

# Heat Transfer in Ferrofluid Flow over a Stretching Sheet with Radiation

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**Abstract** - An investigation is carried out to study the effect of radiation and heat transfer of ferrofluid on the boundary layer flow over a stretching sheet. It is found that radiation has significant impact in controlling the rate of heat transfer in the boundary layer region. It is also noted that the effect of magneto-thermomechanical interaction decelerates the fluid motion thus increasing the skin friction and influencing the rate of heat transfer. In studying the heat transfer characteristics two types of boundary conditions are considered namely, Prescribed Surface Temperature (PST) and Prescribed Heat Flux (PHF) boundary conditions.

**Keywords**— *Ferromagnetic liquid; Magnetic dipole; Stretching sheet; Radiation; Prandtl number.*

## INTRODUCTION

The heat transfer analysis due to a stretching surface moving continuously through an ambient fluid has been investigated widely and reported in the literature. Properties of the product obtained through engineering processes like metallurgical process, polymer extrusion process, mainly depend on the nature of ambient liquid and on the rate of stretching. As the rapid stretching or the erratic change in temperature of the extrudate may destroy the expected properties of the final product and hence the heat transfer rate needs to be regulated carefully. With this view point, ambient fluids having better electromagnetic properties are of much interest as their flow can be regulated by external magnetic field. Ferrofluids have promising potential for such applications. Ferrofluids are artificially synthesized fluids that consist of highly concentrated colloid suspensions of fine magnetic particles in a non-conducting carrier fluid. This fluid behaves like a normal fluid except that it experiences a force due to magnetization. Ferrohydrodynamics deals with mechanics of magnetic fluid motion influenced by strong forces of magnetic polarization.

Neuringer [1] worked on saturated ferrofluids under the combined influence of thermal and magnetic field gradients. The flow of Newtonian viscous fluid past a linearly stretching surface in otherwise quiescent

surrounding was first considered by Crane [2] and subsequently extended to non-newtonian fluids. Anderson and Valnes [3] extended Crane's problem for a viscous non-conducting ferrofluid. They studied the influence of the magnetic field due to a magnetic dipole on a shear driven motion (flow over a stretching sheet) and concluded that the primary effect of the magnetic field was to decelerate the fluid motion as compared to the hydrodynamic case. Tzirtzilakis et al. [4] studied the forced and free convective boundary layer flow of a magnetic fluid over a flat plate under the action of a localized magnetic field. Thermal radiations is one of the important factors controlling the heat transfer in a non-isothermal system. Cortell [5, 6] investigated the effects of viscous dissipation and radiation over a stretching sheet. Siddheshwar and Mahabaleshwar [7], Abel and Mahesha [8], Sujit Kumar Khan [9], Sajid and Hayat [10], Hayat et al. [11,12] Biliiana et al. [13], Norfifah Bachok et al. [14], Anuar Ishak et al. [15], Jat and Gopi Chand [16] have reported the effects of radiation in various situations.

Motivated by a forementioned studies it is proposed to study the effects of radiation on heat transfer in a ferrofluid flow over a stretching sheet subjected to an external magnetic field due to a magnetic dipole.

## 1. MATHEMATICAL FORMULATION

Consider a steady two-dimensional flow of an incompressible, viscous and electrically non-conducting ferrofluid driven by an impermeable stretching sheet. By applying two equal and opposite forces along the  $x$  - axis, the sheet is stretched with a velocity  $u_w(x)$  which is proportional to the distance from the origin. A magnetic dipole is located at some distance from the sheet. The centre of the dipole lies on the  $y$  - axis at a distance 'a' from the  $x$  - axis and whose magnetic field points are in the positive  $x$  - direction giving rise to a magnetic field of sufficient strength to saturate the ferrofluid. The temperature  $T_w$  at the stretching sheet is assumed to be less than the Curie temperature  $T_c$ ,

while the fluid elements far away from the sheet are assumed to be at temperature  $T = T_\infty$ , where  $T_\infty < T_w < T_c$ . Above  $T_c$  the fluid is incapable of being magnetized. The boundary layer equations governing the flow and heat transfer in a ferrofluid are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2} + \frac{\mu_0}{\rho} M \frac{\partial H}{\partial x}, \quad (2)$$

