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Heatline And Isothermallines Visualization Of Forced Convection In A Porous Medium With Radiation And Varying Wall Temperature

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Abstract

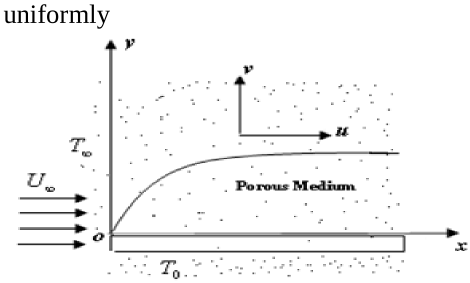
The aim of the present paper is to study the pattern of Isothermal lines and Heat lines in a forced convection problem at a horizontal plate in a porous medium under the effect of radiation and varying wall temperature. The temperature of the plate is assumed to vary along the length of the plate as a power function of distance. A similarity transformation is used to reduce the partial differential equations governing the problem into ordinary differential equations and the equations are solved analytically subject to appropriate boundary conditions to find the temperature function, Heatfunction and hence draw the related Isothermallines and the Heatlines. Heatlines and Isothermallines are drawn for both hot wall and cold wall cases. With the effect of radiation and varying wall temperature, significant patterns of Heatlines and Isothermallines are displayed in the discussion. The lines are drawn by the use of 'MATHEMATICA'.

Key Words: Forced Convection, Darcy model, Isothermal lines, Heatlines and Radiation.

1. Introduction

During the last three decades much insightful work has been done on Heatlines, Streamlines and Masslines. These tools are used to visualize the heat flow and mass transfer occurring in the boundary layers. One can as well find more complete picture of involved transport phenomenon as well as the true conditions where the boundary layer hypothesis is applied. Heatlines and Streamlines are found to be adequate to visualize and understand heat energy distribution and thermal mixing occurring inside or near a boundary layer of the given geometry.

The Heatline concept was first introduced by Kimura and Bejan [6] and appeared for the first time in a book on convective heat transfer by Bejan [1]. Prior to that, convective heat transfer process was analyzed using mainly the isotherms. On the other hand, the adequate visualizing tools in fluid dynamics are the streamlines and not the isobars, the same occurring in the field of convective heat transfer, where the adequate visualizing tools are the Heatlines and not Isotherms as pointed out in Costa [3]. For the first time Heatline visualization of forced convection in saturated porous media was presented by Morega and Bejan [8]. The development of the similarity version of the Heatfunction for the laminar natural convection near a vertical wall, isothermal or under constant heat flux was presented for the first time in Costa [4]. The streamlines and Heatlines are the most effective ways to visualize the paths followed by mass and heat flowing in two-dimensional problems without source terms (refer Costa [3]). In Dash [5] Heatline concept was also applied when dealing with heat transfer in turbulent flows through the introduction of turbulent fluxes into the Heatfunction differential equation. If the heat transfer problem under consideration occurs in the turbulent boundary layer near a wall, the Heatline concept can be applied in a simpler way, by using an effective diffusion coefficient for heat, which includes the eddy diffusivity effect of the turbulent transport (refer Bejan[2]). Heatline approach for visualization of heat flow and efficient thermal mixing with discrete heat sources was studied by Ram Satish et.al. [11]. Raja Rani & CNB Rao[10] studied the effect of radiation and magnetic field on mixed convection at a vertical plate embedded in a porous medium with variable fluid properties and varying wall temperature. In recent works Pawan and Supot [9] studied a natural convection flow in a rectangular enclosure filled with porous medium when the walls are insulated except the left wall which is non-uniformly heated.



Physical model and coordinate system
Fig-1

The aim of the present paper is to investigate the pattern of Heatlines and Isothermallines in a forced convection problem at a horizontal plate in a porous medium under the effect of radiation and varying wall temperature. The temperature of the plate T_0 is assumed to vary as a power function of distance along the plate as, $T_0 = T_\infty + Ax^\lambda$, where T_∞ is the temperature of the ambient fluid, '*A*' is constant and λ is a real number.

The hot wall $(T_0 > T_\infty)$ can be obtained for 'A' positive and the cold wall $(T_0 < T_\infty)$ can be obtained for 'A' is negative. The physical coordinate system of the problem is presented in fig.1. The governing partial differential equations are transformed into ordinary differential equations by applying similarity transformation and the equations are solved analytically subject to appropriate boundary conditions to find the Heatlines and Isothermallines. Present work has agreed well with Morega and Bejan [8] for Heatlines of both cold wall as well as hot wall for isothermal plate and without radiation effect $(\lambda = 0 \; \& \; Rd = 0)$.

