Heat and Mass Transfer with Radiation and Dissipation over a Fixed Vertical Plate

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***Abstract*:-This paper investigates the effect of heat and mass transfer with radiation and dissipation over a fixed vertical plate. The dimensionless governing equations are solved using perturbation techniques. The effects of velocity, temperature and concentration are studied for different parameters like modified Grashof number, Grashof number, Suction/injection parameter, Schmidt number and Prandtl number, Eckert number.**

Key words: heat transfer, mass transfer, vertical plate, radiation, dissipation

1. INTRODUCTION

In the processes involving high temperature, the radiation heat transfer in combination with conduction, convection and also mass transfer plays very important role in the design of pertinent equipment's in the areas such as nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites, and space vehicles. Reference [1] presented radiation and mass transfer effects on transfer free convectional flow of a dissipative fluid past semi-infinite vertical plate with inform heat and mass flux. Viscous mechanical dissipation effects are important in geophysical flows and also in certain industrial and are usually characterized by the Eckert number. Reference [2] reported the influence of viscous heating dissipation effects in natural convective flows, showing that the heat transfer rates are reduced by an increase in the dissipation parameter. Soret and Dufour effects are important for intermediate molecular weight gases in coupled heat and mass transfer in binary system, often encountered in chemical process engineering and also in high-speed aerodynamics. Reference [3] used network simulation methods (NSM) to study the effect of viscous dissipation, Soret and radiation on unsteady MHD free convection flow past a vertical porous plate .Reference [4] presented radiation and mass transfer effects on an unsteady MHD free convection flow past a heated vertical plate with viscous dissipation. Reference [5] investigated the thermal radiation effects on Magnetohydrodynamic (MHD) flow past a semi-infinite vertical plate in the presence of mass diffusion. Reference [6] investigated network simulation method applied to radiation and viscous dissipation and Soret effects on MHD unsteady free convection over vertical porous plate. Reference [7] examined the dispersion of a chemically non-reacting and chemically reacting solute in a micro polar fluid, for circular pipe geometry. Reference [8] analyzed the radiation effects on an unsteady two dimensional hydromagnetic free convective boundary layer flow of a viscous incompressible fluid past a semi-infinite vertical plate with mass transfer in the presence of heat source or sink. Reference [9] studied effects of variable suction and thermophoresis on steady MHD combined free forced convective heat and mass transfer flow over a semi-infinite permeable inclined plate in the presence of thermal radiation. Reference [10] investigated free convection flow past a vertical plate. Reference [11] reported effects of varying viscosity and thermal Conductivity on steady free Convective flow and heat transfer along an isothermal vertical plate in the presence of heat Sink. Reference [12] studied heat transfer to unsteady MHD flow past an infinite vertical moving plate with variable Suction.

2. MATHEMATICAL FORMULATION

Consider the transient free convective flow of a viscous fluid in a vertical channel with the walls at a constant distance *d* apart. The x-axis is taken along one of the wall of the channel and y-axis is normal to it. It is also considered that there is radiation only from the fluid. The fluid is a grey, emitting, and absorbing radiation, but non-scattering medium and the Roseland approximation is used to describe the radiative heat flux in the *x* direction is assumed negligible in comparison the channel walls ($y^* = 0$) while the other wall at $y^* = d$ is maintained at a constant temperature T_d, which causes free convection current in the channel. Under usual Boussinesq's approximation the mathematical model for the above free convection flow in the channel is stated below in equations (1), (2) and (3)

$$\frac{\partial U'}{\partial t'} - v_0 \frac{\partial U'}{\partial y'} = \frac{\upsilon \partial U'}{\partial y'} + g\beta(T' - T_d) + g\beta^*(C' - C'_d) \tag{1}$$

$$\frac{\partial c'}{\partial t'} - v_0 \frac{\partial c'}{\partial y'} = D^* \frac{\partial^2 c'}{\partial y'^{12}} \tag{2}$$

$$\frac{\partial T'}{\partial t'} v_0 \frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^{2}} + \frac{1}{\rho c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 \quad (3)$$

