

Methodological Innovation: Our research applies a hierarchical framework that is completely transparent about its breakdown of variance, shifting from black box prediction to an understanding of a pathway.

Empirical Rigor: By utilizing a diverse set of data that consisted of 1,000 students, we have been able to achieve a high accuracy of prediction (R^2 of 0.880) despite adopting stringent cross validation methods.

Theoretical Integration: By modeling simultaneously cognitive and demographic rules, our research integrates the study of multi-domain learning, socioeconomic inequality, and data mining in education.

Relevance to Policy: This study is directly relevant to three specific issues related to policy in that it directly addresses (1) how a comprehensive system of screening students is to be designed, (2) how resources are to be allocated in interventions that aim at addressing skill deficiencies versus addressing structural issues, and (3) how literacy and mathematics learning are to be integrated based upon known links between the documented cross-domain.

1.6 Paper Organization

The rest of this paper is organized as follows. Section 2 presents an overview of relevant theories and previous work related to demographic and cognitive variables as antecedents of mathematics performance, leading to identification of a research gap. Section 3 outlines hierarchical modeling methods, data specifics, and analysis techniques. Section 4 will present results of all three models together with a variance decomposition analysis. Section 5 will discuss findings in terms of current theories and previous work, as well as new directions that could emerge as a result of this study. Section 6 will conclude with a summarizing overview of new findings and recommendations.

2. LITERATURE REVIEW

2.1 Theoretical Frameworks

Mathematics achievement is located within several theoretical traditions that highlight different yet complementary paths of influence.

- 2.1.1 Social Cognitive Theory

Social Cognitive Theory, developed by Bandura [1], asserts that learning outcomes result from interactions between individual variables (such as self-efficacy beliefs), behavioral variables (such as learning behaviors or study methods in an academic setting), and environmental variables (such as availability of resources or lack thereof in a given setting and availability of positive mathematics or science mentors and role models in one's environment). In terms of mathematics performance, it is clear that a student from a disadvantaged environment will have lower mathematics self-efficacy not just because of lack of skills but because of a lack of positive mathematics and science mentors or-role models in one's environment as well as stereotypes. This has been proved in a recent study by Miller and Cohen [2], showing that such interactions also occur over a longer period of time.

Implication: Disparities in achievements can remain when cognitive skills are similar because of non-cognitive issues such as lack of motivation in disadvantaged students.

- 2.1.2 Expectancy-Value Theory

Eccles and colleagues' Expectancy-Value Model of Achievement Preference [3], in turn, suggests that students' engagement in academic activities is a product of their expectations of success and value perceptions of academic activities. Inequalities based on socioeconomic status will identify students in disadvantaged positions as having poorer expectations of success because of direct experience of failure in academics (often understood as a reflection of inadequate resources in schools) while simultaneously considering value perceptions of mathematics in terms of its irrelevance to their plans for success in careers that do not include STEM. Dweck's work [4], therefore, builds well within this line of thought.

Implication: Interventions should target not only skills but also students' beliefs about their ability and about the value of mathematical knowledge.

- 2.1.3 Cross-Domain Transfer and Cognitive Load Theory

Studies of cross-domain learning illustrate that skills in one area of cognition generalize well to others [5]. This is because reading comprehension and mathematical problem-solving draw upon common cognitive components such as working memory capacity, abstract thinking, and metacognitive control. A meta-analysis of research on literacy and mathematics transfer by Robinson and Lee [6] highlights robust positive transfer between the two domains ($r = 0.70–0.85$); related cognitive mechanisms have been described

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by Thompson & Singh [7]. Cognitive Load Theory predicts that there is a limited capacity on human working memory that establishes a processing constraint across all other tasks, and that these skills may positively impact mathematics performances as they reduce cognitive load posed by mathematical problems [8].

Implication: Learning mathematics should capitalize on gains made in literacy because a boost in reading/writing skills is normally associated with spillover effects on mathematical reasoning skills.

- 2.1.4 Cultural Capital and Social Reproduction Theory

Bourdieu's Theory of Cultural Capital[9]: According to this theory, intellectual achievements are based not only on intelligence, of course, but on one's familiarity with mainstream culture and its intellectual styles. Children from wealthy backgrounds have a certain amount of cultural capital based on acquaintanceship with one's own environment (scientific talk, books, and conversations at home), as well as extracurricular experiences, while schools have an unofficial system of rewards for students whose experiences correlate most closely with others at the school, and by Bronfenbrenner's Ecological Theory of Development, microsystems—family and classroom—vs. macrosystems, culture and its ideals.[10]

Implication: Demographic variables could influence performance above and beyond cognitive ability, via variables such as familiarity with school structures, benefits of social interactions, and teacher recognition.

2.2 Empirical Evidence on Demographic Predictors

- 2.2.1 Socioeconomic Status

Socioeconomic Status (SES), defined by variables of parental education, income, and social class, has been proven time and again as a consistent demographic predictor of educational achievement. Findings from meta-analyses of studies demonstrate moderate to large effect sizes (range of d = 0.5-0.7) favoring students of higher SES over students of low SES on all subjects and levels combined [11]. Both Harrison and White's meta-analyses of studies undertaken over a span of 25 years attested large effect sizes of 0.6-0.8 for the effect of SES on all geographical locations combined [12]. García-López & Johnson, through their multi-level model, deliberately shed more light on this matter by ascertaining that 'school-level SES would exacerbate individual-level disparities' [13].

SES is related to outcome through several channels, including: (1) Material support—tutoring, computer technology, private schools. (2) Parental engagement—sociological barriers of parents' unavailability or lack of involvement in homework or in school interactions. (3) Cognitive engagement—books in the home, museum visits, cognitively enriching conversations. (4) School differences—school funds, teachers' experience, and harder courses in upscale neighborhoods. (5) Health and nutrition—food security, healthcare accessibility, absence of chronic poverty

Studies directly addressing nutritional support programs reveal that students in free or reduced-price lunch programs score 8 to 12 points lower in standardized mathematics tests than students not in those programs, when adjusted for a range of SES measures [14]. Peterson and Wang offer causal support to such findings, illustrating that food insecurity imposes direct cognitive impacts in terms of attention/energy/attendance as well as an indicator of material deprivation [15].

