Heat and Mass Transfer for a Chemically Reacting Permeable Fluid in a Vertical Double Passage Channel

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Abstract

The heat and mass transfer characteristics of mixed convection about a vertical double passage channel in a saturated porous medium is presented. The governing equations of continuity, momentum, energy and concentration which are coupled and nonlinear ordinary differential equations are solved analytically using perturbation technique and using differential transform method. Numerical calculations for the analytical expressions are carried out and the results are shown graphically. The effects of the various dimensionless parameters such as porous parameter, thermal Grashoff number, mass Grashoff number, Brinkman number and chemical reaction parameter on the velocity, temperature and concentration fields are discussed in detail at all the baffle positions.

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Keywords: Baffle, first order chemical reaction, mixed convection, porous medium, perturbation method, Differential Transform method.

INTRODUCTION

Combined heat and mass transport in porous media is gaining attention due to its many interesting applications in engineering. The flow phenomenon in this case is relatively complex compared to that of pure thermal or solutal transport. Some examples of particular interest are the migration of moisture in fibrous insulation, the transport of contaminants in saturated soil, drying processes or solute transfer in the mushy layer during the solidification of binary alloys.

Combined heat and mass transfer by free convection under boundary layer approximations has been studied by Bejan and Khair [1], Lai and Kulacki [2] and Murthy and Singh [3]. Coupled heat and mass transfer by mixed convection in Darcian fluid saturated porous media has been analyzed by Lai [4]. The free convection heat and mass transfer in a porous enclosure has been studied by Angirasa et al. [5]. The combined effects of thermal and mass diffusion in channel flows has been studied by Nelson and Wood [6, 7], and others [8,9]. Das et al. [10] have studied the effect of mass transfer on the flow started impulsively past an infinite vertical plate in presence of wall heat flux and chemical reaction. Muthucumaraswamy and Ganeshan [11, 12] have studied the impulsive motion of a vertical plate with heat flux/mass flux/suction and diffusion of chemically reactive species. Seddeek [13] has studied the finite element method for the effect of chemical reaction, variable viscosity, thermophoresis, and heat generation/absorption on a boundary layer hydro magnetic flow with heat and mass transfer over a heat surface. Kandaswamy et al. [14, 15] have examined the effects of chemical reaction, heat and mass transfer with or without MHD flow in the presence of heat source/suction. Recently Prathap Kumar et al. [16-19] studied homogeneous and heterogeneous chemical reactions of immiscible fluids.

The well-known concepts of passive techniques of convective heat transfer enhancement [20, 21] are to increase the heat transfer area and/or the convective heat transfer co-efficient in terms of (a) mixing the main flow and/or the flow in the wall region and by using rough surface, inserts, etc. (b) reducing the flow boundary layer thickness by using offset strip fins, jet impingement, etc. (c) creating the rotating and/or the secondary flow by using swirl flow device, duct rotation, etc. (d) raising the turbulence intensity by using rough surface, turbulence promoter, etc. The most commonly used technique for internal cooling enhancement is the placement of periodic ribs. Ribs are generally mounted on the heat transfer surface, which disturb the boundary layer growth and enhancement of the heat transfer between the surface and the fluid. In addition to ribs and impingement, a third common internal cooling enhancement technique is the placement of internal flow swirls, tape twisters, or baffles. The convective heat transfer in a vertical channel could be enhanced by using special inserts which can be specially designed to increase the included angle between the velocity vector and temperature gradient vector rather than to promote turbulence. This increases the rate of heat transfer without a considerable drop in the pressure [22]. A plane baffle may be used as an insert to enhance the rate of heat transfer in the channel. A thin and perfectly conductive baffle is used so as to avoid a considerable increase in the transverse thermal resistance into the channel. Candra et al. [23, 24] investigated the use of ribbed walls, Han et al. [25] used V-shaped

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turbulence promoters, and Beitelmal et al. [26] demonstrated the effect of jet impingement mechanism. Recently Prathap Kumar et al. [27, 28] and Umavathi [29] studied mixed convection of non-Newtonian fluid in a vertical double-passage channel.

One of the semi-exact method which does not need small parameters is the DTM. The concept of the DTM was first proposed by Zhou [30], who solved the linear and nonlinear problems in electrical circuit problems. The main advantage of this method is that it can be applied directly to non-linear differential equations. It is a semi-analytical-numerical technique that formulizes Taylor series in a very different manner. This method constructs, for differential equations, an analytical solution in the form of a polynomial. This method is well addressed in [31-34].

The mixed convection in a vertical double passage channel saturated with porous medium in the presence of first order chemical reaction has not been reported in the literature. This type of investigation is useful in understanding heat and mass transfer characteristics for solidification and centrifugal casting of metals, petroleum industry, biomechanics and geophysical problems. This work aims to study a mathematical model for the steady mixed convection heat and mass transfer saturated with porous medium in a vertical channel by inserting a perfectly thin conducting baffle. Regular perturbation method is employed to solve the nonlinear system of governing equations which are valid for small values of Brinkman number. To relax this constraint the nonlinear equations are solved by differential transform method. The solutions obtained by DTM and PM are tabulated and are found to be in good agreement in the absence of viscous dissipation.

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MATHEMATICAL FORMULATION

The physical configuration (Fig. 1) consists of two infinite, vertical parallel plates maintained at different constant temperatures, extending in the X and Y directions. The X-axis is taken vertically upward, and parallel to the direction of buoyancy, and the Y-axis is normal to it. Walls are maintained at a constant temperature and the fluid properties are assumed to be constant. The channel is divided into two passages by means of thin, perfectly conducting plane baffle and each stream will have its own pressure gradient and hence the velocity will be individual in each stream. The fluid properties are assumed to be constant. The channel is filled with porous matrix and the fluid in steam-1 is concentrated after introducing the baffle.

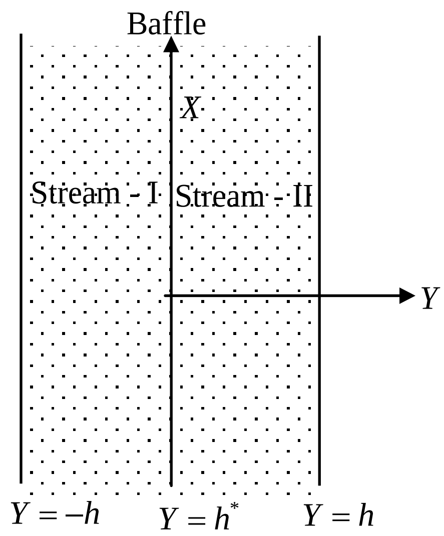

Figure 1. Physical configuration.

The stream wise and the transverse momentum balance equations using Brinkman model yields [35,36].

Stream-I

$$\rho g \beta_T (T_1 - Tw_2) + \rho g \beta_C (C_1 - C_0) - \frac{\partial P}{\partial x} + \mu \frac{d^2 U_1}{dy^2} - \frac{\mu}{\kappa} U_1 = 0 \quad (1)$$

$$\frac{d^2 T_1}{dy^2} + \frac{\mu}{K}\left(\frac{dU_1}{dy}\right)^2 + \frac{\mu}{K\kappa} U_1^2 = 0 \quad (2)$$

$$D\frac{d^2 C}{dy^2} - K_c C = 0 \quad (3)$$

Stream-II

$$\rho g \beta_T (T_2 - Tw_2) - \frac{\partial P}{\partial x} + \mu \frac{d^2 U_2}{dy^2} - \frac{\mu}{\kappa} U_2 = 0 \quad (4)$$

$$\frac{d^2 T_2}{dy^2} + \frac{\mu}{K}\left(\frac{dU_2}{dy}\right)^2 + \frac{\mu}{K\kappa} U_1^2 = 0 \quad (5)$$

It is assumed that the fluid viscosity and the effective viscosity are same. This assumption has been made to restrict the number of parameters governing the system. In reality, as pointed out by one of the reviewers and also by the experimental observations of Givler and Altobelli [37], they are not equal.

The boundary conditions on velocity are no-slip conditions requiring that the velocity must vanish at the wall. The boundary conditions on concentration are C_1 and C_2. The boundary

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conditions on temperature are isothermal conditions. In addition, the continuity of temperature and heat flux at the baffle is assumed.