2. Formulation and Solution

Let a semi infinite plate be embedded horizontally in a porous medium saturated with an incompressible homogeneous fluid. The fluid is assumed to be a gray medium that emits, absorbs but do not scatter thermal radiation. It is also assumed that radiation from the fluid is only taken in to consideration which is present in the form of unidirectional flux, transverse to the horizontal wall. Let *x*-axis be taken along the wall and *y*-axis perpendicular to it. The orientation of the wall is shown in fig-1. The equations governing the forced convection boundary layer flow for the Darcy model are written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2.1)$$

$$\frac{\partial P}{\partial x} + \frac{\mu}{K} u = 0 \qquad (2.2)$$

$$\frac{\partial P}{\partial y} + \frac{\mu}{K} v = 0 \qquad (2.3)$$

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_m \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \qquad (2.4)$$

where *u*, *v* are fluid velocity components, *T* is fluid temperature, *K* is Permeability, k_m is effective thermal conductivity of the porous medium, q_r is radiative heat flux, transverse to the vertical plate. The Rosseland approximation is used in the energy equation to describe the thermal radiative heat transfer. It may be noted that by the use of Rosseland approximation, the applicability of the present analysis is limited to optically thick fluids only.

The appropriate boundary conditions are:

$$\left. \begin{array}{l} T_0 = T_\infty + A x^\lambda, \; v = 0, \; at \;\; y = 0, \;\; x \ge 0 \\ T \to T_\infty, \;\; u \to U_\infty, \;\; as \;\; y \to \infty, \; x \ge 0 \end{array} \right\} \qquad (2.5)$$

where U_∞ is free stream velocity parallel to the plate, which is taken as $U_\infty = B x^\lambda$ where B is a constant. A stream function ψ, $Pe'clet$ number (Pe_x), non-dimensional functions f, θ; a similarity variable η and radiative flux q_r are introduced through the relations (2.7) given below.

It may be noted that the flow and temperature fields in the thermal boundary layer region are expressible in closed form by introducing $u = U_\infty$ and $v = 0$. (2.6)

$$\left. \begin{array}{l} Pe_x = \dfrac{U_\infty x}{\alpha_m} \\ f(\eta) = \dfrac{\psi}{\alpha_m Pe_x^{\frac{1}{2}}} \\ \theta(\eta) = \dfrac{T - T_\infty}{T_0 - T_\infty} \\ \eta = \dfrac{y}{x} Pe_x^{\frac{1}{2}} \\ q_r = -\dfrac{16 \sigma_s T_\infty^3}{3 k_c} \left(\dfrac{\partial T}{\partial y} \right) \end{array} \right\} \qquad (2.7)$$

Boundary conditions can be easily written in terms of f, θ as

$$\left. \begin{array}{l} at \;\; \eta = 0, \quad \theta = 1, \quad f = 0, \\ as \;\; \eta \to \infty, \; \theta \to 0, \; f' \to 0 \end{array} \right\} \qquad (2.8)$$

By eliminating fluid pressure from (2.2) and (2.3) and introducing (2.7) and (2.8), the governing equation is obtained as

$$f''(\eta) = 0 \qquad (2.9)$$

Equation (2.9) can be integrated once using the condition on f' at $\eta = \infty$ to get

$$f' = 1 \qquad (2.10)$$

$$\Rightarrow f = \eta \qquad (2.11)$$

Now $x^* = \dfrac{x}{L}$, $y^* = \dfrac{y (Pe_L)^{\frac{1}{2}}}{L}$ are introduced, where

$$Pe_L = \frac{U_\infty L}{\alpha_m}. \qquad (2.12)$$

To find the temperature function and Heatfunction for boundary layer near a hot varying wall temperature plate, equation (2.4) is rewritten as

$$\rho c_p\left(u\frac{\partial(T-T_{ref})}{\partial x}+v\frac{\partial(T-T_{ref})}{\partial y}\right)=k_m\frac{\partial^2(T-T_{ref})}{\partial y^2}+\frac{16\sigma_s T_\infty^3}{3k_c}\frac{\partial^2(T-T_{ref})}{\partial y^2} \quad (2.13)$$

2.1.1 Boundary Layer near a hot varying wall temperature plate:

$T_{ref}=T_\infty$ for hot varying wall i.e., $T_0 > T_\infty$. In this case the transfer of heat is from wall to fluid and introducing (2.6), (2.8) and (2.12) in (2.13) one can get

$$\theta = Erfc\left[\frac{(1-\lambda)^{\frac{1}{2}}\eta}{\sqrt{2}(1+1.333Rd)^{\frac{1}{2}}}\right] \quad (2.14)$$

The form of equation (2.4) demands a special definition for Heatfunction, *H* which is valid inside the boundary layer region and

$$\frac{\partial H}{\partial y}=\rho c_p u(T-T_{ref}) \quad (2.15)$$

$$-\frac{\partial H}{\partial x}=\rho c_p v(T-T_{ref})-k_m\frac{\partial T}{\partial y} \quad (2.16)$$

where $H^* = \dfrac{H}{k_m(T_0-T_\infty)Pe_L^{\frac{1}{2}}}$ (2.17)