The initial and boundary conditions

$$\left.\begin{array}{l} U' = 0, T' = T_d' + (T' - T_d'), \\ C' = C_d' + (C' - C_d') \quad at \quad y = 0 \\ U' = 0, T' = T_w', C' = C_w' \quad at \quad y' = a \end{array}\right] \quad (4)$$

Where u is the axial Velocity, t is the time ,T is the fluid temperature. T_ω and T_0 are walls temperatures P the pressure, g the gravitational force, q the radiative heat flux, C the fluid concentration, C_ω and C_0 are walls concentrations D is the mass diffusivity, d is a constant, ω is the frequency of the oscillation, β is the coefficient of thermal expansion, β^* is the coefficient of concentration expansion, ρ the density of the fluid, v is the kinematics viscosity coefficient, v_0 means suction velocity, which is a non-zero positive constant and the minus sign indicate that the suction is toward the plate.

Dimensionless variable and parameters of the above three formulated problems are.

$$\left.\begin{array}{l} Gc = \dfrac{g\beta^*(C_d' - C_d')a^2}{\upsilon u}, \\ y = \dfrac{y'}{a},\ \theta = \dfrac{T' - T_d'}{T_\varpi' - T_d'}, \\ C = \dfrac{C' - C_d'}{C_\varpi' - C_d'},\ \eta = \dfrac{V_0 a}{\upsilon} \\ t = \dfrac{t\upsilon}{a^2},\ U = \dfrac{u'}{u}, \\ Gr = \dfrac{g\beta(T_\varpi' - T_d')}{\upsilon u}, \\ Sc = \dfrac{\upsilon}{D^*},\ \Pr = \dfrac{\upsilon\rho C_p}{k}, \\ Ec = \dfrac{U^2}{\upsilon\rho Cp}, \end{array}\right\} \quad (5)$$

Where U is the flow mean velocity.

Assume solution of the form

$$\frac{\partial U}{\partial t} - \eta\frac{\partial U}{\partial y} = \frac{\partial^2 U}{\partial y^2} + Gr\theta + GcC \quad (6)$$

$$\frac{\partial C}{\partial t} - \eta\frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \quad (7)$$

$$\frac{\partial \theta}{\partial t} - \eta\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + Ec\left(\frac{\partial u}{\partial y}\right)^2 \quad (8)$$

With the boundary conditions

$$\left.\begin{array}{l} U = 0, \theta = 1, C = 1 \quad at \quad y = 0 \\ U = 0, \theta = 0, C = 0 \quad at \quad y = 1 \end{array}\right] \quad (9)$$

$$\left.\begin{array}{l} U(y,t) = U_0(y) + U_1(y)e^{i\omega t} \\ C(y,t) = C_0(y) + C(y)e^{i\omega t} \\ \theta(y,t) = \theta_0(y) + \theta_1(y)e^{i\omega t} \end{array}\right\} \quad (10)$$

3. METHODOLGY

Equation (6) to (8) are coupled, non-linear partial differential equations and these cannot be solved in closed-form. However, these equations can be reduced to a set of ordinary differential equations,
which can be solved analytically
Where ω is the frequency of the oscillation. Substituting (10) into (6) to (9) we obtain

$$U_0'' + \eta U_0' = -Gr\theta_0 - GcC_0 \quad (11)$$

$$U_1'' + (\eta - iw)U_1' = -Gr\theta_1 - GcC_1 \quad (12)$$

$$C_0'' + \eta ScC_0' = 0 \quad (13)$$

$$C_1'' + \eta ScC_1' - i\omega ScC_1 = 0 \quad (14)$$

$$\theta_0'' + \eta\Pr\theta_0' = -Ec\Pr U_0'^2 \quad (15)$$

$$\theta_1'' + \eta\Pr\theta_1' - iw\Pr\theta_1 = -2Ec\Pr U_1' U_1' \quad (16)$$

Equation (11) to (16) are solved with help of (17), the solutions for fluid temperature , concentration and velocity are given as follows