$U_1 = 0$, $T_1 = T_{W_1}$, $C = C_1$, at $Y = -h$

$U_2 = 0$, $T_2 = T_{W_2}$, at $Y = h$

$$U_1 = 0 \ , U_2 = 0 \ , \ T_1 = T_2 \ , \frac{dT_1}{dY} = \frac{dT_2}{dY} \ , C = C_2 \ , \text{at } Y = h^* \tag{6}$$

Equations (1) – (5) along with boundary conditions (6) are made dimensionless by using the following transformations:

$$u_i = \frac{U_i}{\bar{\bar{U}}_1}, \ \theta_i = \frac{T_i - T_{W_2}}{T_{W_1} - T_{W_2}}, \ Gr = \frac{g\beta_T \Delta T h^3}{\upsilon^2}, \ Gc = \frac{g\beta_C \Delta C h^3}{\upsilon^2}, \ \phi = \frac{C_i - C_0}{C_1 - C_0}, \ \text{Re} = \frac{\bar{U}_1 h}{\upsilon}, \ Br = \frac{\bar{U}_1^2 \mu}{k\Delta T},$$

$$Y^* = \frac{y^*}{h}, \ p = \frac{h^2}{\mu \bar{U}_1} \frac{\partial p}{\partial X} \ , \ \Delta T = T_{W_2} - T_{W_1} , \ \Delta C = C_1 - C_0 , \ Y = \frac{y}{h}, \ \sigma^2 = \frac{h^2}{\kappa} \tag{7}$$

The nondimensional momentum, energy and concentration equations corresponding to stream-I and stream-II are as follows

Stream-I

$$\frac{d^2 u_1}{dy^2} + GR_T \theta_1 + GR_C \phi - p - \sigma^2 u_1 = 0 \tag{8}$$

$$\frac{d^2 \theta_1}{dy^2} + Br\left(\left(\frac{du_1}{dy} \right)^2 + \sigma^2 u_1^{\,2} \right) = 0 \tag{9}$$

$$\frac{d^2 \phi}{dy^2} - \alpha^2 \phi = 0 \tag{10}$$

Stream-II

$$\frac{d^2 u_2}{dy^2} + GR_T \theta_2 - p - \sigma^2 u_2 = 0 \tag{11}$$

$$\frac{d^2 \theta_2}{dy^2} + Br\left(\left(\frac{du_2}{dy} \right)^2 + \sigma^2 u_2^2 \right) = 0 \tag{12}$$

The nondimensional form of the velocity, temperature and concentration boundary conditions (6) become:

$u_1 = 0$, $\theta_1 = 1$, $\phi = 1$, at $y = -1$

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$u_2 = 0$, $\theta_2 = 0$, at $y = 1$

$$u_1 = 0,\ u_2 = 0,\ \theta_1 = \theta_2,\ \frac{d\theta_1}{dy} = \frac{d\theta_2}{dy},\ \phi = n, \text{at } y = y^* \tag{13}$$

where $GR_T = \frac{Gr}{\mathrm{Re}}, GR_C = \frac{Gc}{\mathrm{Re}}, \alpha = \frac{K_c h^2}{D},\ n = \frac{C_2 - C_0}{C_1 - C_0}$.

SOLUTIONS

The exact solutions for concentration distribution is found using equations (10) subject to the boundary conditions (6) and is given by

$$\phi_1 = B_1 Cosh(\alpha y) + B_2 Sinh(\alpha y) \tag{14}$$

Perturbation method

Equations (8), (9), (11) and (12) are coupled non-linear ordinary differential equations. Approximate solutions can be found by using the regular perturbation method and Differential Transform method. The perturbation parameter we considered is the Brinkman number Br. Adopting this method, solutions for velocity and temperature are assumed in the form

$$u_i(y) = u_{i0}(y) + Br u_{i1}(y) + Br^2 u_{i2}(y) + \dots \tag{15}$$

$$\theta_i(y) = \theta_{i0}(y) + Br\theta_{i1}(y) + Br^2\theta_{i2}(y) + \dots \tag{16}$$

where $i = 1, 2$ represents stream-I and stream-II respectively.

Substituting Equations (15), (16) and (17) in Equations (8-12) and equating the coefficients of like power of Br to zero and one, we obtain the zeroth and first order equations as

Stream-I

Zeroth order equations

$$\frac{d^2 u_{10}}{dy^2} + GR_T\theta_{10} + GR_C\phi - p - \sigma^2 u_{10} = 0 \tag{18}$$

$$\frac{d^2\theta_{10}}{dy^2} = 0 \tag{19}$$

First order equations

$$\frac{d^2 u_{11}}{dy^2} + GR_T\theta_{11} - \sigma^2 u_{11} = 0 \tag{20}$$

$$\frac{d^2\theta_{11}}{dy^2} + \left(\frac{du_{10}}{dy}\right)^2 + \sigma^2 u_{10}{}^2 = 0 \qquad (21)$$

Stream-II

Zeroth order equations

$$\frac{d^2u_{20}}{dy^2} + GR_T\theta_{20} - p - \sigma^2 u_{20} = 0 \qquad (22)$$

$$\frac{d^2\theta_{20}}{dy^2} = 0 \qquad (23)$$

First order equations

$$\frac{d^2u_{21}}{dy^2} + GR_T\theta_{21} - \sigma^2 u_{20} = 0 \qquad (24)$$

$$\frac{d^2\theta_{21}}{dy^2} + \left(\frac{du_{20}}{dy}\right)^2 + \sigma^2 u_{20}{}^2 = 0 \qquad (25)$$

The corresponding boundary conditions reduces to

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Zeroth-order conditions

$u_{10} = 0$, $\theta_{10} = 1$, at $y = -1$

$u_{20} = 0$, $\theta_{20} = 0$, at $y = 1$

$$u_{10} = 0,\ u_{20} = 0,\ \theta_{10} = \theta_{20},\ \frac{d\theta_{10}}{dy} = \frac{d\theta_{20}}{dy},\ \text{at } y = y^* \qquad (26)$$

First order conditions

$u_{11} = 0$, $\theta_{11} = 0$ at $y = -1$

$u_{21} = 0$, $\theta_{21} = 0$ at $y = 1$

$$u_{11} = 0,\ u_{21} = 0,\ \theta_{11} = \theta_{21},\ \frac{d\theta_{11}}{dy} = \frac{d\theta_{21}}{dy}\ \text{at } y = y^* \qquad (27)$$

Basic concepts of the differential transform method

The analytical solutions obtained in Section 3.1 are valid only for small values of Br. In many practical problems mentioned earlier, the values of Br are usually large. In that case analytical solutions are difficult, and hence we resort to semi-numerical-analytical method known as Differential Transform method (DTM).

The general concept of DTM is explained here: The k[th] differential transformation of an analytical function $F(k)$ is defined as (Zhou [20])

$$\bar{U}(k)=\frac{1}{k!}\left[\frac{d^k u(y)}{dy^k}\right]_{y=0}, \tag{28}$$

and the inverse differential transformation is given by

$$u(y)=\sum_{k=0}^{\infty}\bar{U}(k)y^k, \tag{29}$$

where $\bar{U}(k)$ is the transformed form of $u(y)$.

In real applications the sum $\sum_{k=n}^{\infty}\bar{U}(k)y^k$ is very small and can be neglected when n is sufficiently large. So $u(y)$ can be expressed by a finite series, and equation (29) may be written as

$$u(y)=\sum_{k=0}^{n}\bar{U}(k)y^k \quad , \tag{30}$$

where the value of n depends on the convergence requirement in real applications. Table 1 lists the basic mathematics operations frequently used in the following analysis.