$H(x,y)$ can be easily obtained from (2.15) and (2.16) and it satisfies the energy equation (2.4), provided T_{ref} is a constant. By using (2.6), (2.14) and (2.17) it can be seen that

$$\frac{\partial H^*}{\partial y^*}=\theta(\eta) \text{ and } \frac{\partial H^*}{\partial x^*}=\frac{\partial\theta}{\partial y^*} \quad (2.18)$$

Equation (2.18) can be integrated by using (2.6) and (2.14) to get

$$H^*(x^*,y^*)=x^{*\frac{\lambda+1}{2}}(\eta\theta+2\theta')$$

$$H^*(x^*,y^*)= x^{*\frac{\lambda+1}{2}}\left(\eta\, Erfc\left(\frac{(1-\lambda)}{2(1+1.333Rd)}\right)^{\frac{1}{2}}-\frac{2}{\sqrt{\pi}}\exp\left(-\frac{\eta^2(1-\lambda)}{2(1+1.333Rd)}\right)\right) \quad (2.19)$$

2.1.2 Boundary Layer near a cold varying wall temperature plate:

In this case the transfer of heat is from fluid to the wall. Here the wall is colder than the porous medium and also it can be observed that the pattern of heat lines will differ from hot wall case. Here to find $H^*(x^*,y^*)$, $T_{ref}=T_0$ is taken and a procedure similar to that of the above is used. In this case

$$H^*(x^*,y^*)= x^{*\frac{\lambda+1}{2}}\left(\eta\, Erf\left(\frac{(1-\lambda)}{2(1+1.333Rd)}\right)^{\frac{1}{2}}+\frac{2}{\sqrt{\pi}}\exp\left(-\frac{\eta^2(1-\lambda)}{2(1+1.333Rd)}\right)\right) \quad (2.20)$$

and $$\theta = Erf\left[\frac{(1-\lambda)^{\frac{1}{2}}\eta}{\sqrt{2}(1+1.333Rd)^{\frac{1}{2}}}\right] \quad (2.21)$$

By taking θ = constant, Isothermallines can be drawn and by taking $H^*(x^*,y^*)$ = constant, Heatlines can be drawn.

3. Solution of the Problem

3.1.1 Parameters of the Problem and Their Effect on heatlines and isothermallines

The flow and heat transfer depend on the parameter λ and *Rd* where λ is power of index of the wall temperature and *Rd* is the radiation parameter. $T_0 > T_\infty$ corresponds to hot wall and $T_0 < T_\infty$ corresponds to cold wall respectively. To determine certain important values for λ , the total heat convected in the flow, *Q(x)* at any downstream location *x* is considered.

$$Q(x)=\int_0^\infty \rho c_p\,\beta(T-T_\infty)u\,dy$$

This can be seen to be proportional to $x^{\frac{3\lambda+1}{2}}$. For uniform heat flux surface, *Q(x)* should vary linearly with *x* and so $\lambda = \frac{1}{3}$. For an adiabatic surface, *Q(x)* can be independent of *x* and so $\lambda = \frac{-1}{3}$. Zero value of λ corresponds to the isothermal case. In this study solutions are found for the values of -0.3, 0, 0.5 and 1.0 of λ.

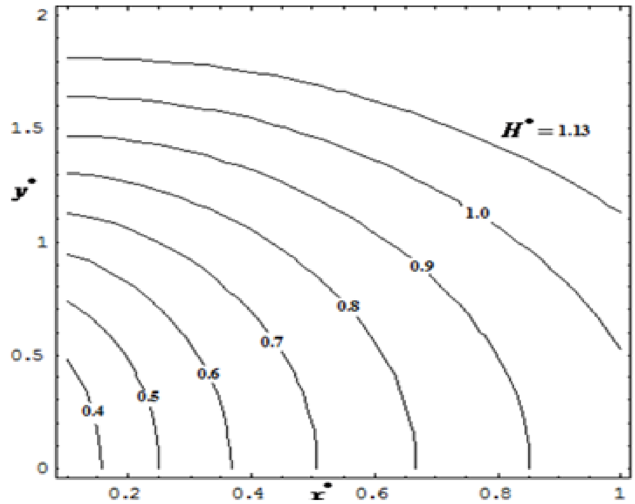
When transfer of heat energy through radiation is neglected, the parameter *Rd* takes zero value and for increasing intensity of the thermal radiation, the parameter takes larger values. Solutions are found for 0.5 and 5 of the parameter *Rd*, whereas the intensity of thermal radiation decreases in case of -0.5 of *Rd*. As λ increases the value of heatfunction increases in case of both hot wall as well as cold wall.
The effects of simultaneous variation of the values of the parameters on the heatlines and isothermal lines are presented in the discussion.