$$\left.\begin{array}{l} U_0(y)=U_1(y)=0,\ \theta_0(y)=\theta_1(y)=0 \\ C_0(y)=C_1(y)=0 \qquad at\ \ y=0 \\ U_0(y)=U_1(y)=0,\ \theta_0(y)=1,\ \theta_1(y)=0 \\ C_0(y)=1\ ,C_1(y)=0 \qquad at\ \ y=1 \end{array}\right] \quad (17)$$

$$\left.\begin{array}{l} \theta(y,t)=\dfrac{1}{1-e^{-\eta\Pr}}\left[e^{-\eta\Pr y}-e^{-\eta\Pr}\right]+ \\ Ec\left(\begin{array}{l} b_1e^{-2\eta\Pr y}+b_2e^{-(\eta Sc+\eta\Pr)y}+b_3ye^{-\eta\Pr y} \\ +b_4e^{-(\eta\Pr+\eta)y}+b_5e^{-2\eta Scy}+b_6e^{-\eta Scy}+ \\ b_7e^{-(\eta Sc+\eta)y}+b_8y+b_9+b_{10}e^{-2\eta y}+ \\ b_{11}e^{-\eta y}+\dfrac{a_{16}}{\eta\Pr}+a_{17}e^{\eta\Pr y} \end{array}\right) \end{array}\right\} \quad (18)$$

$$C(y,t)=\frac{1}{1-e^{-\eta Sc}}\left[e^{-\eta Scy}-e^{-\eta Sc}\right] \quad (19)$$

$$\begin{array}{l} U(y,t)=\dfrac{a_1e^{-\eta\Pr y}}{\eta\Pr(\eta\Pr-\eta)}+\dfrac{a_2e^{-\eta Scy}}{\eta Sc(\eta Sc-\eta)} \\ +\dfrac{a_3y}{\eta}-\dfrac{a_3}{\eta^2}+\dfrac{a_4}{\eta}+a_5e^{-\eta y}+Ec+ \\ -\dfrac{Grb_1e^{-2\eta\Pr y}}{2\eta\Pr(2\eta\Pr-\eta)}-\dfrac{Grb_2e^{-(\eta\Pr+\eta Sc)y}}{(\eta\Pr+\eta Sc)(\eta\Pr+\eta Sc-\eta)} \\ +\dfrac{Grb_3ye^{-\eta\Pr y}}{\eta\Pr(\eta\Pr-\eta)}+\dfrac{Grb_3e^{-(\eta\Pr-\eta)y}}{\eta^2\Pr^2(\eta\Pr-\eta)}-\dfrac{Grb_4e^{-(\eta\Pr+\eta)y}}{(\eta\Pr+\eta)\eta\Pr} \\ -\dfrac{Grb_5e^{-2\eta Scy}}{2\eta Sc(2\eta Sc-\eta)}-\dfrac{Grb_6e^{-\eta Scy}}{\eta Sc(\eta Sc-\eta)}-\dfrac{Grb_7e^{-(\eta Sc+\eta)y}}{(\eta Sc+\eta)\eta Sc}+ \\ \dfrac{Grb_8y^2}{2\eta}+\dfrac{Grb_8y}{\eta^2}+\dfrac{Grb_8}{\eta^3}-\dfrac{Grb_{10}e^{-2\eta y}}{2\eta^2}+\dfrac{Grb_{11}y}{\eta} \\ -\dfrac{Gr_{17}e^{-\eta\Pr y}}{\eta\Pr(\eta\Pr-\eta)}-\dfrac{Gr}{\eta}\left(b_7+\dfrac{a_{16}}{\eta\Pr}\right)y+ \\ \dfrac{Gr}{\eta^2}\left(b_7+\dfrac{a_{16}}{\eta\Pr}\right)+\dfrac{q_1}{\eta}+q_2 \end{array} \quad (20)$$