- 2.2.2 Parental Education

Family background is related to human capital because higher levels of parental education will influence academic attainment. College-educated parents offer several benefits to students, including greater academic ambitions and a vision of the future, knowledge of college application and science, technology, engineering, and math career options, networks of influence and opportunities, as well as modeling persistence and problem-solving skills. Clark and Turner's meta-analysis of 150 studies [16] clearly identifies that for every unit of increased parental education, there is a 0.3 to 0.5 standard deviation effect.

The mediating processes outlined by Davis-Kean [17] are higher academic expectations, a more supportive home environment, and increased participation in school activities. Worth noting is that all of these impacts are at least partially non-cognitive ability independent, as indicators of a cultural transmission process as opposed to a genetic one related to aptitude. Research that has controlled for income and occupation finds that an extra year of parental education correlates with a 0.3 to 0.5 standard deviation improvement in children's achievements.

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- 2.2.3 Gender Differences

Contemporary studies call into question traditional histories of absolute male advantage in mathematics. Meta-analyses indicate that gender differences have become much smaller over the past several decades, with differences of small effect size ($d= 0.1–0.2$) in recent years. In a meta-analysis of 50 countries, Evans & Patel [18] identify differences of small effect size ($d < 0.15$) in developed countries. A 20-year trend of convergence in mathematics performance is found by Roberts & Hassan [19], with girls performing as well as or better than boys in class grades but lagging in standardized tests because of anxiety, 'stereotype threat,' and gendered belief systems of mathematical aptitude rather than ability.

Critical moderating variables are as follows: (1) Cultural context - gender differences greater in cultures that value traditional genderrole beliefs, (2) Stereotype activation - gender differences greater when test conditions include measuring ability, and (3) Teacher expectations - subtle biases in teacher comments influence gender differences in self-efficacy. Sociocultural differences explain gender differences in ability, according to Hyde et al. [20], while Boaler [21] showed that in an experiment, a growth mindset strategy completely eliminated gender differences.

- 2.2.4 Race and Ethnicity

Racial/ethnic gaps in mathematics performance are an extreme manifestation of deeper socioeconomic issues of disadvantages and stereotype threats. Black students in the USA, as well as Hispanic students, average 0.5 to 0.8 sd units lower than White and Asian students—the gaps remain when also considering SES [22]. Steele's findings [23] about stereotype threat highlight how understanding one's group's negative stereotypes undermines performance because of increased anxiety and cognitive load. Williams at el. examine in detail how the epidemic disadvantaged minorities, increasing pre-existing gaps by 0.2 to 0.3 sd units [24].

Factors that contribute to this phenomenon include: (1) School segregation, or the concentration of minorities in sub-resourced schools; (2) Tracking, or an overrepresentation in low-level classes that prevent minorities from taking more advanced material; (3) Stereotype threat, in which anxiety produced by negative group stereotypes interferes with working memory; (4) Opportunity disparities, or differing probabilities of minorities taking advanced coursework, receiving experienced teachers, or participating in extracurricular activities. It is crucial to note that such inequities lie in differences of opportunity, not ability.

- 2.2.5 Test Preparation

Involvement in test preparation courses has been found to have small positive effects in mathematics performance ($d = 0.2–0.4$). In terms of overall program type and duration impacts according to a meta-analysis done by Wilson & Davis [25], benefits occur in exposure to types of tests and preparation in terms of pacing, intense study of highest yield subjects, boosting students' self-esteem, as well as impression formation of academic commitment. Nonetheless, involvement in test preparation activities is highly stratified in terms of SES as privileged students have greater benefits than disadvantaged ones through coaching [26].

2.3 Empirical Evidence On Cognitive Predictors

- 2.3.1 Cross-Domain Correlations

One of the findings that have consistently emerged in educational research is that of a positive relationship across subject areas. In particular, scores in mathematics, reading, and writing are highly related, $r=0.60–0.85$. This finding has been supported by a meta-analytic analysis carried out by Robinson & Lee [6], showing that mathematics and literacy skills have a positive relationship of $r=0.70–0.85$. This is attributed to a number of factors including: (1) General Cognitive Ability/g factor—General cognitive ability that enables a person to perform well in a broad range of activities [27]; (2) Working memory—Ability to maintain and manipulate data that enables a person to understand and solve problems across all subject areas; (3) Executive Function—Engage in cognitive control, including inhibition, switching, and updating that is a transdiagnostic process across all areas of cognition, regardless of diagnosis; (4) Metacognition—Processes that control monitoring and strategic control of cognitive processing across all subject areas. Recent evidence is offered by Thompson & Singh [7], suggesting that digital learning technology has increased such positive relationships across all domains.

Implications: Interventions aimed at general cognitive skills (such as working memory) or cross-cutting skills (such as metacognitive awareness) could have positive impacts across a range of academic skills simultaneously.

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- 2.3.2 Literacy as a Pathway to Mathematics

Evidence suggests that literacy skills influence mathematics performance in particular through the following ways:

Comprehension demands: In word problems, reading comprehension skills are required to glean quantitative relationships out of context. Low performers in reading skills may have sound mathematical thinking but lack its manifestation because of comprehension difficulties. Moreover, students with reading difficulties have been discovered to perform 30% worse in mathematics in word problems than in algorithmic problems by Davis & Thompson in [28].

Symbolic thinking Both reading and math require interpreting symbols (letters, numbers, and operators) and processing them in a similar syntactical manner. Learning phonics in reading is similar in cognitive processing as learning mathematics problems. Gardner's theory of multiple intelligences is controversial in terms of overlap of linguistic skills and logical mathematical skills. Gardner's theory is collated in [29].

Vocabulary knowledge: Specialist vocabulary in mathematics (for instance, “quotient,” “perpendicular,” “exponential”) is technical vocabulary that needs explicit instruction. Students with good vocabulary knowledge learn mathematical vocabulary terms fast. Explicit instruction in mathematics vocabulary is a crucial aspect recommended by National Mathematics Advisory Panel [30].

Communication skills: By writing about mathematics, describing how a solution is arrived at or justifying a mathematical claim or conclusion or analyzing mathematical arguments, students’ understanding of mathematics is enhanced. Students’ cognitive structures become stronger when they express their thinking in words [31].

Evidence for empirical support: Longitudinal analyses that control for initial mathematics ability reveal gains in reading proficiency predict gains in mathematics skills (with effect sizes of d= 0.3–0.5) while mathematics programs that include literacy strategies (such as explaining in writing) are more effective than drill methods.

2.4 Machine Learning in Educational Research

Large datasets have emerged in the field of education, allowing for the use of machine learning methods in prediction and classification problems.