3.1.2 Discussion of the Results

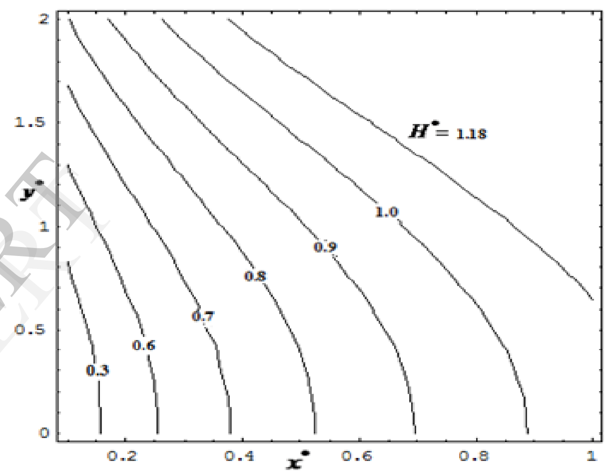
The visualization of both heatlines and isothermallines for both hot wall and cold wall are presented in the Figures 2-20. Heatlines near a cold wall case are presented for different values of parameters of λ and *Rd* in the Figures- 2to 8. Heat lines near a hot wall are presented for different values of parameters of λ and *Rd* in the Figures-9 to 14. Isothermallines for both cold and hot wall cases for different values of λ and *Rd* are presented in Figs- 15 to 20.

The pattern of heatlines of Fig-2 and Fig-9 have well agreed with Morega and Bejan [8] for λ=0 and *Rd*=0. Heatlines for an isothermal wall are presented in Fig-2 to 4. For an isothermal wall as the values of '*Rd*' increase from 0 to 0.5 the values of heatfunction also increase.

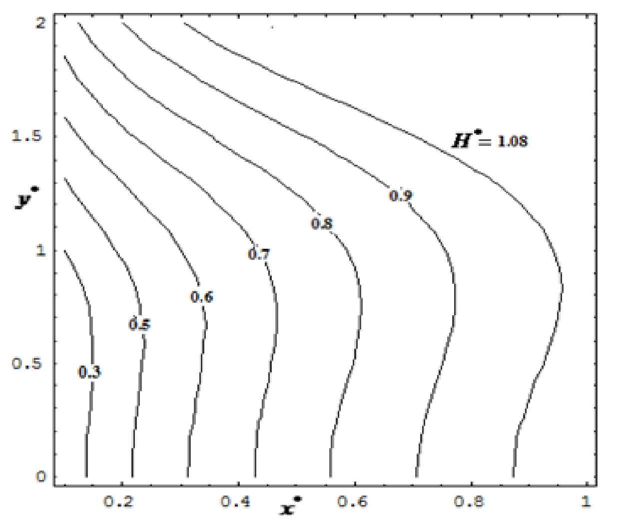
Dimensionless heatfunction can be seen to increase from heat lines of figures 2, 3 and 4. The heatlines can be seen to be perpendicular to the plate near the plate while they are not so at distances away from the plate. Since the wall is colder than the distant medium and $T_{ref} = T_0$, it follows that at the wall $\frac{\partial H}{\partial y} = 0$. The perpendicularity of the heatlines at the wall is due to the fact that $T_{ref} = T_0$. From Fig.3 it can be observed that as the heatfunction increases from H^*=0.3 at the tip to H^*=1.18 at the trailing edge. One can find same heatline pattern in Fig.2 and Fig.4.