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The solutions of zeroth and first order equations (18) to (25) using the boundary conditions (26) and (27) are

Zeroth-order solutions

Stream-I

$$\theta_{10}=C_1 y+C_2 \tag{31}$$

$$u_{10}=A_1\cosh(\sigma y)+A_2\sinh(\sigma y)+r_1+r_2 y+r_3\cosh(\alpha y)+r_4\sinh(\alpha y) \tag{32}$$

Stream-II

$$\theta_{20}=C_3 y+C_4 \tag{33}$$

$$u_{20}=A_3\cosh(\sigma y)+A_4\sinh(\sigma y)+r_5+r_6 y \tag{34}$$

First order solutions

Stream-I

$$\begin{aligned}\theta_{11} = {} & E_2 + E_1 y + q_1 y^2 + q_2 y^3 + q_3 y^4 + q_4 \cosh(\alpha y) + q_5 \sinh(\alpha y) + q_6 \cosh(2\alpha y) \\ & + q_7 \sinh(2\alpha y) + q_8 \cosh(2\sigma y) + q_9 \sinh(2\sigma y) + q_{10} \cosh(\sigma y) + q_{11} \sinh(\sigma y) \\ & + q_{12} y \cosh(\sigma y) + q_{13} y \sinh(\sigma y) + q_{14} y \cosh(\alpha y) + q_{15} y \sinh(\alpha y) \\ & + q_{16} \cosh(\alpha+\sigma) y + q_{17} \cosh(\alpha-\sigma) y + q_{18} \sinh(\alpha+\sigma) y + q_{19} \sinh(\alpha-\sigma) y\end{aligned} \quad (35)$$

$$\begin{aligned}u_{11} = {} & E_5 \cosh(\sigma y) + E_6 \sinh(\sigma y) + H_1 + H_2 y + H_3 y^2 + H_4 y^3 + H_5 y^4 + H_6 \cosh(\alpha y) \\ & + H_7 \sinh(\alpha y) + H_8 \cosh(2\alpha y) + H_9 \sinh(2\alpha y) + H_{10} \cosh(2\sigma y) + H_{11} \sinh(2\sigma y) \\ & + H_{12} y \cosh(\sigma y) + H_{13} y \sinh(\sigma y) + H_{14} y \cosh(\alpha y) + H_{15} y \sinh(\alpha y) \\ & + H_{16} \cosh(\alpha+\sigma) y + H_{17} \cosh(\alpha-\sigma) y + H_{18} \sinh(\alpha+\sigma) y \\ & + H_{19} \sinh(\alpha-\sigma) y + H_{20} y^2 \cosh(\sigma y) + H_{21} y^2 \sinh(\sigma y)\end{aligned} \quad (36)$$

Stream-II

$$\begin{aligned}\theta_{21} = {} & E_4 + E_3 y + F_1 y^2 + F_2 y^3 + F_3 y^4 + F_4 \cosh(2\sigma y) + F_5 \sinh(2\sigma y) \\ & + F_6 \cosh(\sigma y) + F_7 \sinh(\sigma y) + F_8 y \cosh(\sigma y) + F_9 y \sinh(\sigma y)\end{aligned} \quad (37)$$

$$\begin{aligned}u_{21} = {} & E_7 \cosh(\sigma y) + E_8 \sinh(\sigma y) + H_{22} + H_{23} y + H_{24} y^2 + H_{25} y^3 + H_{26} y^4 + H_{27} \cosh(2\sigma y) \\ & + H_{28} \sinh(2\sigma y) + H_{29} y \cosh(\sigma y) + H_{30} y \sinh(\sigma y) + H_{31} y^2 \cosh(\sigma y) + H_{32} y^2 \sinh(\sigma y)\end{aligned} \quad (38)$$

Taking differential transform of Eqs. (8)-(12), one can obtain the transformed equations as

Stream-I

$$\bar{U}_1(k+2) = -\frac{1}{(k+1)(k+2)}\left(GR_T \bar{\Theta}_1(k) + GR_c \bar{\Phi}(k) - \sigma^2 \bar{U}_1(k) - p\,\delta(k)\right) \quad (39)$$

$$\bar{\Theta}_1(k+2) = -\frac{Br}{(k+1)(k+2)}\left(\sum_{r=0}^{k}(k-r+1)(r+1)\bar{U}_1(k-r+1)\bar{U}_1(r+1) + \sigma^2 \sum_{r=0}^{k} \bar{U}_1(k-r)\bar{U}_1(r)\right) \quad (40)$$

$$\bar{\Phi}(k+2) = \frac{\alpha^2 \bar{\Phi}(k)}{(k+1)(k+2)} \quad (41)$$

Stream-II

$$\bar{U}_2(k+2) = -\frac{1}{(k+1)(k+2)}\left(GR_T \bar{\Theta}_2(k) - \sigma^2 \bar{U}_2(k) - p\,\delta(k)\right) \quad (42)$$

$$\bar{\Theta}_2(k+2) = -\frac{Br}{(k+1)(k+2)}\left(\sum_{r=0}^{k}(k-r+1)(r+1)\bar{U}_2(k-r+1)\bar{U}_2(r+1) + \sigma^2 \sum_{r=0}^{k} \bar{U}_2(k-r)\bar{U}_2(r)\right) \quad (43)$$

The following are the transformed initial conditions

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$$\bar{U}_1(0) = c_1,\ \bar{U}_1(1) = c_2,\ \ \bar{U}_2(0) = c_3,\ \bar{U}_2(1) = c_4,$$
$$\bar{\Theta}_1(0) = d_1,\ \bar{\Theta}_1(1) = d_2,\ \ \bar{\Theta}_2(0) = d_3,\ \bar{\Theta}_2(1) = d_4,$$
$$\bar{\Phi}(0) = e_1,\ \bar{\Phi}(1) = e_2 \tag{44}$$

Using the boundary condition (13), we can evaluate c_1, c_2, c_3, c_4, d_1, d_2, d_3, d_4, e_1 and e_2.

RESULTS AND DISCUSSIONS

The problem of mixed convection heat and mass transfer in a vertical double passage channel with filled with porous matrix is investigated. The analytical solutions are found using regular perturbation method considering Brinkman number as the perturbation parameter and Differential Transform method. The effects of governing parameters such as porous parameter σ, thermal Grashoff number GR_T, mass Grashoff number GR_C, Brinkman number Br and first order chemical reaction parameter α for the velocity, temperature and concentration are shown graphically.

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The effect of porous parameter σ on the flow is shown in Figs. 2a,b,c and 3a,b,c. As the porous parameter increases, the velocity and temperature decreases in both the streams at all baffle positions. As expected, the velocity profile decreases for increasing values of σ in both passages, showing the dragging effect of the Darcy resistance. When the baffle is at hot wall the maximum velocity is in stream-II ,when the baffle is at the cold wall the maximum velocity is seen in stream-I and when the baffle is in center of the channel the maximum velocity is seen in stream-I. The temperature decreases in both the streams at all the baffle positions as the porous parameter increases. Since we consider the continuity of temperature and continuity of heat flux at the baffle position, the temperature profiles are unique in both the passages.

The effect of thermal Grashoff number GR_T on the velocity and temperature field is shown in Figs. 4a,b,c and 5a,b,c at all three different baffle positions. As the thermal Grashoff number increases the velocity and temperature increases in both the streams at all baffle positions. This is an expected result because increase in thermal Grashoff number results in increase of bouncy force and hence increases the flow in both the streams at all baffle positions.

The effect of mass Grashoff number GR_C on the velocity and temperature field is shown in Figs. 6a,b,c and 7a,b,c respectively. As mass Grashoff number increases the flow is enhanced in both the streams at all the baffle positions. The enhancement on velocity is significant in stream-I when compared to stream-II .This is due to the fact that the fluid is concentrated only in stream-I. Though the fluid is not concentrated in stream-II still its effect is observed in stream-II when the baffle position is in the center of the channel, this is due to the fact that we have considered the baffle to be conducting. That is say that there is heat transfer from stream-I to stream-II but there is no mass transfer from stream-I to stream-II. Due to transfer of heat from stream-I to stream-II results in increase of both thermal buoyancy force and concentration buoyancy force and hence velocity increase in stream-II slightly as mass Grashoff number increases as seen in Fig. 6. As the mass Grashoff number increases the temperature increases significantly in stream-I when the baffle position is at the center of the cold wall. There is no much variation on the temperature field when the baffle position is near the hot wall.

The effect of Brinkman number Br is shown in Figs. 8a,b,c and 9a,b,c on the velocity and temperature fields respectively. As the Brinkman number increases, the velocity and temperature increases in both the streams at all the baffle positions. This is due to the fact that increase in Brinkman number increases the viscous dissipation and hence the flow is enhanced.

The effect of first order chemical reaction parameter on the velocity, temperature and concentration fields are displayed in Figs. 10a,b,c ,11a,b,c and 12a,b,c respectively. As α increases the velocity and temperature decreases in stream-I, and remains constant in stream-II. This is due to the fact that the fluid in stream-I is concentrated. The maximum value of velocity and temperature is seen in stream-II for the baffle position at $y^* = -0.8$ and in stream-I at baffle position at $y^* = 0 \,\&\, 0.8$. The effect of chemical reaction parameter α on the concentration is shown in Fig. 12a,b,c at different baffle positions. Since the fluid in stream is not concentrated, the concentration profiles are drawn in stream-I only at different baffle positions. It is observed that as α increases the concentration decreases. The similar result was also obtained by Srinivas and Muturajan [38] for mixed convective flow in a vertical channel in the absence of baffle.