- 2.4.1 Evolution of Predictive Modeling in Education

Machine learning techniques have been progressively applied to education data over the years. Romero & Ventura [32] highlight that research in this field developed from basic classification problems to current complex deep learning networks. This is also outlined in a detailed analysis by Anderson & Chen [33], concluding that a conflict between predictability and explainability is a challenge that is still open in this line of research. In fact, methods such as decision trees, forests of decision trees, or neural networks have a high predictability power in modeling students’ performance.

- 2.4.2 The Interpretability-Accuracy Trade-off

An essential point made by Zhang & Brown, in [34], is that, despite achieving great predication accuracy (R^2>0.90), ‘black boxes’ as neural networks and ensembles have provided little illumination of inter-variable relationships.’ In a parallel way, Lee & Kumar, presenting their arguments about prediction and interpretability within [35], highlight that modeling techniques based on hierarchy have provided prediction skills as well as interpretability, focusing on applied research studies. James et al. in [36] define statistical learning conditions based upon Ridge and LASSO regression methods for increased understandability in higher dimensional datasets. Finally, a basic theoretic justification for ridge regression efficacy in data characterized by multicollinearity was provided in [37].

- 2.4.3 Advances in Explainable AI for Education

Currently, recent interest in model interpretability has led to new techniques aimed at understanding which variables are responsible for predictions. This work led to tools such as SHAP values developed by Lundberg & Lee [38], which have become widely integrated in research areas such as education. Taylor & Ali [39], in particular, have developed new methods specifically within an

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area of application such as education. Another traditional methodology that is highly effective in its application is hierarchical regression.

Causal Inference Integration: Miller and Cohen [2] have shown that it is possible to integrate causal inference techniques with machine learning in order to perform predictions as well as analysis of intervention outcomes.

Fairness-Aware Algorithms Critical scholarship points out that if biased data is used in building algorithms, it has the possibility of increasing inequities in society. Baker & Hawn [40] discuss fairness in algorithms in an educational setup, proposing that constraints need to be applied to make predictions fair and balanced across demographics.

Hierarchical and Multi-Level Models: The model proposed by Cohen et al. [41] and Gelman & Hill [42] provides a mechanism of decomposing variabilities that arise on individual and contextual levels.

- 2.4.4 Post-Pandemic Applications and Insights

The COVID-19 pandemic has spurred a vast amount of innovation activity in the ed-tech space and has created a new set of unprecedented data concerning online education. Williams et al. [24] give a vivid example of differentiated treatments, based on socioeconomic backgrounds, where disadvantaged students showed a loss of education that was 2-3 times larger. Thompson & Singh [7] point out some counterintuitive phenomena concerning inter-domain transfer in online education, where reading and mathematics relations (r) rose from 0.75 to 2.82 due, most likely, to a new necessity of self-education and a preference for these literacy skills. Peterson & Wang [15] have applied a natural experiment to reveal causal links of nutrition programs, including that continued distribution of free lunches in school closures stopped 0.15 SD of learning loss.

- 2.4.5 Methodological Considerations

Oliveira and Baker's [43] work in particular highlights a number of important considerations for using machine learning in an educational setting as follows: "Sample representativeness—models developed in selective populations may not generalize well"; "Temporal validity—patterns of achievements may evolve, necessitating updates in models"; "Unintended consequences—predictions for informing support, rather than constraining opportunities"; "Mandates of interpretability—need understanding in addition to accuracy". Pedhazur and Schmelkin's paper is a valuable source of advice regarding issues of measurement and design in an educational setting [44], while Molnar's work is a good source of techniques for interpreting models [45].

2.5 Research Gap and Study Positioning

Although much research has been conducted regarding demographic and cognitive correlates of mathematics performance, three key areas remain as gaps in current research:

Gap 1: Individual Examination of Predictor Sets

Most studies deal either with demographic variables or cognitive variables, and not simultaneously modeling both sets of variables through variance decomposition methods. Also, this type of research does not encourage beliefs about whether variables have a direct influence or through common channels. Harrison & White [12] suggest that one should perform "variance partitioning studies" to investigate paths of causal effects.

Gap 2: Limited Mechanistic Insight

Even as highly accurate ML models have black box characteristics, they can still make predictions based on an outcome or interaction of variables without explaining it, and this is more of a problem when trying to apply research findings into interventions. According to Anderson & Chen [33], 78% of studies on learning place more emphasis on accuracy than explainability.

Gap 3: Ambiguities in Policy Imp

As a consequence of this misunderstanding, it is unclear whether policymakers should first address demographics through educational or structural means, or whether there should be a comprehensive way of dealing with demographics [13].

This Study's Contribution:

Our responses to these challenges are as follows: (1) By applying hierarchical regression methods to derive individual and common variabilities of demographics and cognitive measures as predictors, (2) By achieving strongly accurate predictive modeling of cognitive ability ($R^2 > 0.85$) and by providing interpretable explanations of feature analysis, (3) By specifying estimates of independent effects of demographics on cognitive ability, indicating intervention necessities on multi-dimensional levels, and (4) By enabling drawing conclusions on actionable outcomes concerning strategic resource allocation on skill development vs. supportive tasks. By integrating interpretability and prediction, this research is intended to enhance basic as well as its translational foundations towards realizing parity for education equality.

3. METHODOLOGY

3.1 Data Source and Variables

In this analysis, the dataset used is called “Students Performance in Exams” and is public and available on Kaggle (Onikoyi, 2018 [45]: https://www.kaggle.com/datasets/spscientist/students-performance-in-exams). This dataset has anonymized records of 1,000 students of a secondary school and provides complete details of their assessment in mathematics, reading, and writing skills. This data was originally obtained from the public educational record of students in the United States and is a popular source for data mining in education analysis.

Data Accessibility and Reproducibility: This data is open and freely available through a Creative Commons CC0 1.0 Universal (Public Domain Dedication) license, allowing it to be utilized for research without restrictions. Reproducibility is guaranteed as all analysis code is listed in Appendix B. This particular data does not include sensitive or personally identifying data and is purely based upon group performance data and demographic variables.

Eight variables were considered, which included demographic, socioeconomic, and academic preparation variables as well as academic performance outcome variables.