$$\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) + \mu_0 T \frac{\partial M}{\partial T} \left( u \frac{\partial H}{\partial x} + v \frac{\partial H}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \quad (3)$$

where  $u$  and  $v$  are the velocity components along  $x$  and  $y$  directions respectively  $\rho$  is the fluid density,  $\mu$  the dynamic viscosity,  $\nu = \frac{\mu}{\rho}$  is the kinematic viscosity,  $c_p$  is

the specific heat at constant pressure,  $k$  is the thermal conductivity,  $q_r$  is the radiative flux,  $\mu_0$  is the magnetic permeability,  $M$  is the magnetization,  $H$  is the magnetic field and  $T$  is the temperature of the fluid. The assumed boundary conditions for solving the above equations are

$$\left. \begin{aligned} u(x,0) &= cx, \quad v(x,0) = 0, \\ T(x,0) &= T_w = T_\infty + A \left( \frac{x}{L} \right) \quad \text{in PST} \\ -k \frac{\partial T}{\partial y}(x,0) &= q_w = D \left( \frac{x}{L} \right) \quad \text{in PHF} \\ u(x,\infty) &\rightarrow 0, \quad T(x,\infty) \rightarrow T_\infty \end{aligned} \right\} \quad (4)$$

where  $A$  and  $D$  are positive constants and  $L = \sqrt{\frac{\nu}{c}}$  is the characteristic length

The flow of ferrofluid is affected by the magnetic field due to the magnetic dipole whose magnetic scalar potential is given by

$$\phi = \frac{\alpha'}{2\pi} \left( \frac{x}{x^2 + (y+a)^2} \right), \quad (5)$$

where  $\alpha'$  is the magnetic field strength at the source. The components of the magnetic field  $H$  are

$$H_x = -\frac{\partial \phi}{\partial x} = \frac{\alpha'}{2\pi} \left\{ \frac{x^2 - (y+a)^2}{(x^2 + (y+a)^2)^2} \right\} \quad (6),$$

$$H_y = -\frac{\partial \phi}{\partial y} = \frac{\alpha'}{2\pi} \left\{ \frac{2x(y+a)}{(x^2 + (y+a)^2)^2} \right\}. \quad (7)$$

Since the magnetic body force is proportional to the gradient of the magnitude of  $H$ , we obtain

$$H = \left[ \left( \frac{\partial \phi}{\partial x} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2 \right]^{\frac{1}{2}}, \quad (8)$$

$$\frac{\partial H}{\partial x} = -\frac{\alpha'}{2\pi} \left( \frac{2x}{(y+a)^4} \right),$$

$$\frac{\partial H}{\partial y} = \frac{\alpha'}{2\pi} \left( \frac{-2}{(y+a)^3} + \frac{4x^2}{(y+a)^5} \right) \quad (9)$$

Variation of magnetization  $M$  with temperature  $T$  is approximated by a linear equation

$$M = K(T - T_\infty) \quad (10)$$

where  $K$  is the pyromagnetic coefficient.

## 2. SOLUTION PROCEDURE

We now introduce the non-dimensional variables as assumed by Andersson (1998):

$$(\xi, \eta) = \left( \frac{c}{\nu} \right)^{\frac{1}{2}} (x, y), \quad (U, V) = \frac{(u, v)}{\sqrt{c\nu}}, \quad (11)$$

$$\theta(\xi, \eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \left. \begin{aligned} &= \begin{cases} \theta_1(\eta) + \xi^2 \theta_2(\eta) & \text{in PST case} \\ \phi_1(\eta) + \xi^2 \phi_2(\eta) & \text{in PHF case} \end{cases} \end{aligned} \right\} \quad (12)$$

where  $T_w - T_\infty = A \left( \frac{x}{L} \right)$  in PST case,

$$T_w - T_\infty = \frac{DL}{k} \left( \frac{x}{L} \right) \text{ in PHF case.}$$