A comparison of the numerical values of the dimensionless velocity and temperature for the present model using DTM and PM method is given in Tables 2(a,b,c), 3(a,b,c) and 4(a,b,c). An excellent agreement is observed, which validates our results. It is observed from Tables 2a, 3a and 4a that the values of velocity and temperature obtained by DTM agree with the exact solution (in the absence of dissipation). The values as given in Table 2(b,c) shows that the results of velocity and temperature agree to the order of 10^{-3} for small values of viscous dissipation

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(Brinkman number) whereas it agree to the order of 10^{-1} for large values of viscous dissipation. In Tables 3(b,c) and 4(b,c) we observed that the results of DTM agree well with the PM in the presence of viscous and Darcy dissipations in shorter stream whereas in the wider stream the results agree to the order of 10^{-2}.

CONCLUSIONS

In this study, flow, heat and mass transfer of fluid saturated with porous medium in a vertical double-passage channel was investigated. The governing equations were expressed in the non-dimensional form and are solved analytically by using regular perturbation method and Differential Transform method. The features of flow and heat transfer characteristics were analyzed by plotting graphs and discussed in detail. The main findings are concluded as follows:

1. Increasing the values of porous parameter reduces the flow field where as increase in the thermal Grashoff number and mass Grashoff number enhance the flow in both the streams at different baffle positions
2. With increase in the Brinkman number, the velocity and temperate are enhanced in both the streams.
3. Increase in the chemical reaction parameter suppresses the velocity and temperate in stream-I and remains invariant in stream-II. The chemical reaction parameter suppresses the concentration in stream-I
4. The use of baffle in the flow channel resulted in the heat transfer enhancement as high as compared to the heat transfer in a straight channel.
5. The numerical study indicated that the DTM gives more accurate in comparison to PM method. The method has been applied directly without requiring linearization, discretization, even perturbation. The obtained results certify the reliability of the algorithm and give it a wider applicability to nonlinear differential equations

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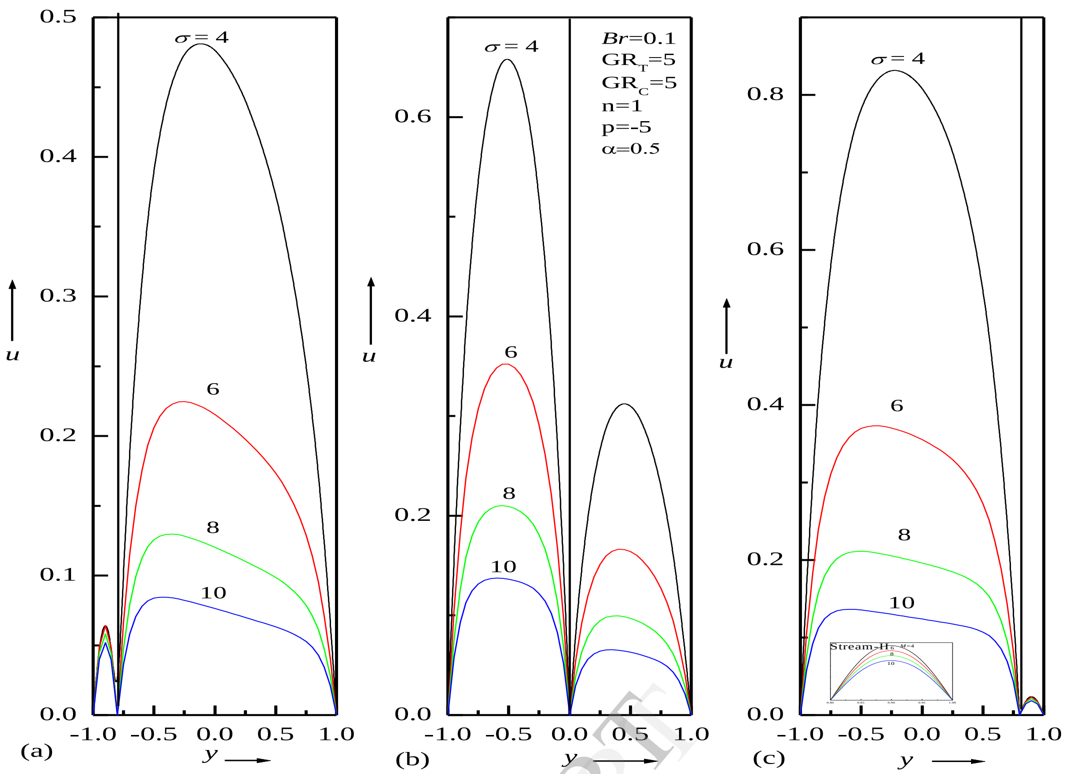

Fig. 2. Velocity profile for differnt values of porous parameter σ
(a) $y^*=-0.8$ (b) $y^*=0$ (c) $y^*=0.8$

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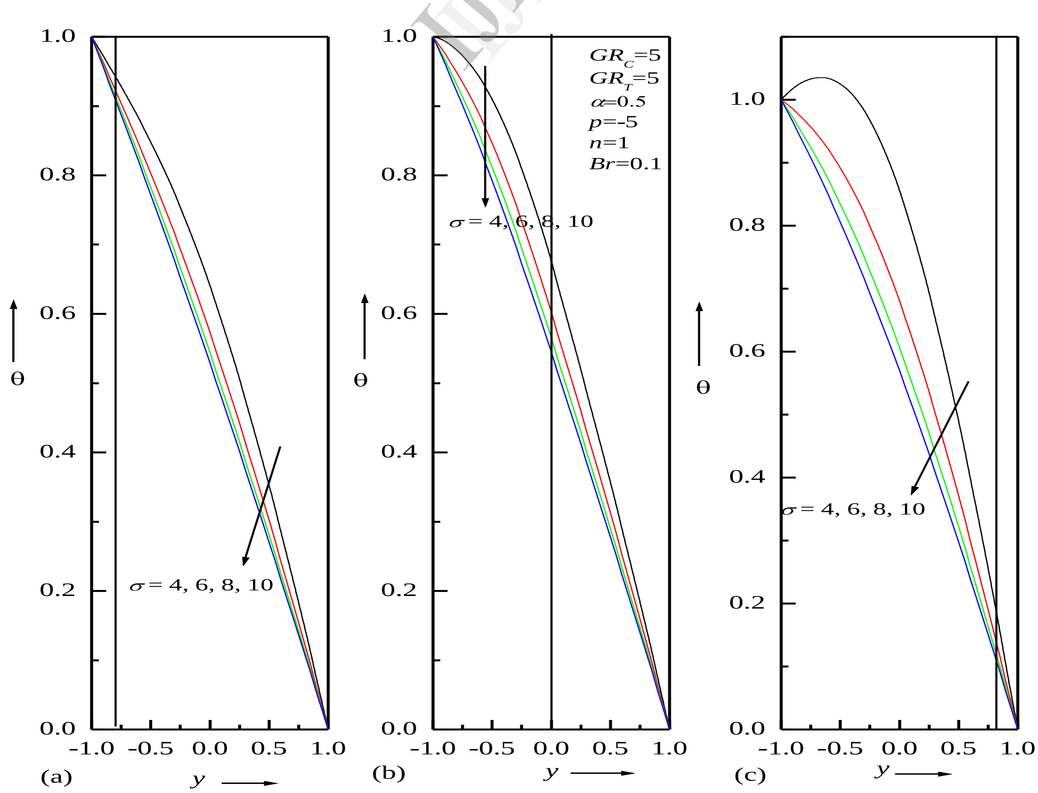

Fig. 3. Temperature profile for differnt values of porous parameter σ
(a) $y^*=-0.8$ (b) $y^*=0$ (c) $y^*=0.8$