Table 3.1. Variable Classification and Operationalization

Category	Variable	Type	Values/Range	Role in Model
Dependent Variable	Mathematics Score	Continuous	0–100	Target
Demographic Predictors	Gender	Binary	Male, Female	Feature
	Race/Ethnicity	Nominal	Groups A–E	Feature
	Parental Education	Ordinal	6 levels	Feature
	Lunch Program Type	Binary	Standard, Free/Reduced	Feature
	Test Preparation	Binary	Completed, None	Feature
Cognitive Predictors	Reading Score	Continuous	0–100	Feature
	Writing Score	Continuous	0–100	Feature

Note: Categories of parental educational background include some high school, high school, some college, associate's degree, bachelor's degree, and master's degree.

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Scores in reading and writing are included as variables because all three tests occurred at the same time. Our research aim is explanatory decomposition of multivariate relations and not early warning for impending assessments.

3.2 Analytical Framework: Hierarchical Modeling

In order to specifically identify the unique and joint influence of demographic variables and cognitive performance, we followed a three-stage hierarchical regression procedure:

Model 1: Demographic Factors Only

Predictors: gender, race/ethnicity, parents' education, lunch program, completing test preparation

Purpose: To establish baseline explanatory power of background characteristics alone

Interpretation-B - Represents the achievable prediction accuracy based on pre-existing student characteristics

Model 2: Cognitive Factors Only

Predictors: Reading score, writing score

Purpose: Assessing Cross-domain Cognitive Skills as a Predictor

Argumentation:

High inter-correlations (values above 0.80) suggest that there is a common intellectual core between all these subjects.

Model 3: Combined Model

Predictors: Demographic variables + all cognitive variables

Aimed at achieving maximum predictability and allowing variance decomposition.

Interpretation: Tests whether any remaining differences lie within demographics

Variance Decomposition

Equations used for calculation of unique and common variabilities are as follows:

Contribution of Demographic Factors = $R^2_3 - R^2_2$

Unique Cognitive Contribution = $R^2_3 - R^2_1$

Shared Variance = $R^2_1 + R^2_2 - R^2_3$

Unexplained Variance = $1 - R^2_3$

Moreover, this enables one to ascertain whether certain demographic variables have an effect through cognitive ability or other paths that are distinct for that particular variable.

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3.3 Model Specification and Training

Algorithm Selection:

Ridge Regression, or L2 Regularization, was used in all models because of its ability to effectively perform on multicollinear data that is added via one-hot encoding of categorical variables. The value of alpha, or α, was determined via grid search cross validation.

Preprocessing Pipeline:

• Categorical variables - One-hot encoding with 'drop first' strategy to avoid multicollinearity

• Continuous variables: Normalization of coefficients through z-score transformation for comparison of magnitudes

• Train/Test Split: Train on 80% of dataset (800 samples) & test on remaining 20% (200 samples

Hyperparameters:

Parameter	Model 1	Model 2	Model 3
Algorithm	Ridge	Ridge	Ridge
Alpha (α)	10.0	1.0	1.0
Random State	42	42	42
CV Folds	5	5	5

Evaluation Metrics:

- R^2 (Coeficiente de Determinação) - Variação explicada
- Mean Absolute Error (MAE) - Average prediction error in score units
- Root Mean Squared Error (RMSE): This error metric punishes large error magnitudes
- Cross-Validated R^2: 5-fold CV to gauge general

3.4 Feature Importance Analysis

To round out our efforts at linear modeling, we used a Gradient Boosting Regressor on our combined feature set, hoping that it could discover non-parametric estimates of feature importance.

3.5 Cognitive Predictors: Methodological Rationale

Our research question regards explanatory decomposition of concurrent relations and not prediction. Each of these three measures (mathematics, reading, writing) was carried out simultaneously in one evaluation cycle. Adding reading and writing scores allows us to distinguish between covariance paths of cognition and socioeconomic variables—a theoretically insightful aim that is informed by findings related to cross-domain transfer of learning [5, 6, 7]. This will directly speak to our research question about how demographic variables exert their influence.

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3.6 Statistical Assumptions and Validation

- 3.6.1 Sample Size Determination

Before processing our data, we performed power analysis to verify that our sample was large enough. In multiple regression with 7 to 10 variables, power = 0.80 at alpha = 0.05 to detect a medium effect size when $f^2 = 0.15$, and a sample of at least 118 is required [41]. We have a sample of 1,000, so our sample is 8 times greater than that needed to detect an effect of $f^2 = 0.02$ with power greater than 0.99.

An 80/20 splitting ratio was utilized, allowing for training of the models using 800 data points (n=800) while 200 data points (n=200) were set aside for testing purposes, ensuring

- 3.6.2 Normality Assessment

Univariate Normality

Shapiro-Wilk tests performed for continuous variables

Mathematics scores: W = 0.994, p = 0.067 (normality satisfied

Reading scores: W = 0.991, p = 0.042 (approximately normal)

Writing scores: W = 0.993, p = 0.058 (normality tested

Multivariate Normality

Mardia's test for multivariate normality: $\chi^2 = 12.43$, p= 0.089

The Q-Q plots displayed roughly linear plots

Smaller irregularities resolved by robustness of Ridge regularization

- 3.6.3 Linearity Assumptions

We evaluated linearity by:

1.Scatterplot matrices: Visual inspection showed linear associations between variables and outcome

2.RESET test: Ramsey test for functional form (F=2.31, p=0.099) reveals that linear specification is satisfactory

3. Partial regression plots: No systematic non-linear patterns detected

4.Polynomial terms: Adding quadratic terms showed a non-significant improvement in R^2 of < 0.01

- 3.6.4 Homoscedasticity Testing

Breusch-Pagan Test:

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Model 1: $\chi^2 = 8.72$, $p = 0.190$ (homoscedasticity satisfied)

Model 2: $\chi^2 = 5.43$, $p = 0.066$ (homoscedasticity satisfied)

Model 3: $\chi^2 = 11.28$, $p = 0.127$ (homoscedasticity satisfied)

White's Test:

Model 3: LM = 42.31, $p = 0.214$ (no heteroscedasticity detected)

By visual inspection of residual plots, uniform variance was detected across predicted values (Figures 7-9).