Heatlines near a cold wall case for $\lambda = 0$ & $Rd = 0$
Fig.2

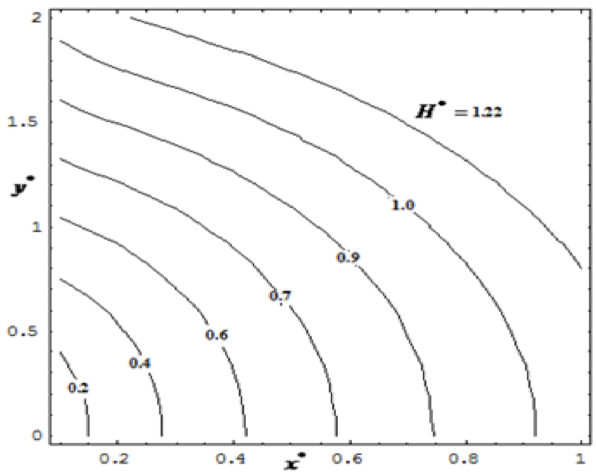


Heatlines near a cold wall case for $\lambda = 0$ & $Rd = 0.5$
Fig.3

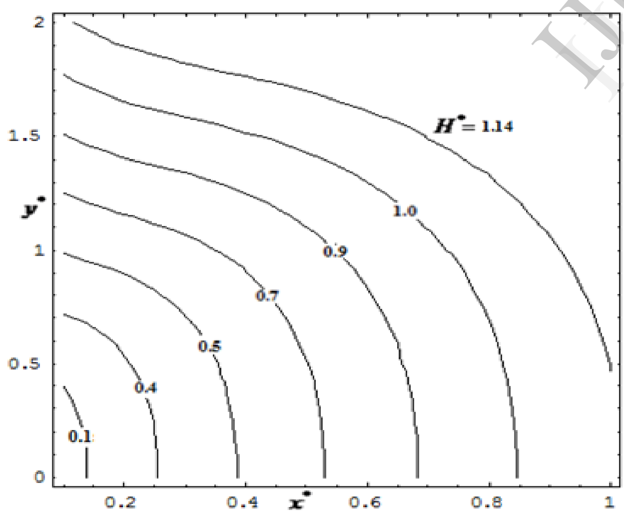


Heatlines near a cold wall case for $\lambda = 0$ & $Rd = -0.5$
Fig-4

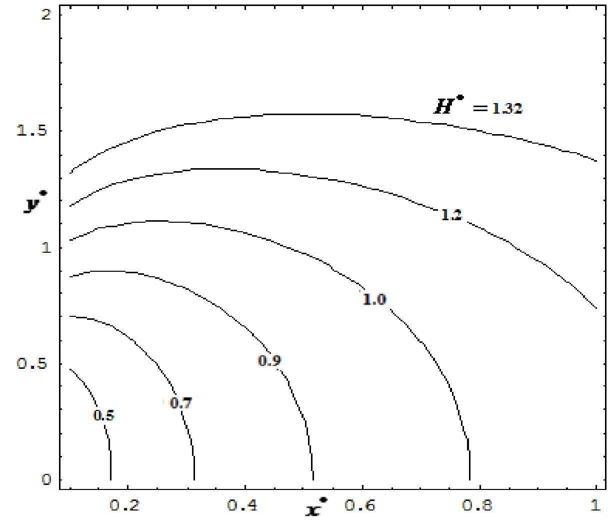
As λ increases from 0 to 0.5 heatfunction increases from H^*=1.13 to H^*=1.22. Also it can be observed from Fig-7 that the pattern of the lines for λ= - 0.3 are entirely different from λ=0 and 0.5 as it is independent of x as already discussed in §3.1.1 and the heat function increases to 1.32. From Fig.8 it is observed that in this case denser heatlines are not found and also the value of the heatfunction nearer to x^* = 0 is more as compared to that of the other figures.



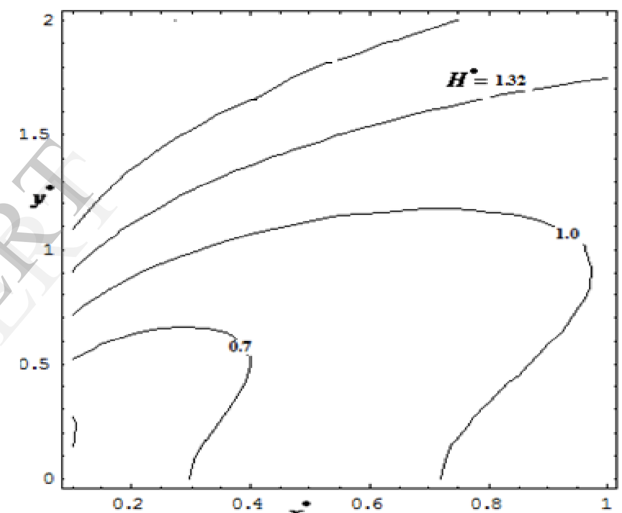
Heatlines near a cold wall case for $\lambda = 0.5$ & $Rd = 0$
Fig.5



Heatlines near a cold wall case for $\lambda = 0.5$ & $Rd = -0.5$
Fig. 6



Heatlines near a cold wall case for $\lambda = -0.3$ & $Rd = 0$
Fig.7



Heatlines near a cold wall case for $\lambda = -0.3$ & $Rd = -0.5$
Fig.8

The pattern of heatlines near an isothermal hot wall can be visualized in Figs-9 and 10. As radiation effect increases (as Rd increases) the heatfunction decreases. This is because the wall is warmer than the distant porous medium ie., $T_0 > T_\infty$. The pattern of the heatlines for hot wall is just opposite to cold wall with radiation effect. From Figures 9 and 11 it is observed that as λ increases, heat function increases as in cold wall case. The denser heatlines near x^*=0 indicate the higher fluxes near the leading edge of the heated section of the boundary. As x^* increases the wall heat function $H^*(x^*,0)$ decreases due to the fact that wall loses heat to the boundary layer in case of hot wall. The total heat released by wall can be seen from the heatlines at the trailing edge of y^*.