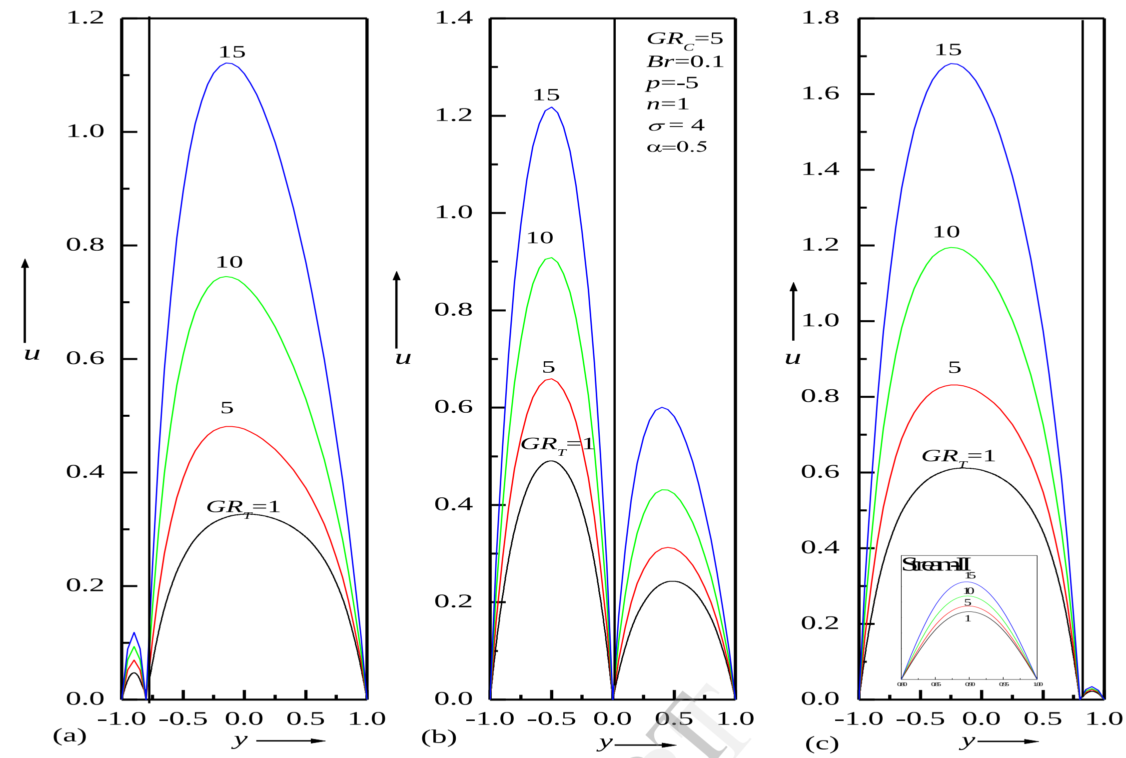

Fig. 4. velocity distribution for different values of thermal Grashoff number GR_T at (a) y*=-0.8 (b) y*=0.0 (c) y*=0.8

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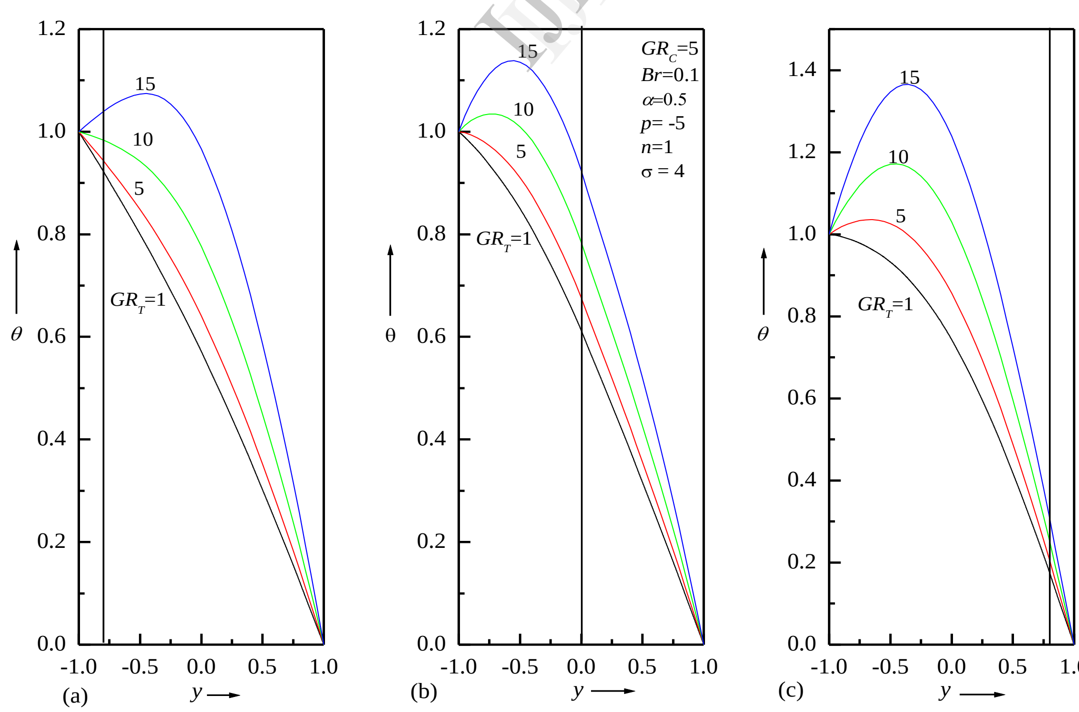

Fig. 5. Temperature profile for different values of thermal Grashof number GR_T at (a) y*=-0.8 (b) y*=0 (c) y*=0.8

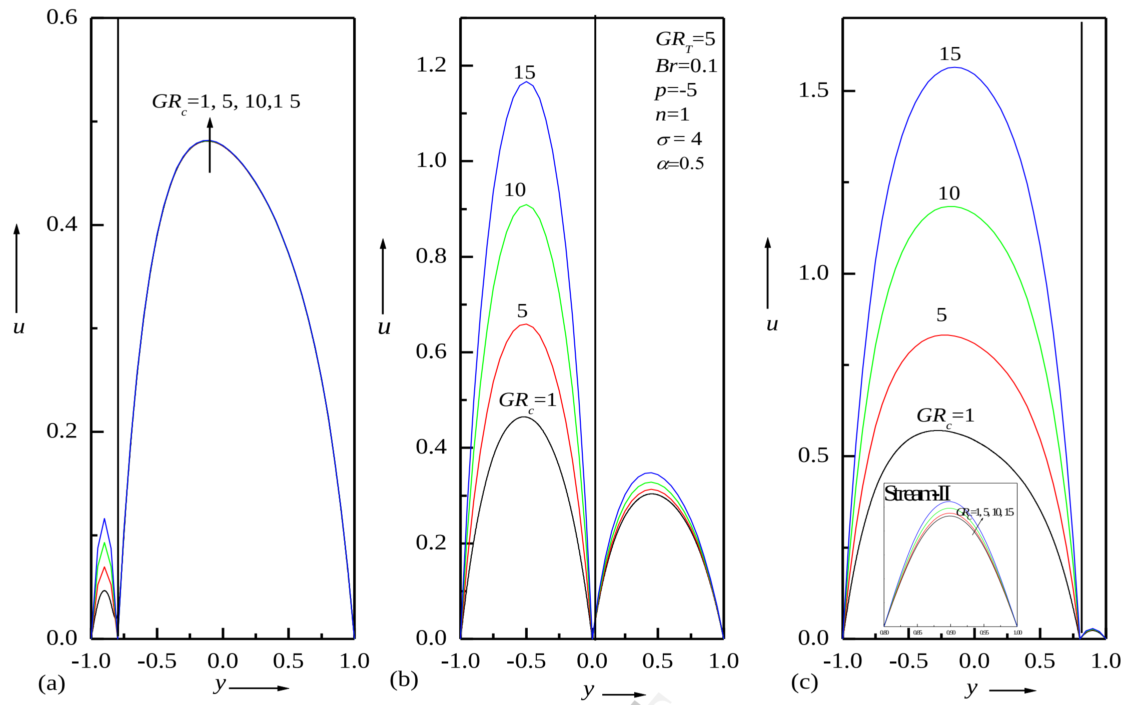

Fig. 6. Velocity profile for different values of concentration Grashof number GR_C at (a) y*=-0.8 (b) y*=0 (c) y*=0.8