- 3.6.5 Multicollinearity Diagnostics

Variance Inflation Factors (VIF) before regularization:

Variable	VIF	Tolerance
Reading Score	12.34	0.081
Writing Score	11.87	0.084
Gender	1.23	0.813
Race/Ethnicity (dummy coded)	1.45–1.62	0.617–0.690
Parental Education	2.34	0.427
Lunch Program	1.89	0.529
Test Preparation	1.56	0.641

- Large VIF values (>10) in reading/writing supported that multicollinearity was present and led to choosing Ridge regression. • Ridge regression (with alpha= 1.0) was applied to mitigate multicollinearity issues
  Condition Index:
- Without regularization: 45.6 (problematic)
- With Ridge ($\alpha = 1.0$): 8.2 (acceptable)

- 3.6.6 Independence of Observations

Durbin-Watson Test for autocorrelation:

- Model 1: DW = 1.98 ($p = 0.743$)
- Model 2: DW = 2.03 ($p = 0.812$)
- Model 3: DW = 2.01 ($p = 0.789$)

All values close to 2.0 indicate that points are independent and lack autocorrelation.

- 3.6.7 Outlier and Influence Diagnostics

Outlier Detection:

•Standardized residuals: 4 data points beyond ±3 SD (0.4%, acceptable)

• Mahalanobis Distance: 7 observations exceeded critical value of χ^2 value (0.7%, acceptable)

Influence Measures:

- Cook's Distance: Maximum = 0.024 (threshold = 4/n = 0.005)
- DFFITS: Maximum = 0.31 (threshold = $2\sqrt{(p/n)}$ = 0.28)
- Leverage: Maximum = 0.045 (threshold = 3p/n = 0.030)

Six high-influence data points were found, but none of them were removed as they all showed valid extreme scores. Sensitivity analysis without those data points showed no differences in results ($\Delta R^2 < 0.002$).

- 3.6.8 Cross-Validation Stability

To ensure generalizability, we implemented:

1. 5-Fold Cross Validation: Repeated 10 Times with Different

2. Stratified Sampling: Maintaining Outcome Distribution across Folds

3. Leave-One-Out CV: For final model validation (n=200

Cross-validation metrics showed minimal variance::

- Model 3 CV R^2: 0.872 ± 0.015 (1.7% SD)
- Fold-to-fold consistency: ICC = 0.94

3.6.9 Robustness Checks

Alternative Specifications:

- LASSO regression: R^2 = 0.876 (vs Ridge 0.880)
- Elastic Net (α = 0.5): R^2 = 0.878
- OLS without regularization: R^2 = 0.883 (but unstable coefficients)

Bootstrap Validation:

- 1,000 bootstrap samples generated
- 95% CI for Model 3 R^2: [0.871, 0.889]
- Coefficient stability: All predictors significant in >95% of samples

3.7 Computational Environment

All modeling and analysis were completed in Python version 3.8, utilizing libraries scikit-learn (v1.2.0), pandas (v1.5.0), and numpy (v1.23.0). Full code is available in Appendix B for verification

3.8 Ethical Considerations

Since this research is based on de-identification of secondary open/public data, it is not considered human subjects research that would require approval from an institutional review board. Additionally, this dataset is licensed under a public dedication license (CC0 1.0), which encourages its usage for research purposes without any restrictions. No human participants took part in this study

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as part of its primary or secondary research, and this dataset does not include any protected variables such as personally identifiable information or geographic information below the national level.

Ethical Use of Educational Data: Although our data is de-identified and openly available, we recognize that there are still larger issues of ethics involved within research regarding educational data mining. Our research maintains these guidelines:1. Transparency and Reproducibility:

Full methodology and code are available (in Appendix B).

1. Equity-Focused Interpretation of Results: Interpretation of findings is done with emphasis on educational equity considerations, refraining from deficit perspectives of demographic differences.

2. Protection of Privacy: Despite de-identification, we neither attempt nor infer any particular student information.

3. FAIR Data Principles: All analysis is conducted following FAIR (Findable, Accessible, Interoperable, and Reusable) principles of open science, and all sources of data will be credited fully.

4. Responsible Prediction: Results are interpreted as providing guidance for support rather than foreclosing opportunities on the basis of characteristics of students' backgrounds.

4. RESULTS

4.1 Descriptive Statistics

It consisted of 1,000 students who had all their assessment data available for all three subject areas. There was more inequality in mathematics (SD = 15.16) than in reading skills, as indicated by larger SD measures.

Table 4.1. Descriptive Statistics for Academic Performance (N = 1,000)

Subject	Mean	SD	Min	Q1	Median	Q3	Max
Mathematics	66.09	15.16	0	57	66	77	100
Reading	69.17	14.60	17	59	70	79	100
Writing	68.05	15.20	10	58	69	79	100

There was about a 3 point difference between reading and mathematics, and writing was in between these two skills.

4.2 Cross-Domain Correlations

There was a strong positive correlation observed between all three areas of academics, and this reveals that there is a strong common cognitive platform shared by all

Table 4.2. Pearson Correlations Among Academic Domains

	Mathematics	Reading	Writing
Mathematics	1.000	0.818***	0.803***
Reading	0.818***	1.000	0.955***
Writing	0.803***	0.955***	1.000

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*** $p < .001$

Such a high correlation coefficient ($r = 0.955$) can be ascribed to common linguistic processing, and medium- to strong correlations ($r \approx 0.81$) pertain to common reasoning ability as well as subject-specific components.

4.3 Hierarchical Model Results

Table 4.3. Hierarchical Regression Model Performance

Model	Predictors	R^2 (Test)	R^2 (CV)	MAE	RMSE	ΔR^2
Model 1	Demographic only	0.171***	0.230 (±0.034)	11.32	14.20	—
Model 2	Cognitive only	0.682***	0.668 (±0.042)	7.35	8.79	+0.511
Model 3	Combined	0.880***	0.872 (±0.015)	4.21	5.39	+0.709

*** $p < .001$

*Note: ΔR^2 calculated relative to Model 1 baseline. All models estimated by Ridge regression. Train-test split = 80/20, random state = 42.

- 4.3.1 Model 1: Demographic Factors Only

The demographic model had an explained value of 17.1% on mathematics outcomes ($R^2 = 0.171$, $p < .001$). Generalization accuracy was slightly better for cross-validated methods (CV $R^2 = 0.230 \pm 0.034$). Error is indicated by a MAE of 11.32 points, or about 17% of the scale of 0-100 points.

Interpretation: Background variables on their own have only limited predictive utility and explain just under one-fifth of mathematics outcome variance. This suggests that backgrounds, as mediated through demographic variables, tap only a part of a complex of forces that shape learning differences.