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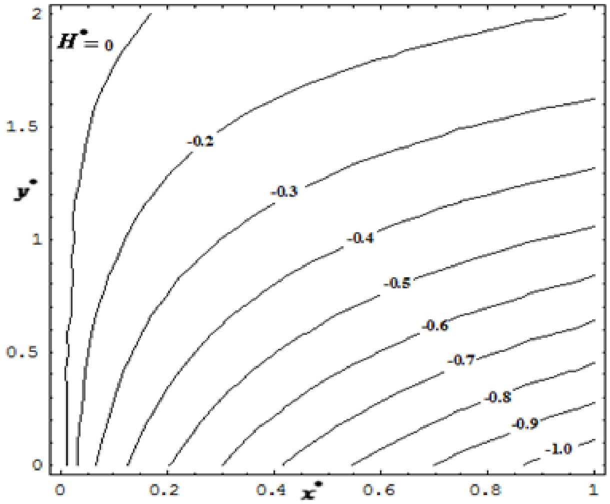
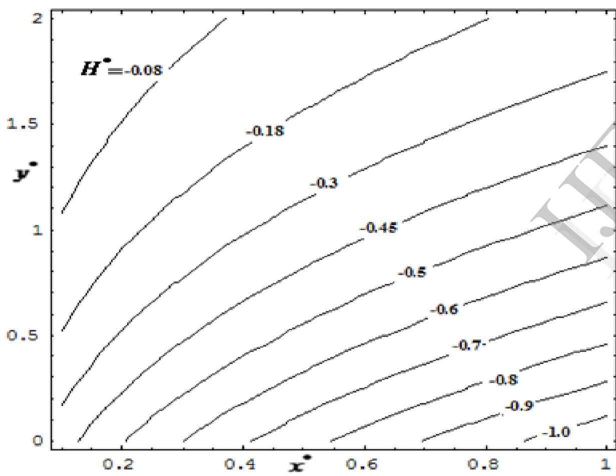
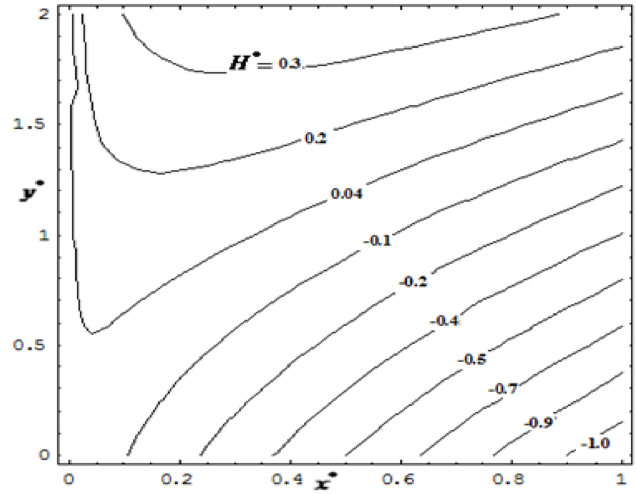
The pattern of heat lines are just opposite to each other for hot wall and cold wall. The values of the heatfunction also change significantly between hot wall and cold wall cases.



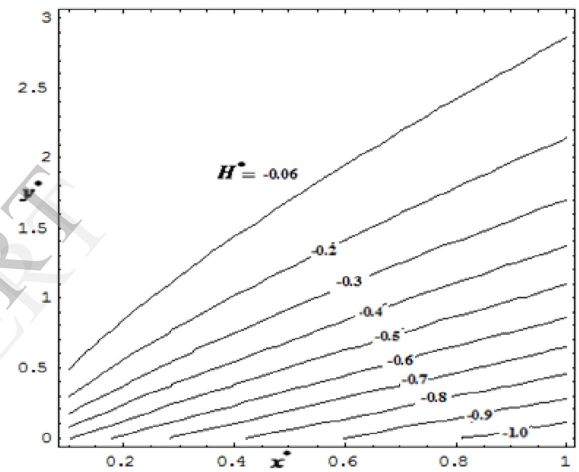
Heatlines near a Hot Wall for $\lambda = 0$, $Rd = 0$
Fig.9



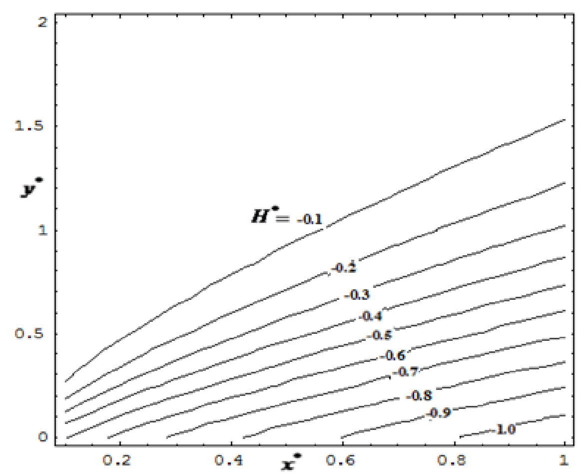
Heatlines near a Hot Wall for $\lambda = 0$, $Rd = 0.5$
Fig.10