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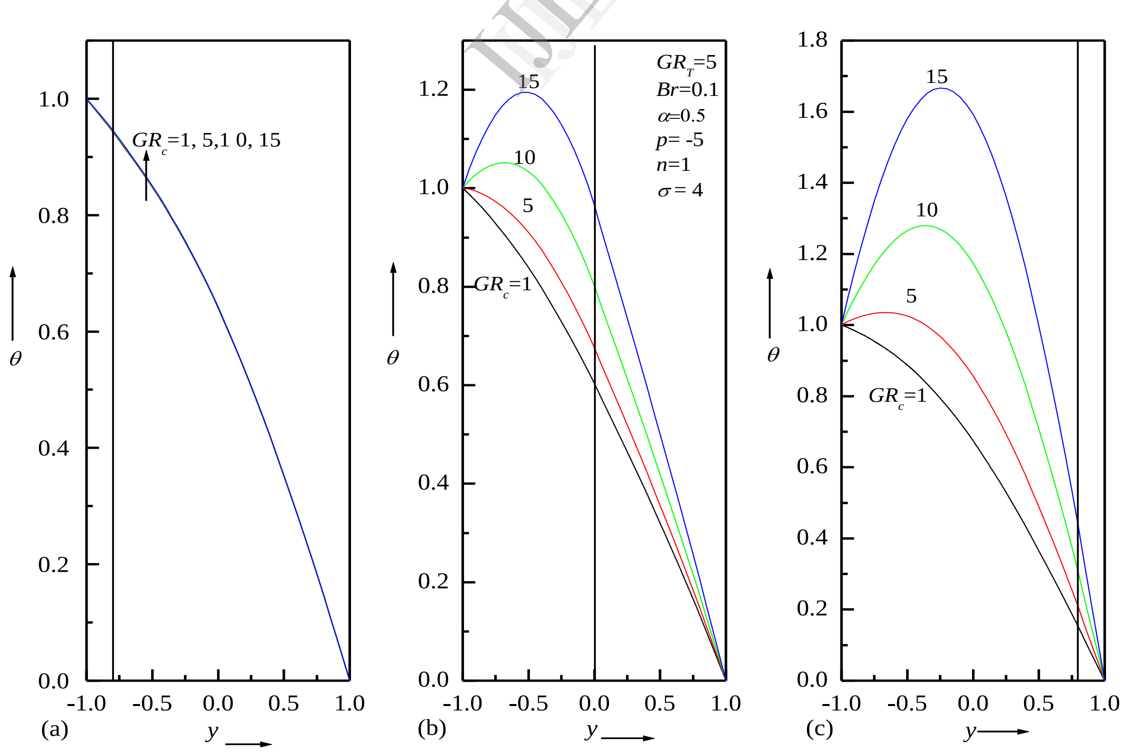

Fig. 7. Temperature profile for different values of concentration of Grashoff number GR_C at (a) y*=-0.8 (b) y*=0 (c) y*=0.8

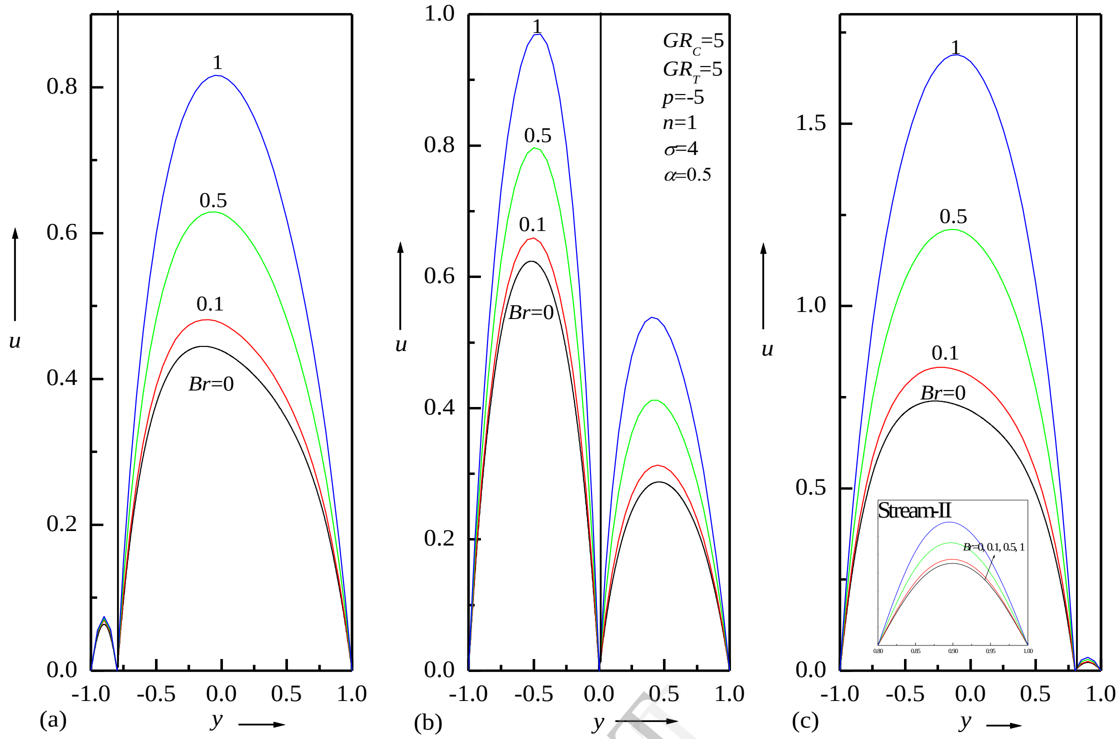

Fig. 8. Velocity for different values of Brinkman number Br
(a) y^*=-0.8 (b) y^*=0 (c) y^*=0.8

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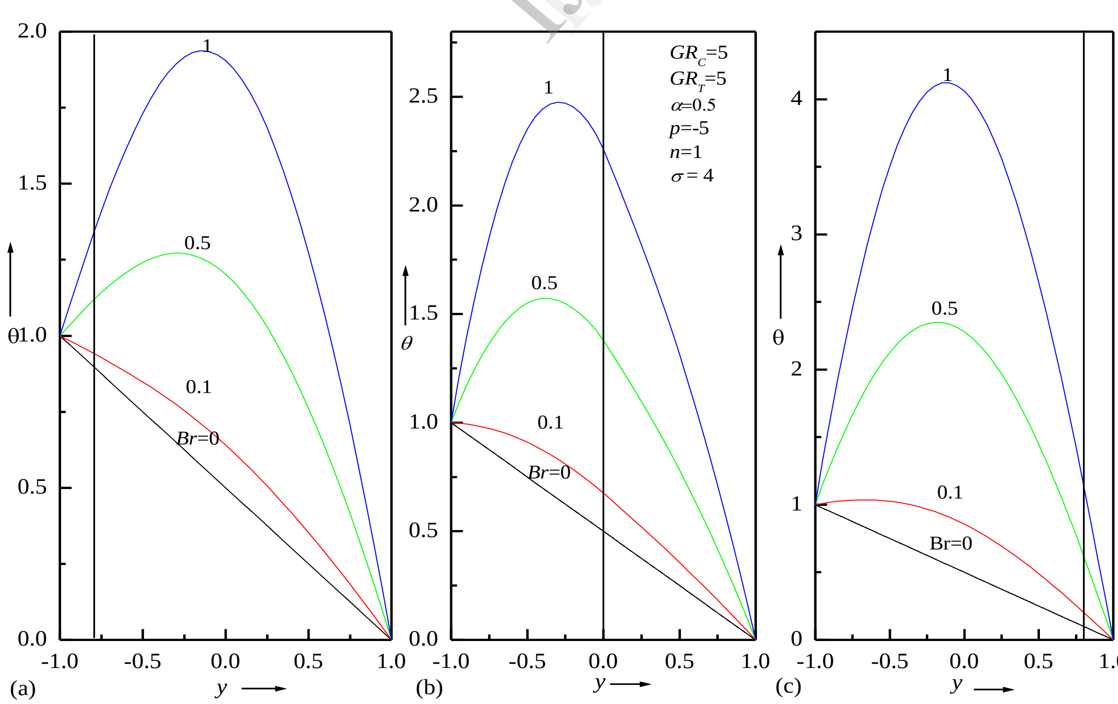

Fig. 9. Temperature profile for differeent values of Brinkman number Br
(a) y^*=-0.8 (b) y^*=0 (c) y^*=0.8