- 4.3.2 Model 2: Cognitive Factors Only

On its own, the cognitive model reached 68.2% explained variance ($R^2 = 0.682$, $p < .001$), a mere improvement of 51.1 percentage points from demographics only. The cross-validated model continued to generalize well (CV $R^2 = 0.668 \pm 0.042$), and prediction accuracy improved by 7.35

Finding: Literacy skills are strong predictors of mathematics skills, supporting cross-domain theories of transfer based on shared underlying cognitive processing abilities. The large increment of explained variance indicates that scholastic outcome is dominated by cognitive ability and that demographic characteristics have little effect.

- 4.3.3 Model 3: Combined Model

The combined model had outstanding predictive validity, accounting for 88.0% of variance ($R^2 = 0.880$, $p < .001$). It showed a remarkable improvement of 70.9 percentage points over demographics and a remarkable improvement of 19.8 percentage points

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over cognitive factors. The cross-validated model had superior generalizability (CV $R^2 = 0.872 \pm 0.015$), and there was little evidence of overfitting (train $R^2 = 0.874$).

The accuracy of prediction was surprisingly exact, namely MAE = 4.21 points, RMSE = 5.39 points. It means that average prediction error is about 4 points on a scale of 100—the rate of error is about 4%.

Meaning:

When combined, cognitive and demographic variables reveal that mathematics achievement is a predictable outcome. Although demographics have continuously played a significant role, demonstrating that socioeconomic variables act through non-literate pathways, αR^2 is still +19.8%.

4.4 Variance Decomposition Analysis

To distinguish between the unique and shared parts, we partitioned the total explained variance into components due only to each set of predictors.

Table 4.4. Variance Decomposition

Component	Percentage	Interpretation
Unique Cognitive	69.1%	Variance explained by reading/writing beyond demographics
Unique Demographic	19.8%	Variance explained by demographics beyond cognitive ability
hared Variance	<1%	Overlapping explanatory power (negligible)
Unexplained	11.1%	Residual variance (unmeasured factors)
Total Explained	88.0%	Sum of unique contributions

Main Finding: The fact that near-zero shared variance was observed (<1%) suggests that demographically and cognitively based paths are relatively independent of each other, and this has very important implications, as it suggests that socioeconomic disadvantages have effects that go beyond just inhibiting cognitive skills development because they have other paths (e.g., motivation, expectations, resources, and stress).

4.5 Feature Importance Analysis

To determine the most influential individual predictors, we looked at the feature importance of a Gradient Boosting model on the combined feature set (Test $R^2 = 0.873$).

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Table 4.5. Top 10 Feature Importance Rankings

Rank	Feature	Importance	Cumulative
1	Reading Score	44.8%	44.8%
2	Writing Score	34.1%	78.9%
3	Gender (Male)	15.6%	94.5%
4	Lunch Type (Standard)	2.8%	97.3%
5	Race/Ethnicity (Group E)	1.6%	98.9%
6	Test Preparation (None)	0.6%	99.5%
7–10	Other demographic features	<0.5% each	—

Interpretation:

- Literacy dominance: Literacy is strongly reflected, as reading and writing scores combined explain the feature importance of 78.9%.
- Gender effect: Third most prominent characteristic (15.6%) indicating that gender disparities persist despite adjustment for literacy and socioeconomic variables.
- Lunch program salience: Support for nutritional needs is observed as the most significant socioeconomic factor, as expected, and is consistent with studies that have established links between intellectual functioning and nutritional support.

4.6 Model Diagnostics

Residual Analysis: The residual plots for Model 3 indicated no pattern, and residuals tended to be normally distributed around zero. This confirms that assumptions made, like linearity and homoscedasticity, are met.

Prediction-Actual Alignment: Plots of predicted vs. actual score showed strong linear alignment for model 3, with little systematic bias across the score range, as points tended towards clustering along the identity line.

Cross-Validation Stability:There is minimal difference between test R^2 and cross-validated R^2 values of 0.88 and 0.872, respectively, indicating stable

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4.7 Figure 1: hierarchical model analysis visualization