Using Rosseland approximation, the radiative flux  $q_r$  is modelled as

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \quad (13)$$

where  $\sigma^*$  is Stefan-Boltzmann constant and  $k^*$  is the mean absorption coefficient. Assuming that the differences in temperature within the flow are such that  $T^4$  can be expressed as a linear combination of temperature; expanding  $T^4$  in Taylor's series about  $T_\infty$  and neglecting the higher order terms, we get

$$T^4 \approx -3T_\infty^4 + 4T_\infty^3 T \quad (14)$$

Introducing the stream function  $\psi(\xi, \eta) = \xi f(\eta)$  that satisfies the continuity equation we obtain,

$$U = \xi f'(\eta), \quad V = -f(\eta). \quad (15)$$

Here the prime denotes differentiation with respect to  $\eta$ . On using 9-15 the equations 2 and 3 along with the boundary conditions 4 give rise to the following boundary value problem (in PST case):

$$f''' + ff'' - (f')^2 - \frac{2\beta\theta_1}{(\eta + \alpha)^4} = 0, \quad (16)$$

$$(1 + Tr)\theta_1'' + Pr(f\theta_1' - f'\theta_1) + \frac{2\beta f \lambda}{(\eta + \alpha)^3}(\theta_1 + \varepsilon) = 0, \quad (17)$$

$$(1 + Tr)\theta_2'' - Pr(3f'\theta_2 - f\theta_2' + N_3 g \theta_2') - \frac{2\lambda\beta f \theta_2}{(\eta + \alpha)^3} + \lambda\beta(\theta_1 + \varepsilon) = 0, \quad (18)$$

$$\left[ \frac{2f'}{(\eta + \alpha)^4} + \frac{4f}{(\eta + \alpha)^5} \right] = 0.$$

$$f(0) = 0, \quad f'(0) = 1, \\ f'(\infty) \rightarrow 0, \quad (19)$$

$$\theta_1(0) = 1, \quad \theta_2(0) = 0, \quad \theta_1(\infty) \rightarrow 0, \quad \theta_2(\infty) \rightarrow 0. \quad (20)$$

The boundary value problem in case of PHF can simply be obtained by replacing  $\theta$  with  $\phi$  in the above equations and making use of following boundary conditions in place of equation 20.

$$\phi_1'(0) = -1, \quad \phi_2'(0) = 0, \\ \phi_1(\infty) \rightarrow 0, \quad \phi_2(\infty) \rightarrow 0 \quad (21)$$

The dimensionless parameters in the above equations denote Prandtl number  $Pr$ , viscous dissipation parameter  $\lambda$ , Curie temperature  $\varepsilon$ , ferrohydrodynamic interaction parameter  $\beta$ , thermal radiation parameter  $Tr$ , dimensionless distance of the centre of the magnetic pole from the origin  $\alpha$  are defined as follows:

$$Pr = \frac{\mu C_p}{k}, \quad \lambda = \frac{c\mu^2}{\rho k(T_w - T_\infty)},$$

$$\varepsilon = \frac{T_\infty}{T_w - T_\infty}, \quad \beta = \frac{\alpha' \rho}{2\pi\mu^2} \mu_0 K (T_w - T_\infty),$$

$$Tr = \frac{16\sigma^* T_\infty^3}{3kk^*}, \quad \alpha = \sqrt{\frac{c\rho a^2}{\mu}}.$$

The three coupled differential equations 16-18, subject to the boundary conditions 19-21 are solved numerically using shooting technique.

### 3. RESULTS AND DISCUSSION

Numerical solution is indeed an obvious and natural choice in the absence of an analytical solution of problem under consideration. The governing boundary layer equations with appropriate boundary conditions are solved using shooting method that uses Runge-Kutta-Fehlberg integration scheme and Newton-Raphson correction scheme. In this section we present the results obtained through the computations and discuss the influence of various physical parameters on the velocity and the temperature fields.