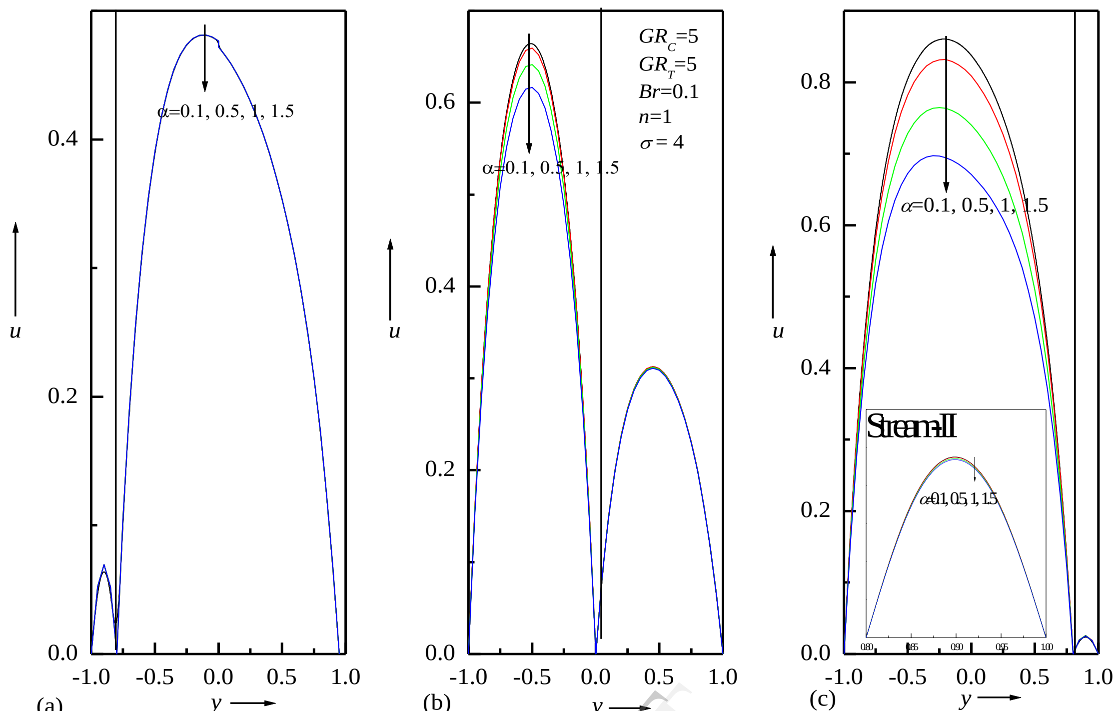

Fig. 10. Velocity profile for different values of chemical reaction parameter α
(a) y^*=-0.8 (b) y^*=0 (c) y^*=0.8

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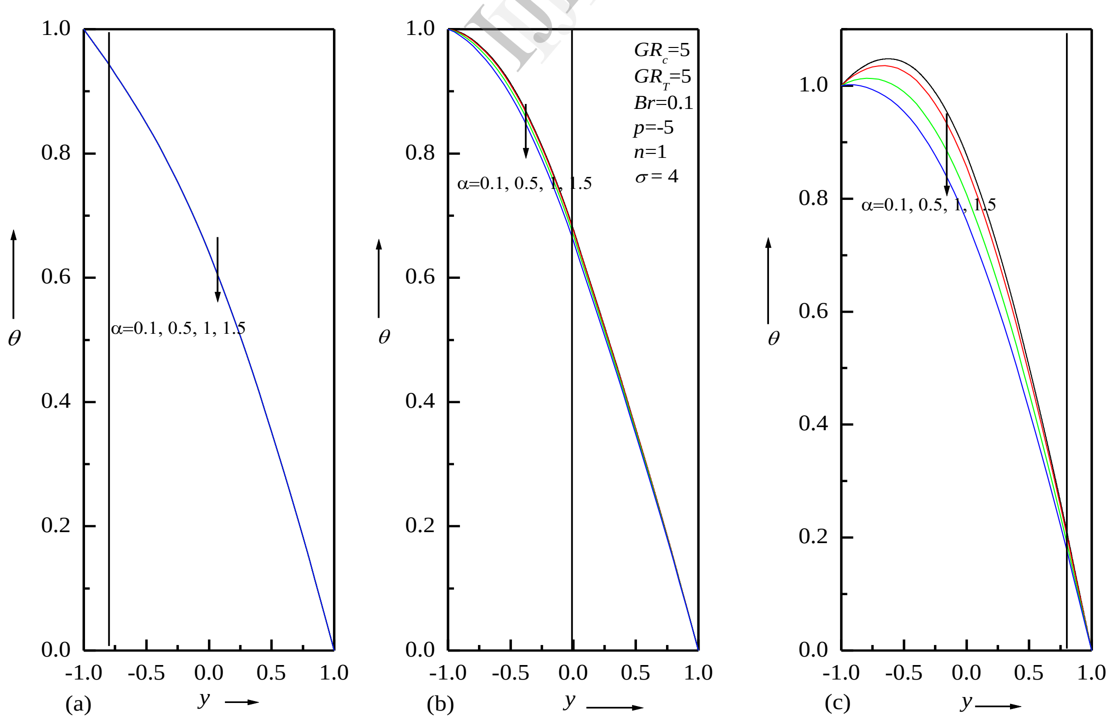

Fig. 11. Temperature profile for different values of chemical reaction parameter α.
(a) y^*=-0.8 (b) y^*=0 (c) y^*=0.8

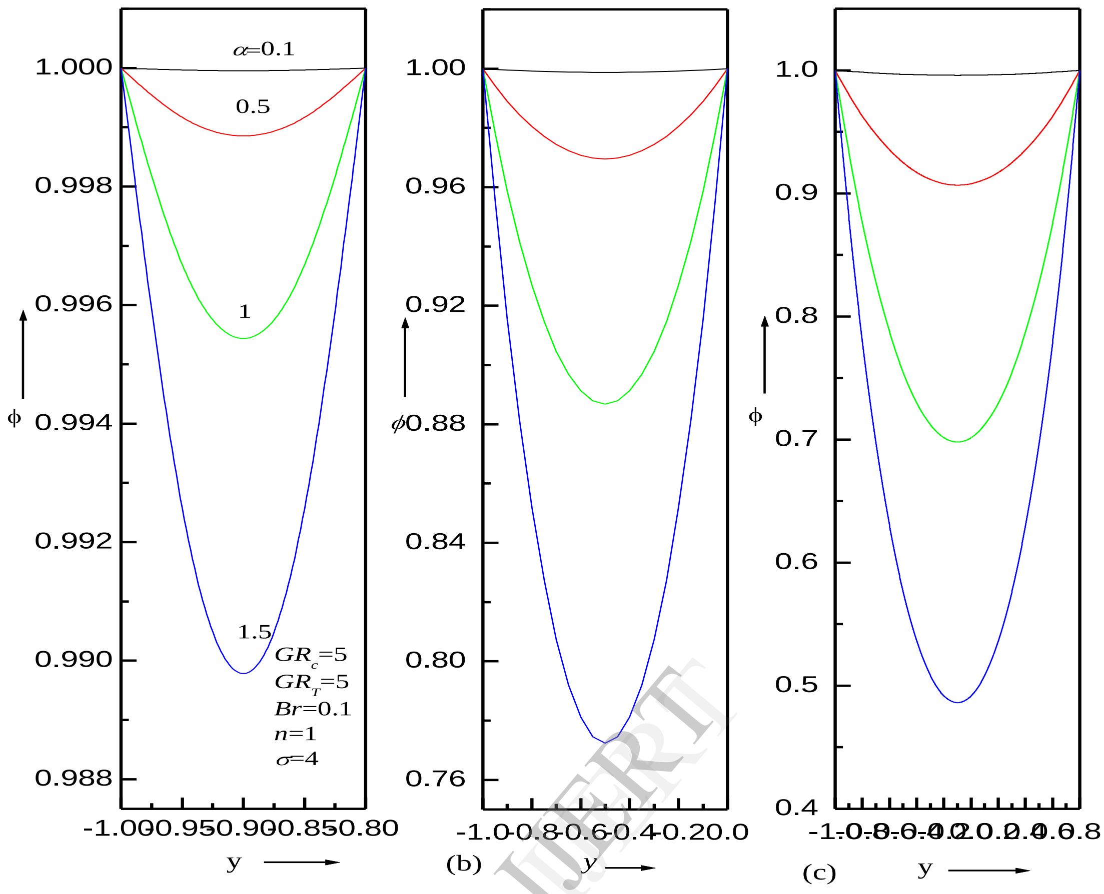

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Fig.12: Concentration profile for different values of chemical reaction parameter α

Table 1. The operations for the one-dimensional differential transform method.