Skin friction number by differentiating (20) taking y=0

$$\tau=-\frac{\partial u}{\partial y}\Big|_{y=0}=-\frac{a_1}{(\eta\Pr-\eta)}-\frac{a_2}{(\eta Sc-\eta)}-\frac{a_3}{\eta^2}+\frac{a_4}{\eta}-a_5$$

$$\begin{array}{l} +Ec\left[+\dfrac{Grb_1}{(2\eta\Pr-\eta)}+\dfrac{Grb_2}{(\eta\Pr+\eta Sc-\eta)}-\dfrac{Grb_3}{\eta^2\Pr^2}+\dfrac{Grb_4}{\eta\Pr}\right. \\ +\dfrac{Grb_5}{(2\eta Sc-\eta)}-\dfrac{Grb_6}{(\eta Sc-\eta)}+\dfrac{Grb_7}{\eta Sc} \\ \left.+\dfrac{Grb_8}{\eta^3}+Grb_{10}+\dfrac{Gr_{17}}{(\eta\Pr-\eta)}+\dfrac{Gr}{\eta^2}\left(b_7+\dfrac{a_{16}}{\eta\Pr}\right)+\dfrac{q_1}{\eta}+q_2\right] \end{array}$$

(21)

Shear wood number by differentiating (19) taking y=0

$$\frac{\partial c}{\partial y\,/}\Big|_{y=0}=\frac{e^{-\eta Sc}}{1-e^{-\eta Sc}} \quad (22)$$

$$\begin{array}{l} \dfrac{\partial\theta}{\partial y}\Big|_{y=0}=\dfrac{e^{-\eta\Pr}}{1-e^{-\eta\Pr}}+Ec \\ -\eta Scb_6-(\eta Sc+\eta)b_7+b_9-2\eta b_{10}-\eta b_{11}+\dfrac{a_{16}}{\eta\Pr} \\ +\eta\Pr a_{17}] \end{array} \quad (23)$$

Where $b_1,b_2,b_3,b_4,b_5,b_6,b_7,b_8,b_9,b_{10},b_{11},a_{16},a_{17}$ are constants, their expression are not presented here for sake of brevity.

4. RESULTS AND DISCUSSION

Except otherwise indicated, we used the following parameters values for the computation Pr = 0.71, Sc =0.01, η = 20, Ec = 0.009, Gr = 2.6, Gc = 1

The Concentration Profiles have been studied and presented in figure 4.2.1 to 4.2.2. The Concentration Profiles for different values of Schmidt number (Sc = 0.1, 0.2, 0.3, 0.4.) are shown in figure 4.2.1. It is observed that the Concentration decreases with the increases of Schmidt number. The Concentration Profiles for different values of Eta number (η = 0.5 , 1.0, 1.5, 2.0.) are shown in figure 4.2.2. It is observed that the Concentration decreases with the increases of Eta number. The Temperature Profiles have been studied and presented in figure 4.2.3 to 4.2.9. The Temperature Profiles for different values of Schmidt number (Sc = 0.1, 0.2, 0.3, 0.4.) are shown in figure 4.2.3. It is observed that the Temperature decreases with the increases of Schmidt number. The Temperature Profiles for different values of Schmidt number (Sc = 1.0, 1.5, 2.0, 2.5.) are shown in figure 4.2.4. It is observed that the Temperature decreases with the increases of Schmidt number. The Temperature Profiles for different values of Schmidt number (Sc = 0.1, 0.2, 0.3, 0.4.) are shown in figure 4.2.5. It is observed that the Temperature decreases with the increases of Schmidt number. The Temperature Profiles for different values of Suction/injection parameter (η = 4.0, 6.0, 8.0, 10.0.) are shown in figure 4.2.6. It is observed that the Temperature decreases with the increases of Eta number. The Temperature Profiles for different values of Eckert number (Ec = 0.01, 0.03, 0.05, 0.07.) are shown in figure 4.2.7. It is observed that the Temperature decreases with the increases of Eckert number. The Temperature Profiles for different values of Prandtl number (Pr = 0.64, 0.68, 0.71, 0.85.) are shown in figure 4.2.8. It is observed that the Temperature increases with the increasing of Prandtl number. The Temperature Profiles for different values of Prandtl number (Pr = 0.64, 0.68, 0.71, 0.85.) are shown in figure 4.2.9. It is observed that the Temperature increases with the increasing of Prandtl number. The Velocity Profiles for different values of Schmidt number (Sc = 0.11, 0.12, 0.13, 0.14.) are shown in figure 4.2.10. It is observed that the Velocity increases with the increasing of Schmidt number. The Velocity Profiles for different values of Prandtl number (Pr = 0.60, 0.64, 0.68, 0.71.) are shown in figure 4.2.11. It is observed that the Velocity decreases with the increasing of Prandtl number. The Velocity Profiles for different values of Eta number (η = 0.11, 0.12, 0.13, 0.14.) are shown in figure 4.2.12. It is observed that the Velocity decreases with the increasing of Eta number. The Velocity Profiles for different values of Grashof number (Gr = 3.0, 3.2, 3.4, 3.6.) are shown in figure 4.2.13. It is observed that the Velocity decreases with the increasing of Grashof number. The Velocity Profiles for different values of modified Grashof number (Gc = 2.0, 2.2, 2.4, 2.6.) are shown in figure 4.2.14. It is observed that the Velocity increases with the increasing of modified Grashof number.