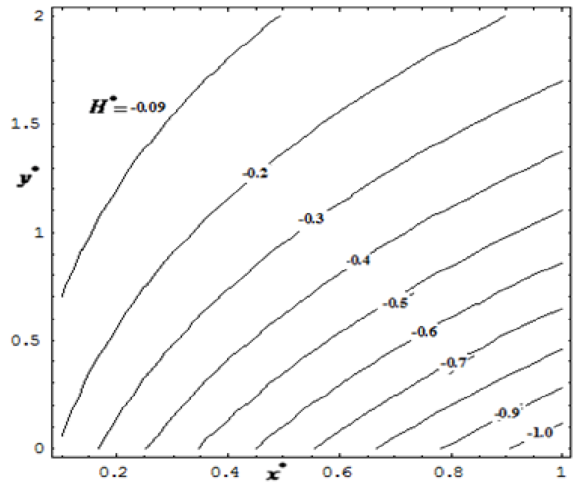
Heatlines near a Hot Wall for $\lambda = 1$, $Rd = 0$
Fig.11



Heatlines near a Hot Wall for $\lambda = -0.3$, $Rd = 0.5$
Fig.12

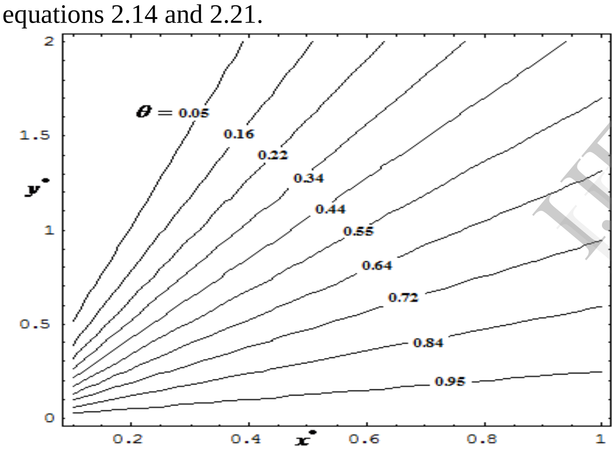


Heatlines near a Hot Wall for $\lambda = -0.3$, $Rd = -0.5$
Fig.13

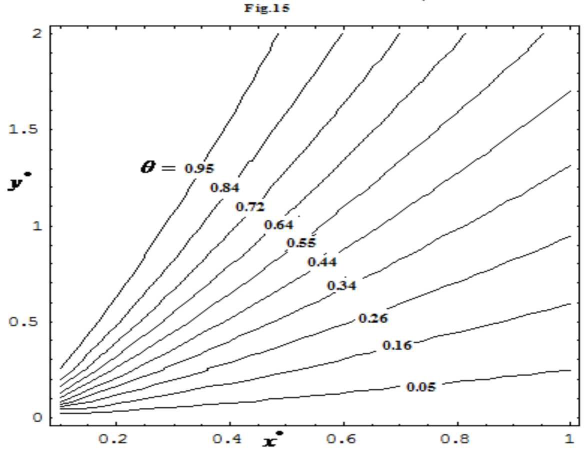


Heatlines near a Hot Wall for $\lambda = 0.5$, $Rd = 0.5$
Fig.14

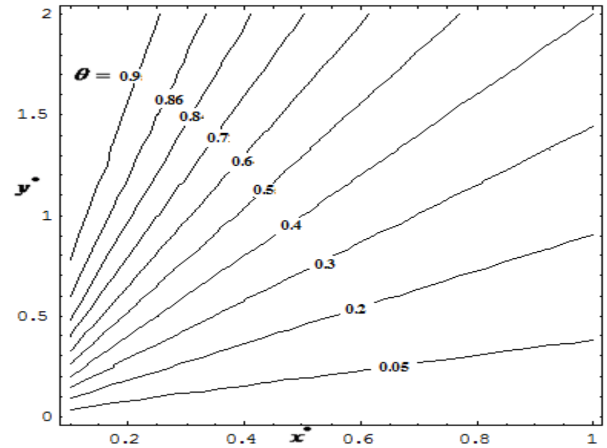
The line along which temperature is a constant is known as an isothermal line. The pattern of the isothermal lines can be seen in Figures 15 to 20. Comparing Fig.15 & Fig.16 it can be observed that the temperature (θ) values are just opposite but not negative as it satisfies the equations 2.14 and 2.21.



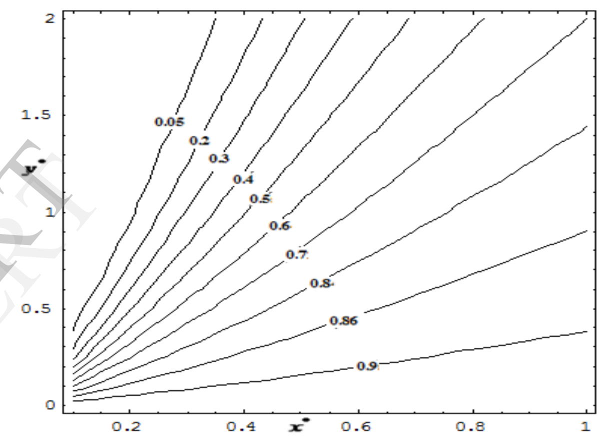
Hot Wall Isothermallines for $\lambda = 0, Rd = 0$
Fig.15