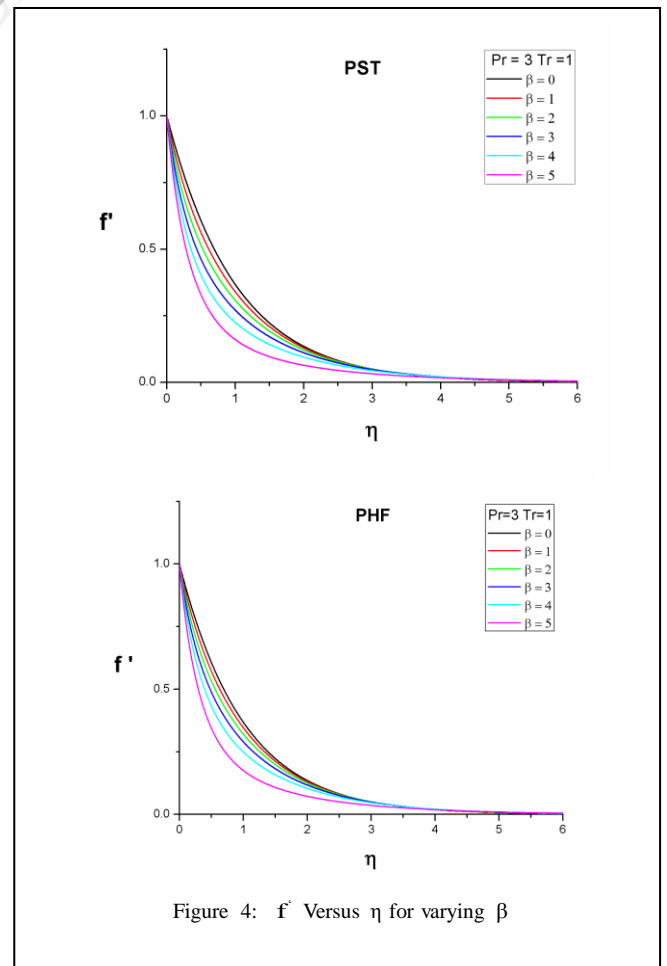
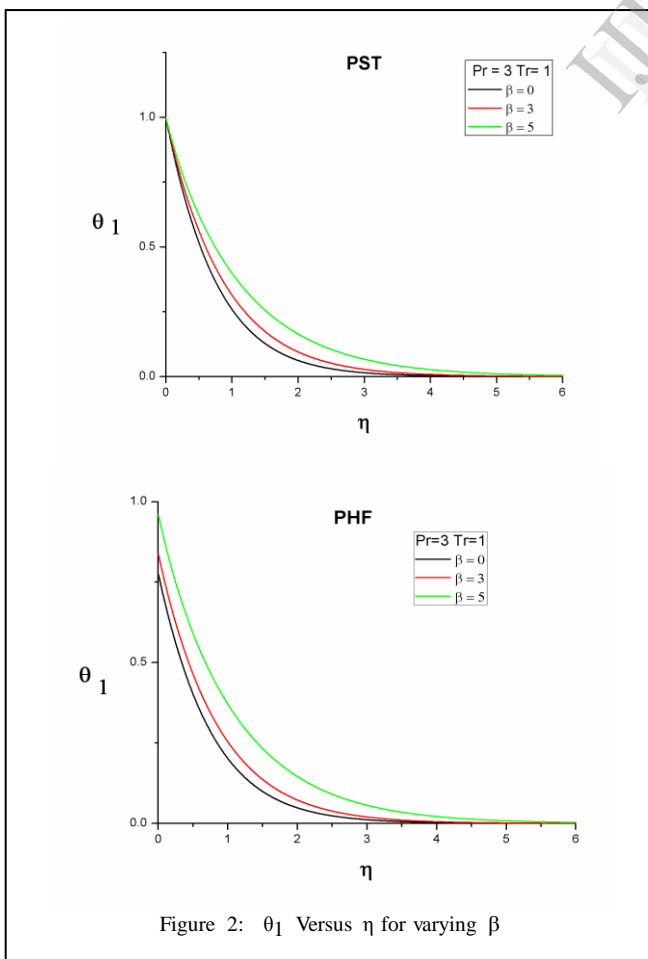
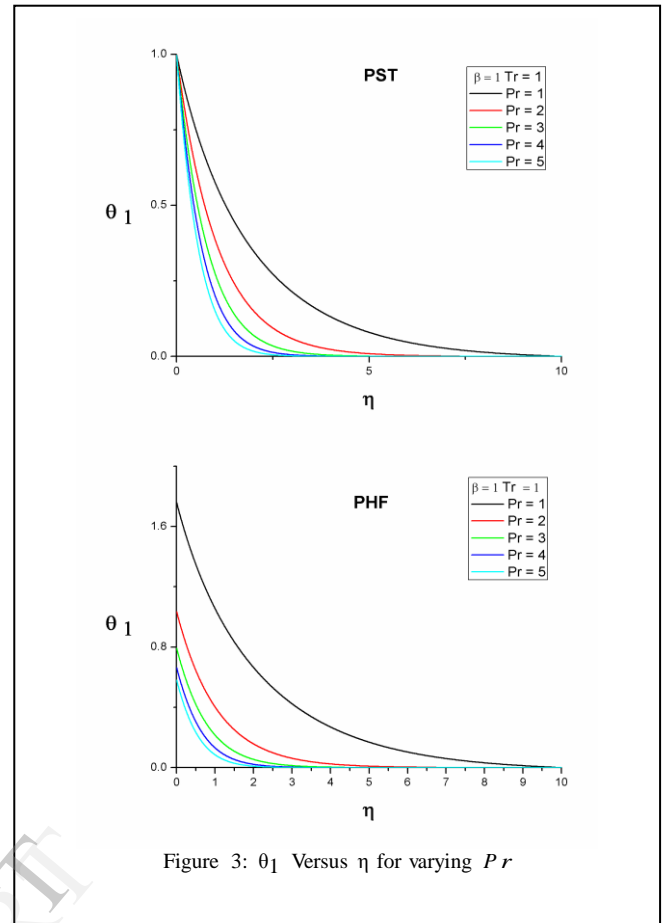
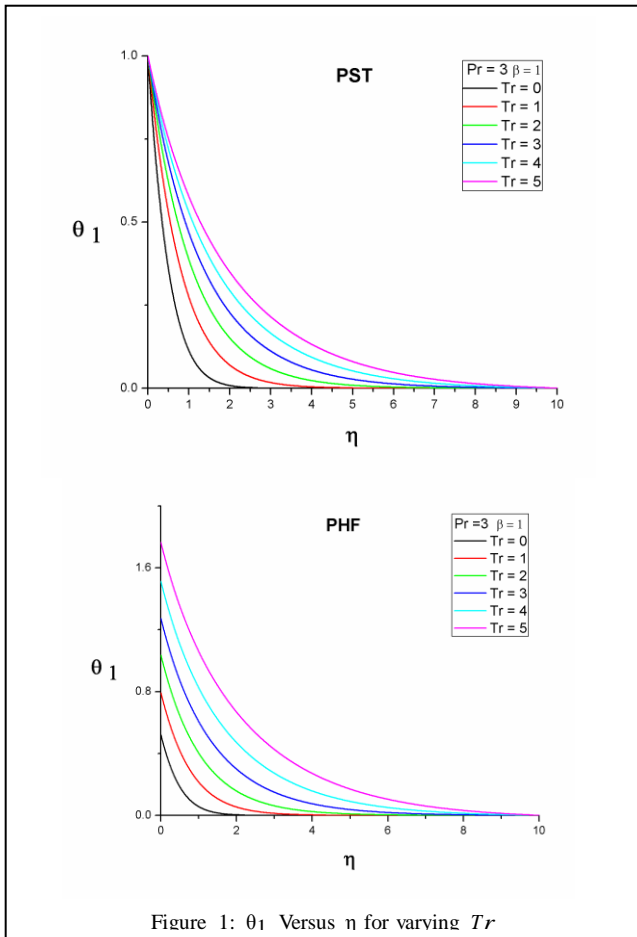
The influence of the physical parameters like, radiation parameter  $Tr$ , ferrohydrodynamic interaction parameter  $\beta$  and Prandtl number  $Pr$  while other parameters held fixed are presented in the figures 1, 2 and 3 respectively. It is evident from figure 1 that the thermal boundary layer thickness increases as the radiation parameter  $Tr$  is increased in both PST and PHF cases. The effect of radiation is to enhance the thermal diffusivity of the medium and hence one can observe pronounced heat generation in the boundary layer.