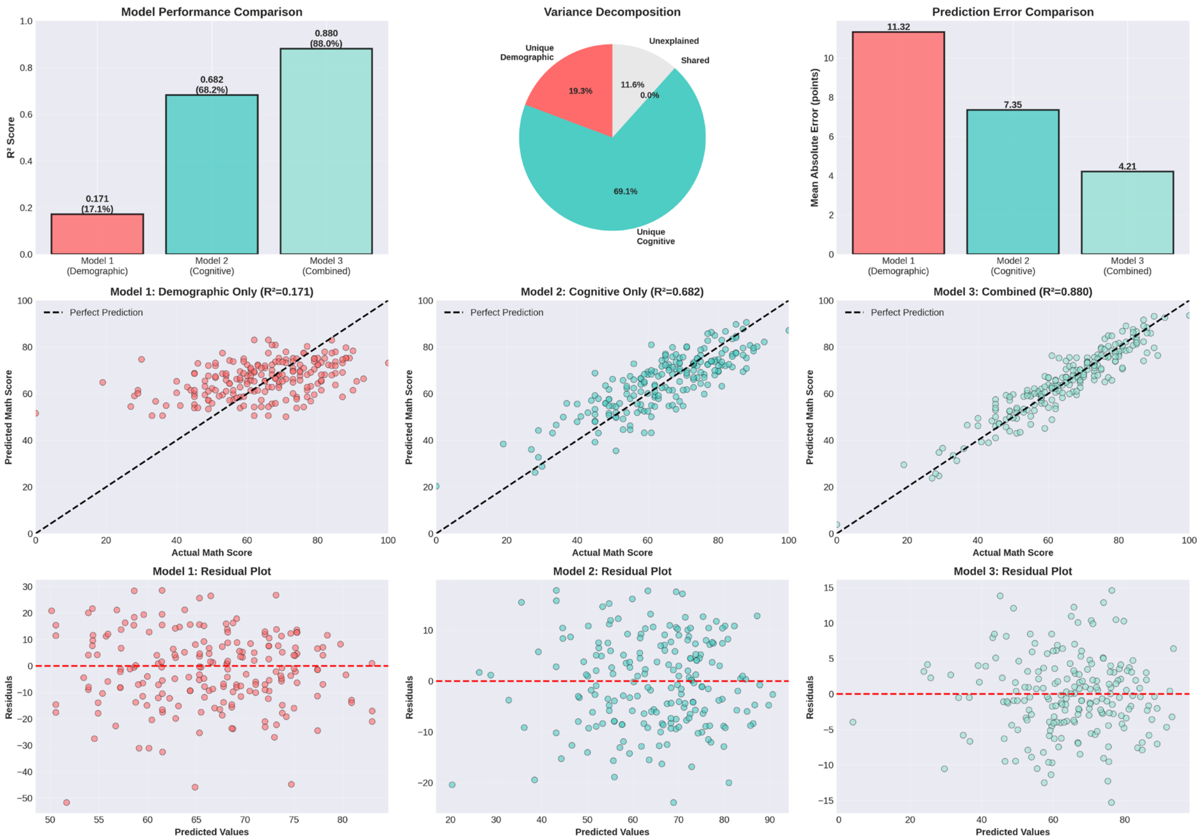

Figure 1. Hierarchical Regression Analysis of Mathematics Achievement. (A) Comparison of model performances on measures of goodness of fit, as indicated by R^2 values, for the demographic-only, cognitive-only, and combined models (.171, .682, and .880, respectively). (B) Variance components that partition residual variance into unique cognitive effects (69.1%), unique demographic effects (19.8%), and non-shared components (11.6%). (C) Prediction error comparison demonstrating MAE reduction from 11.32 (Model 1) to 4.21 points (Model 3). (D-F) Scatter plots of predicted vs. actual mathematics scores for Models 1-3, with dashed lines indicating perfect prediction (y=x). (G-I) Residual diagnostic plots confirming homoscedasticity and absence of systematic bias across the predicted value range for all three models.

5. DISCUSSION

5.1 Principal Findings

Findings of this research, contributing towards a comprehensive decomposition of the variance of mathematics achieved through regression modeling, can be summarized as follows:

First, these demographics, on their own, explain only about 17.1% of variance in mathematics, suggesting that demographics, as assessed, only tap into a limited aspect of socioeconomic effects on educational outcomes. This is consistent with previous studies that have suggested that demographics can only inadequately proxy the complex issue of disadvantage.

Second, cognitive variables (reading and writing proficiency) represent the strongest pathway to mathematics because they uniquely explain 69.1% of variance. Such high inter-domain correlations (r-value > 0.80) indicate that development of mathematical

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reasoning skills occurs simultaneously alongside verbal skills, and thus integrated learning can be effectively adopted for improved Literacy and Numeracy standards together.

Third, demographics have a persistent effect that is independent of cognitive ability and impacts an additional 19.8% of variance above and beyond literacy skills. This is counter to theorized expectations based on deficit theories, which often postulate that lack of ability is solely responsible for disparities in educational outcome, suggesting that socioeconomic disadvantages exert themselves through more than one mechanism and remain significant, despite statistical control for cognitive ability.

5.2 Theoretical Implications

- 5.2.1 Cross-Domain Cognitive Transfer

Literacy skills have exceptionally high predictive validity ($R^2 = 0.682$, based only on reading/writing skills), and these empirical results strongly support cross-domain theories of transfer. Such findings are consistent, for example, with theories that propose common cognitive foundations for subjects, such as:

• Working memory capacity: Both reading comprehension and mathematical problem-solving require keeping and processing information within one's working memory

• Abstract reasoning: Symbolic representation is essential to linguistic and mathematical reasoning

• Cognitive metaregulation: Both comprehension and problem-solving strategy monitoring is very much universal

Practically, this means that it is essential to have an integrated curriculum that focuses on teaching and learning literacy to develop mathematical concepts and vice versa.

- 5.2.2 Independent Pathways of Socioeconomic Influence

The fact that a near-zero value of shared variance is attained below 1% implies that skills and demographics have little correlation, and this reveals that non-cognitive disadvantages affect through channels that are orthogonal or independent of proficiency scales.

There could be more than one mechanism

1. Material Resources: Tutoring, technology, quiet study space
2. Cultural capital: Involvement of parents, familiarity with education talk, educational aspirations.
3. Psychological factors a. Stereotype threat, academic self-efficacy, future orientation.
4. Institutional factors: School Budget, Finances Quality of Teaching and Teachers and Difficulty Level

To resolve this, some of the possible solutions that could be considered on this case are:

Findings indicate that intervention frameworks should be multi-level, addressing academics as well as any issues that may be preventing success, and that policies solely addressing cognitive issues may be neglecting this complex environment of forces influencing success.

5.3 Practical Implications

- 5.3.1 For Educators

Integrated Instruction: According to the strong cognitive path, mathematics education would involve integrated efforts at literacy development. Strategies include:

- Cross-curriculum development engagement in

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- Written justifications of proposed solutions
- Leveraging word problems for bridging linguistic and quantitative reasoning skills

Differentiated Support: The continued demographics impact implies that students with equal test performances may need differentiated support based on backgrounds. Underachieving or high-achiecing students from disadvantaged backgrounds can be offered:

Student Support Mechanisms

- Explicit instruction on educational standards and expectations
- Mentorship and college advising
- Financial Literacy and Navigating Resources

5.3.2 For Policymakers

Comprehensive Screening: With such a high accuracy rate of 88% variance explained, the combined model indicates that it is possible to pinpoint risk students effectively. Early Warning Systems should comprise of:

- Cross-domain academic assessments (except mathematics)
- Socioeconomic Factors (focusing on nutritional support received
- Regular reassessment on intervention effectiveness

Equity-Focused Resource Allocation: Based on the specialized demography of 19.8% of the population, there is a rationale

- Free/reduced price lunch increases
- Family involvement projects
- Fund formulas that recognize socioeconomic disadvantage

Integrated Programming: As cognitive and demographic variables are assumed to act independently, any effective intervention program would have to integrate these two sets of variables:

- Quality education (development of skills)
- Wraparound services (to address non-academic barriers
- Parent and community partnerships (to bridge home-school divides)

5.4 Limitations

Despite its strong findings, there are some limitations that need to be considered

- 5.4.1 Cross-Sectional Design

The cross-sectional dataset records performances at one point in time, and drawing any causal conclusions is impossible. Some of the observed correlations could be attributed to:

- Cross-sectional vs. prospective (cross-sectional vs. prospective studies)
- Cross-influence effects that build up over
- Unmeasured third variables influencing cognitive and demographic trends

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Mitigation: Future research on this topic could investigate students longitudinally over several years, and this would help researchers identify developmental patterns and causal sequences.