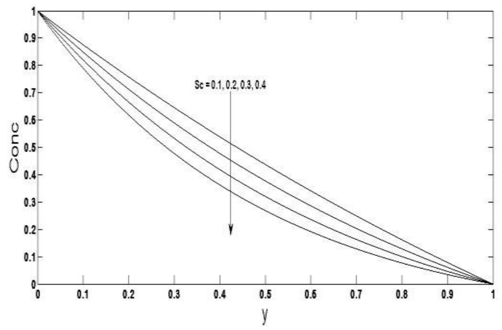

Figure 4.2.1 Concentration profiles for different values of Schmidt number

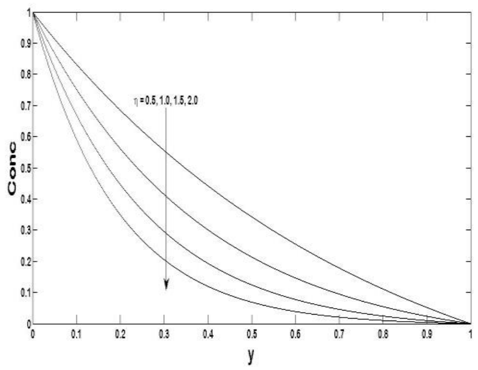

Figure 4.2.2 Concentration profiles for different values of η

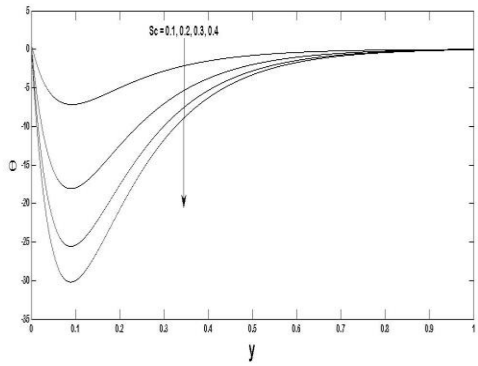

Figure 4.2.3 Temperature profiles for different values of Schmidt number

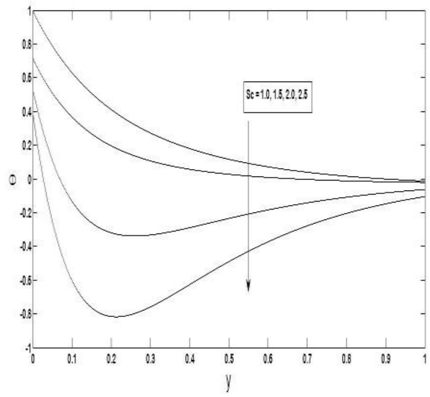

Figure 4.2.4 Temperature profiles for different values of Schmidt number

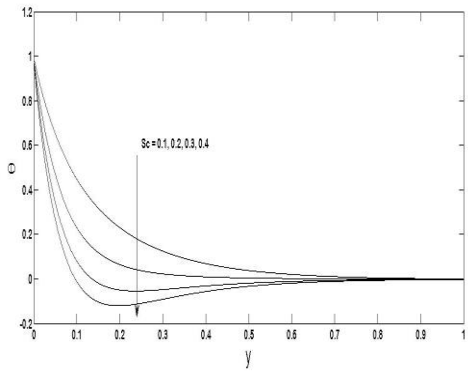

Figure 4.2.5 Temperature profiles for different values of Schmidt number

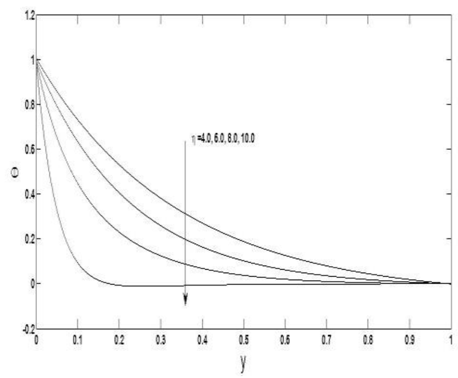

Figure 4.2.6 Temperature profiles for different values of η number

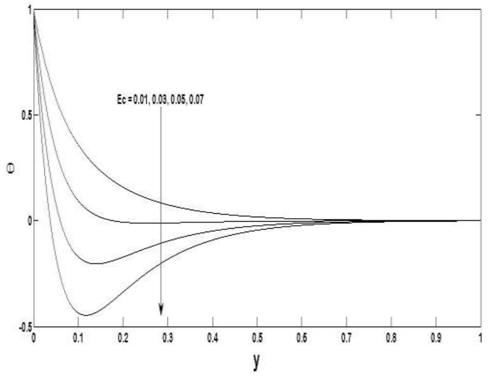

Figure 4.2.7 Temperature profiles for different values of Eckert number