From figure 2 it is clear that increasing values of  $\beta$  will lead to increase the temperature of the fluid in the boundary layer region in both PST and PHF cases. This is because of the interaction between the motion of the fluid and the action of the magnetic field. This interaction decreases the velocity thereby increasing frictional heating between the fluid layers which is responsible for increasing the thermal boundary layer thickness which is evident in the figure.

The Prandtl number has exactly an opposite effect  $Pr$  as compared to  $Tr$  and  $\beta$  on temperature profiles. Figure 3 reiterates the same aspect in both PST and PHF cases. The reason for the decline in the heat transfer lies in the fact that increasing values of  $Pr$  reduces thermal diffusivity thereby reducing the heat diffused away from the heated surface and in consequence increases the temperature gradient at the surface. This phenomena leads to the decreasing of energy ability that reduces the thermal boundary layer thickness.

The ferromagnetic effects in the problem are taken care by the curie temperature  $\varepsilon$ , ferrohydrodynamic interaction parameter  $\beta$  and the dimensionless distance  $\alpha$  of the centre of the magnetic dipole from the origin. The ferromagnetic fluid basically has a Newtonian carrier fluid with micron sized suspended particles of ferrite which enhance the viscosity of the fluid and hence the velocity of the flow is decreased as the values of  $\beta$  are increased, which is seen in figure 4. Due to reduction in motion, heat transfer is also enhanced. Increasing the values of  $Pr$  has no effect on the axial velocity as observed in figure 5, but in figure 6 we notice that  $Tr$  has a decreasing effect on the velocity in both cases of PST and PHF

Figure 7 highlights the effect of thermal radiation and ferrofluid interaction parameters on the local skin-friction coefficient. One can readily see that both  $Tr$  and  $\beta$  have an increasing effect on the skin-friction indicating slowing down of the fluid in the boundary layer.