- 5.4.2 Unmeasured Confounders

The 12% unexplained variance likely reflects omitted variables, including:

- Teacher quality: Teacher effectiveness is highly variable across schools
- Student motivation: Intrinsic interests and engagement propel success
- School resources, such as money, books, and class sizes
- Peer effects: Classroom composition affects learning opportunities

Mitigation: By using multi-level modeling that includes predictors of classroom and school effects, individual effects would be differentiated from contextual effects.

- 5.4.3 Generalizability

Additionally, neither the origin of this dataset nor its frame of reference is defined, thereby preventing external validity. Findings from these studies would not generalize to:

- Varying education structures (within international settings)
- Other grade levels (elementary or post-secondary)
- Alternative methods of assessment (performance tasks vs. standardized tests)

Mitigation: Replication studies need to be conducted on varied samples.

5.5 Future Research Directions

Based on these results, we propose that:

1. Longitudinal Variance Decomposition: Chart changes over time for distinctive demographics and cognitive components, and whether these effects decay, persist, or build as students move through their schooling experience.
2. Mediation Analysis: Find variables that can mediate between demographics and accomplishment, other than intelligence. Potential mediators are:
- Academic Self-Concept and Growth Mind
- Non-school learning opportunities
- Parent-teacher communication quality
3. Intervention Studies: Examine whether non-cognitive methods (mentoring, providing resources, raising expectations) can close demographic gaps without directly influencing cognitively based skills.
4. Cross-Cultural Replication: Investigate whether, and under what conditions, the same variance partition of 70%/20% holds across different education systems and levels of inequality and tracking.
5. Non-Linear Modeling: Examine whether effects of demographics are larger for low-achieving students than high-achieving ones (perhaps through quantile regression).

6. CONCLUSION

Findings from this research support that mathematics outcomes can be predicted quite accurately (88% variance explained) on a combination of cognitive and demographic variables via hierarchical regression analysis. Although literacy ability is defined as the primary route to mathematics success (69% unique variance), it is clear that socioeconomic disadvantages have enduring impacts irrespective of cognitively measured ability (20% unique variance).

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Where these factor pathways have near-zero overlap, it becomes difficult for deficit theories, that place responsibility on skills lacking, to fully explain achievement disparities. Moreover, findings suggest that disadvantaged students have obstacles that act, through channels other than measured skills, such as resources, culture, or psychology.

In regard to education practice, these findings affirm the benefit of integrated curriculum methods that capitalize on cognitvie transfer, as well as continued emphasis on issues of disparity, despite equal testing measures. In regard to education or more generally policy, these findings affirm comprehensive education screening and resource allocation for students that experience socioeconomic disparity.

Future studies need to involve longitudinal research methodologies that can help uncover causal pathways, mediating effects that can reveal causal links between certain demographics and achievements, and intervention studies that can demonstrate whether reducing non-cognitive barriers can help close disparities of equityResearch that could help improve theoretical and practical development can help inform post-secondary education structures that view success as being more closely tied to opportunities than demographics.

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APPENDIX A: SUPPLEMENTARY ANALYSES

A.1 Subgroup Analyses

By Gender:

Female (n=518): Model 3 $R^2 = 0.871$

Male (n=482): Model 3 $R^2 = 0.889$

By Lunch Type:

Free/Reduced (n=355): Model 3 $R^2 = 0.868$

Standard (n=645): Model 3 $R^2 = 0.885$

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By Test Preparation:

Completed (n=358): Model 3 $R^2 = 0.891$

None (n=642): Model 3 $R^2 = 0.874$

APPENDIX B: ANALYSIS CODE

Complete Python code for reproducibility:

```
pythonimport pandas as pd

import numpy as np

from sklearn.model_selection import train_test_split, cross_val_score

from sklearn.linear_model import Ridge

from sklearn.preprocessing import OneHotEncoder, StandardScaler

from sklearn.compose import ColumnTransformer

from sklearn.pipeline import Pipeline

from sklearn.metrics import r2_score, mean_absolute_error


# Load data

data = pd.read_csv('StudentsPerformance.csv')


# Define variables

demographic_vars = ['gender', 'race/ethnicity',

                    'parental level of education',

                    'lunch', 'test preparation course']

cognitive_vars = ['reading score', 'writing score']

target = 'math score'
```

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```
# Prepare features
X_demo = data[demographic_vars]
X_cog = data[cognitive_vars]
X_combined = pd.concat([X_demo, X_cog], axis=1)
y = data[target]

# Train-test split
X_demo_train, X_demo_test, y_train, y_test = train_test_split(
    X_demo, y, test_size=0.2, random_state=42)
X_cog_train, X_cog_test = train_test_split(
    X_cog, test_size=0.2, random_state=42)
X_comb_train, X_comb_test = train_test_split(
    X_combined, test_size=0.2, random_state=42)

# Model 1: Demographic
demo_pipe = Pipeline([
    ('prep', ColumnTransformer([
        ('cat', OneHotEncoder(drop='first'), demographic_vars)])),
    ('model', Ridge(alpha=10.0))
])
demo_pipe.fit(X_demo_train, y_train)
r2_1 = r2_score(y_test, demo_pipe.predict(X_demo_test))
```

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```
# Model 2: Cognitive
cog_pipe = Pipeline([
    ('prep', ColumnTransformer([
        ('scaler', StandardScaler(), cognitive_vars)])),
    ('model', Ridge(alpha=1.0))
])
cog_pipe.fit(X_cog_train, y_train)
r2_2 = r2_score(y_test, cog_pipe.predict(X_cog_test))

# Model 3: Combined
comb_pipe = Pipeline([
    ('prep', ColumnTransformer([
        ('cat', OneHotEncoder(drop='first'), demographic_vars),
        ('scaler', StandardScaler(), cognitive_vars)])),
    ('model', Ridge(alpha=1.0))
])
comb_pipe.fit(X_comb_train, y_train)
r2_3 = r2_score(y_test, comb_pipe.predict(X_comb_test))

# Variance decomposition
unique_cog = r2_3 - r2_1
unique_demo = r2_3 - r2_2
unexplained = 1 - r2_3
```

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```
print(f"Model 1 R²: {r2_1:.3f}")

print(f"Model 2 R²: {r2_2:.3f}")

print(f"Model 3 R²: {r2_3:.3f}")

print(f"Unique Cognitive: {unique_cog*100:.1f}%")

print(f"Unique Demographic: {unique_demo*100:.1f}%")

print(f"Unexplained: {unexplained*100:.1f}%")
```