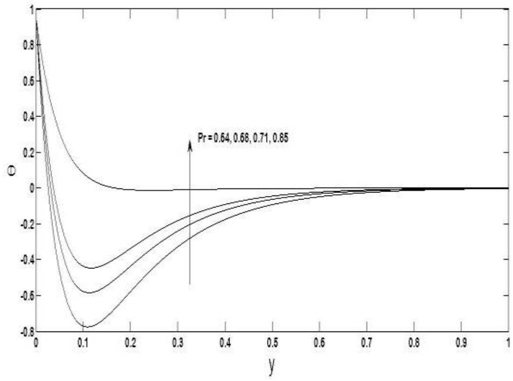

Figure 4.2.8 Temperature profiles for different values of Prandtl number

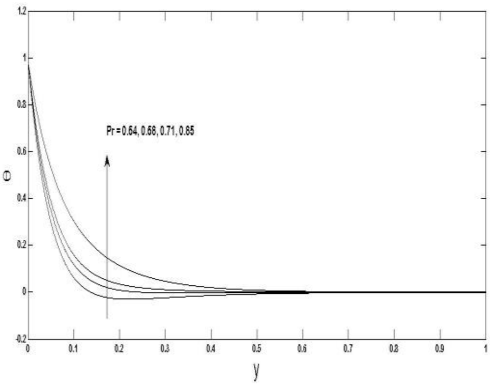

Figure 4.2.9 Temperature profiles for different values of Prandtl number

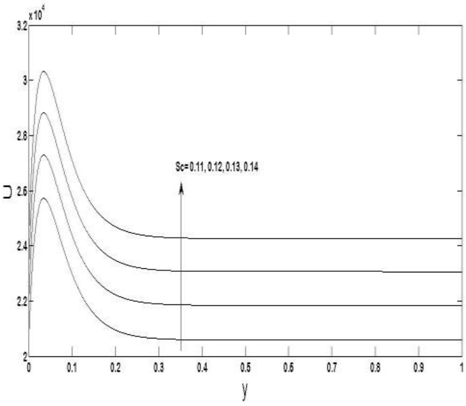

Figure 4.2.10 Velocity profiles for different values of Schmidt number

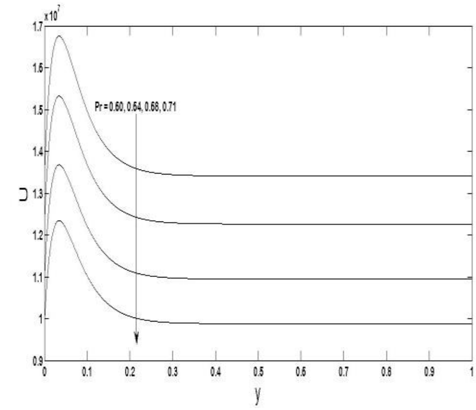

Figure 4.2.11 Velocity profiles for different values of Prandtl number

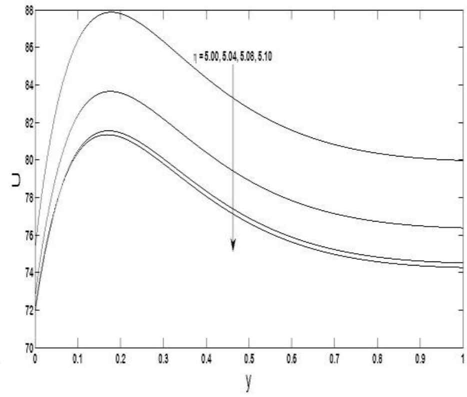

Figure 4.2.12 Velocity profiles for different values of Eta number

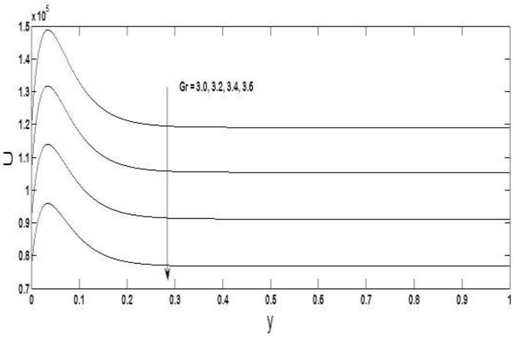

Figure 4.2.13 Velocity Profiles for different values of Grashof number

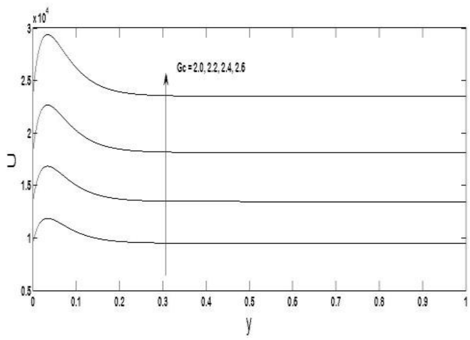

Figure 4.2.14 Velocity profiles for different values of Modified Grashof number

5. CONCLUSION

This paper studied heat and mass transfer with radiation and dissipation over a fixed vertical plate. The dimensionless governing equations are solved using perturbation techniques. The effect of different parameters such as modified Grashof number, Grashof number, Eckert, Schmidt number, Prandtl number and Eta are studied. The conclusions of the study are as follows

(i) The velocity decreases with increases of η ,Gr and Pr
(ii) The velocity increases with the increases of Sc and Gc
(iii) The Concentration decreases with the increases of Sc and η
(iv) The temperature decreases with increases of Sc, η and Ec.
(v) The temperature increases with increases of Pr

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