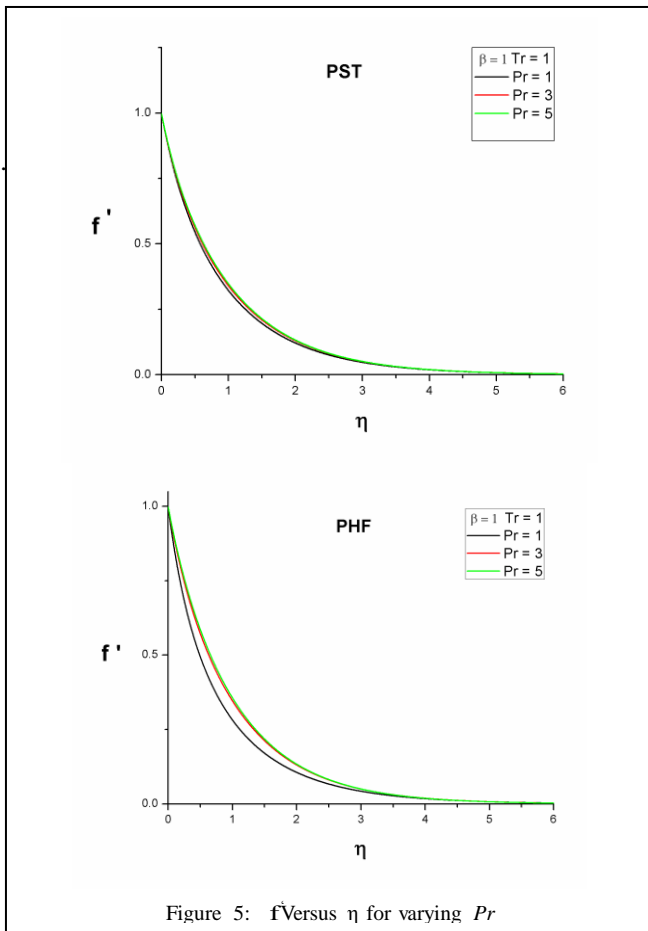


Figure 5:  $f'$  Versus  $\eta$  for varying  $Pr$

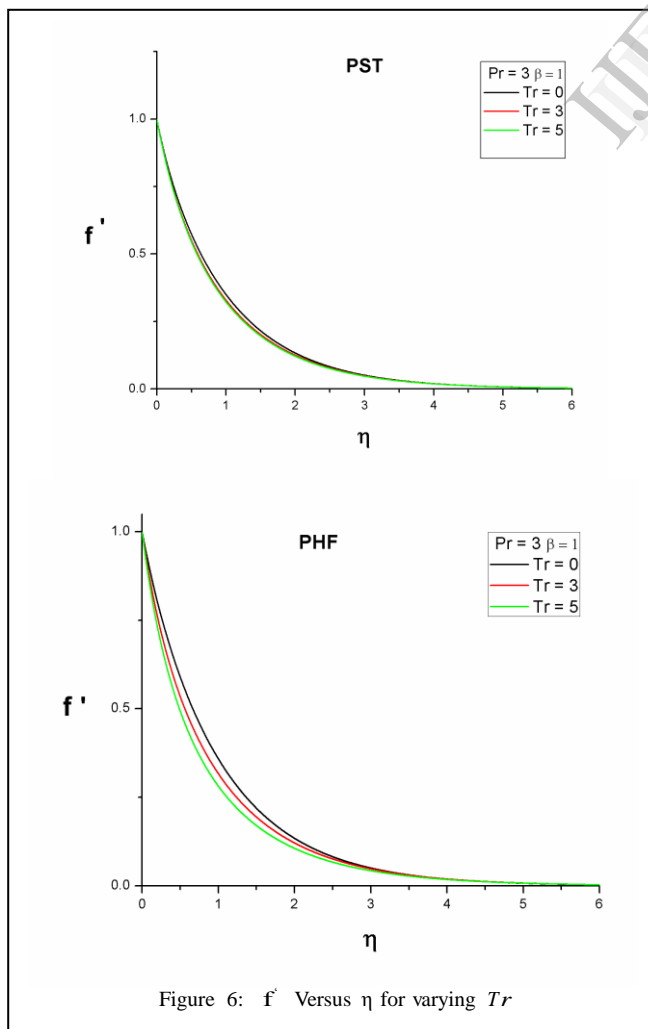


Figure 6:  $f'$  Versus  $\eta$  for varying  $Tr$

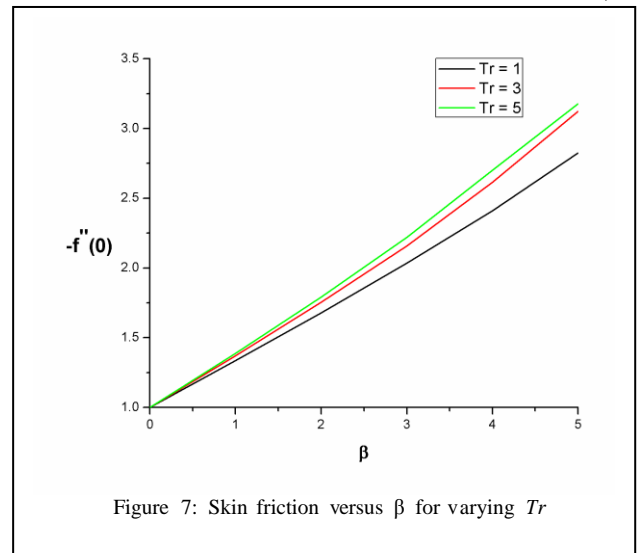


Figure 7: Skin friction versus  $\beta$  for varying  $Tr$

#### 4. CONCLUSION

The effect of radiation on the ferrofluid past a horizontal stretching sheet is analysed in this section. Numerical solutions of the problem obtained by shooting method facilitated with a scientific choice of the missed initial conditions. The important findings of the problem are as follows:

As the value of  $\beta$  increases there is a decrease in the axial velocity there by increasing the heat transfer, whereas as the value of radiation parameter  $Tr$  increases, there is no significant change in the velocity but there is an increase in  $\theta$  indicating the enhanced heat transfer. Hence  $\beta$  and  $Tr$  work together in increasing the heat transfer.

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