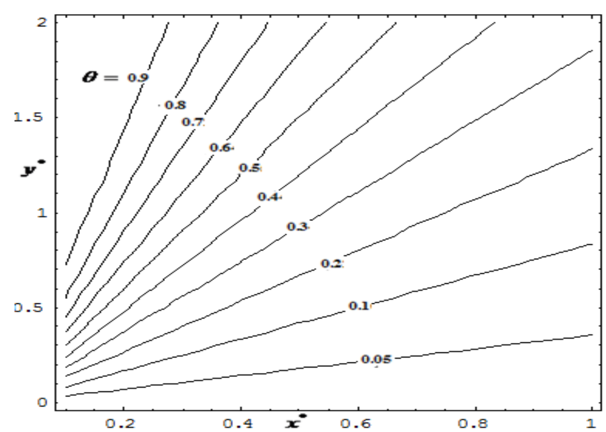
Cold Wall Isothermallines for $\lambda = 0, Rd = 0$
Fig.16



Cold Wall Isothermallines for $\lambda = 0$ & $Rd = 1.0$
Fig-17



Hot Wall Isothermallines for $\lambda = 0$ & $Rd = 1.0$
Fig.18



Cold Wall Isothermallines for $\lambda = 0.5$ & $Rd = 0$
Fig.19

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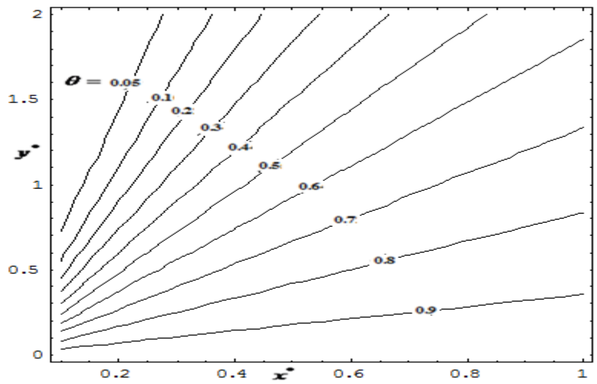

Hot Wall Isothermallines for $\lambda = 0.5$ & $Rd = 0$
Fig.20

Conclusions:

In this paper heatlines and isothermallines for both the hot wall as well as cold wall are presented. As the effect of radiation increases, heatfunction decreases in cases of both the hot wall and cold wall. But as λ (that signifies varying wall temperature) increases value of the heat function also increases for both cold wall as well as hot wall. Significant pattern of heatlines and isothermallines can be visualized from the figures.

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References:

1. Bejan, Adrian., *"Convection heat transfer", 1st ed., Wiley, New York*,1984, pp-22-23, 450,452.
2. Bejan, Adrian., *"Convection heat transfer", A Wiley-Inter Science Publication, John Wiley and Sons, INC*, II-Edition,1995.
3. Costa, V.A.F., "Bejan's Heatlines and Masslines for Convection Visualization and Analysis", *Int.J.of Heat Transfer, Transaction of ASME*, 59,2006, 126-133.
4. Costa, V.A.F.,"Heatline and massline visualization of laminar natural convection boundary layers near a vertical wall", *Int. Journal of Heat and Mass Transfer*, 43,2000, 3765-3774.
5. Dash,S.,K., "Heatline visualization in turbulent flow", *Int.J. Numer.Methods Heat fluid flow*, 5(4), 1996, 37-46.
6. Kimura,s., Bejan,., "The heatline visualization of convective heat transfer", *ASME Journal of heat transfer*, 105, 1983,916-919.
7. Raja Rani, T. , C.N.B.Rao, Lakshmi Prasannam, V., "Free Convection in a Porous medium with Magnetic field, Variable Physical Properties and Varying Wall Temperature", *IJCMI, 1(3),* 2009, 87-91.
8. Morega, Al.M.,and Bejan, A., " Heatline Visualization of Forced Convection in Porous Media",*Int.J.Heat and Fluid flow*, 15(1),1994,42-47.
9. Paween Khansila., Supot Witayangkurn., "Visualization of Natural Convection in Enclosure Filled with Porous Medium by Sinusoidally Temperature on the One Side", *Applied Mathematical Sciences*, 6(97), 2012, 4801-4812.
10. Raja Rani, T., C,N,B,Rao., Lakshmi Prasannam, V., "Effect of Radiation and Magnetic Field on Mixed Convection at a Vertical Plate in a Porous Medium with Variable Fluid Properties and Varying Wall Temperature", *Proceedings of IAENG Conferences-IMECS, Hong-Kong,* March 17-19, III, 2010, 1827-1832.
11. Ram Satish Kaluri., Tanmay Basak., Roy,S., "Heatline approach for visualization of heat flow and efficient thermal mixing with discrete heat source", *Int.J.Heat and Mass Transfer,* 53(15), 2010, 3241-3261.