Original function	Transformed function
$y(x)=g(x)\pm h(x)$	$Y(k)=G(k)\pm H(k)$
$y(x)=\alpha\, g(x)$	$Y(k)=\alpha\, G(k)$
$y(x)=\dfrac{dg(x)}{dx}$	$Y(k)=(k+1)G(k+1)$
$y(x)=\dfrac{d^2g(x)}{dx^2}$	$Y(k)=(k+1)(k+2)G(k+2)$
$y(x)=g(x)h(x)$	$Y(k)=\sum_{l=0}^{k}G(l)H(k-l)$
$y(x)=a^m$	$Y(k)=\delta(k-m)=\begin{cases}1, \text{ if } k=m\\ 0, \text{ if } k\neq m\end{cases}$

Table 2a. Comparison of velocity and temperature with $Br = 0$, $GR_T = 5$, $GR_C = 5$, $p = -5$, $\sigma = 4$, $\alpha = 1$ **and** $y^* = 0.0$.

	Velocity			Temperature		
y	DTM	PM	Error	DTM	PM	Error
-1	0	0	0.00E+00	1.000000	1.000000	0.00E+00
-0.75	0.504494	0.504494	0.00E+00	0.875000	0.875000	0.00E+00
-0.5	0.608520	0.608520	0.00E+00	0.750000	0.750000	0.00E+00
-0.25	0.476998	0.476998	0.00E+00	0.625000	0.625000	0.00E+00
0	0	0	0.00E+00	0.500000	0.500000	0.00E+00
0.25	0.244156	0.244156	0.00E+00	0.375000	0.375000	0.00E+00
0.5	0.286796	0.286796	0.00E+00	0.250000	0.250000	0.00E+00
0.75	0.216661	0.216661	0.00E+00	0.125000	0.125000	0.00E+00
1	0	0	0.00E+00	0	0	0.00E+00

Table 2b. Comparison of velocity and temperature with $Br = 0.05$, $GR_T = 5$, $GR_C = 5$, $p = -5$, $\sigma = 4$, $\alpha = 1$ **and** $y^* = 0.0$.

	Velocity			Temperature		
y	DTM	PM	Error	DTM	PM	Error
-1	0	0	0.00E+00	1.000000	1.000000	0.00E+00
-0.75	0.513963	0.514820	8.57E-04	0.914902	0.921771	6.87E-03
-0.5	0.624144	0.624950	8.06E-04	0.822928	0.826497	3.57E-03
-0.25	0.491508	0.491905	3.97E-04	0.712174	0.713337	1.16E-03
0	0	0	0.00E+00	0.584682	0.584372	3.10E-04
0.25	0.255263	0.255491	2.28E-04	0.443964	0.444442	4.78E-04
0.5	0.297774	0.298287	5.13E-04	0.298960	0.301408	2.45E-03
0.75	0.222933	0.223473	5.40E-04	0.149665	0.153519	3.85E-03
1	0	0	0.00E+00	0	0	0.00E+00

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Table 2c. Comparison of velocity and temperature with $Br = 0.15$, $GR_T = 5$, $GR_C = 5$, $p = -5$, $\sigma = 4$, $\alpha = 1$ **and** $y^* = 0.0$.

	Velocity			Temperature		
y	DTM	PM	Error	DTM	PM	Error
-1	0	0	0.00E+00	1.000000	1.000000	0.00E+00
-0.75	0.569920	0.535472	3.44E-02	1.193360	1.015312	1.78E-01
-0.5	0.701813	0.657811	4.40E-02	1.174845	0.979491	1.95E-01
-0.25	0.557242	0.521720	3.55E-02	1.084629	0.890012	1.95E-01
0	0	0	0.00E+00	0.930151	0.753116	1.77E-01
0.25	0.300572	0.278160	2.24E-02	0.723470	0.583326	1.40E-01
0.5	0.343074	0.321269	2.18E-02	0.499156	0.404223	9.49E-02
0.75	0.249827	0.237098	1.27E-02	0.257645	0.210558	4.71E-02
1	0	0	0.00E+00	0	0	0.00E+00

Table 3a. Comparison of velocity and temperature with $Br = 0$, $GR_T = 5$, $GR_C = 5$, $p = -5$, $\sigma = 2$, $\alpha = 0.5$ and $y^* = -0.8$.

	Velocity			Temperature		
y	DTM	PM	Error	DTM	PM	Error
-1	0	0	0.00E+00	1.000000	1.000000	0.00E+00
-0.95	0.054588	0.054588	0.00E+00	0.975000	0.975000	0.00E+00
-0.9	0.072515	0.072515	0.00E+00	0.950000	0.950000	0.00E+00
-0.85	0.054277	0.054277	0.00E+00	0.925000	0.925000	0.00E+00
-0.8	0	0	0.00E+00	0.900000	0.900000	0.00E+00
-0.5	0.842810	0.842810	0.00E+00	0.750000	0.750000	0.00E+00
-0.2	1.186833	1.186833	0.00E+00	0.600000	0.600000	0.00E+00
0.1	1.229229	1.229229	0.00E+00	0.450000	0.450000	0.00E+00
0.4	1.055271	1.055271	0.00E+00	0.300000	0.300000	0.00E+00
0.7	0.669983	0.669983	0.00E+00	0.150000	0.150000	0.00E+00
1	0	0	0.00E+00	0	0	0.00E+00

Table 3b. Comparison of velocity and temperature with $Br = 0.05$, $GR_T = 5$, $GR_C = 5$, $p = -5$, $\sigma = 2$, $\alpha = 0.5$ and $y^* = -0.8$.

	Velocity			Temperature		
y	DTM	PM	Error	DTM	PM	Error
-1	0	0	0.00E+00	1.000000	1.000000	0.00E+00
-0.95	0.055161	0.054975	1.86E-04	0.992912	0.987655	5.26E-03
-0.9	0.073411	0.073135	2.76E-04	0.985315	0.975231	1.01E-02
-0.85	0.055052	0.054821	2.31E-04	0.977545	0.962795	1.48E-02
-0.8	0	0	0.00E+00	0.969639	0.950280	1.94E-02
-0.5	0.922607	0.907152	1.55E-02	0.892311	0.858004	3.43E-02
-0.2	1.311384	1.290078	2.13E-02	0.777416	0.743716	3.37E-02
0.1	1.363721	1.343996	1.97E-02	0.629571	0.604245	2.53E-02
0.4	1.167606	1.153767	1.38E-02	0.448420	0.434648	1.38E-02
0.7	0.734128	0.727479	6.65E-03	0.237882	0.234267	3.62E-03
1	0	0	0.00E+00	0	0	0.00E+00

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Table 3c. Comparison of velocity and temperature with $Br = 0.1$, $GR_T = 5$, $GR_C = 5$, $p = -5$, $\sigma = 2$, $\alpha = 0.5$ and $y^* = -0.8$.

	Velocity			Temperature		
y	DTM	PM	Error	DTM	PM	Error
-1	0	0	0.00E+00	1.000000	1.000000	0.00E+00
-0.95	0.055729	0.055363	3.66E-04	1.038332	1.000310	3.80E-02
-0.9	0.074695	0.073755	9.40E-04	1.065313	1.000463	6.48E-02
-0.85	0.056332	0.055364	9.68E-04	1.088246	1.000589	8.77E-02
-0.8	0	0	0.00E+00	1.109651	1.000560	1.09E-01
-0.5	1.069778	0.971494	9.83E-02	1.157592	0.966009	1.92E-01
-0.2	1.539664	1.393323	1.46E-01	1.102756	0.887432	2.15E-01
0.1	1.609157	1.458763	1.50E-01	0.956406	0.758491	1.98E-01

0.4	1.371972	1.252264	1.20E-01	0.717323	0.569296	1.48E-01
0.7	0.850546	0.784975	6.56E-02	0.396220	0.318533	7.77E-02
1	0	0	0.00E+00	0	0	0.00E+00

Table 4a. Comparison of velocity and temperature with $Br = 0$, $GR_T = 5$, $GR_C = 5$, $p = -5$, $\sigma = 2$, $\alpha = 0.5$ **and** $y^* = 0.8$.

	Velocity			Temperature		
y	DTM	PM	Error	DTM	PM	Error
-1	0	0	0.00E+00	1.000000	1.000000	0.00E+00
-0.7	1.381736	1.381736	0.00E+00	0.850000	0.850000	0.00E+00
-0.4	1.977978	1.977978	0.00E+00	0.700000	0.700000	0.00E+00
-0.1	2.093787	2.093787	0.00E+00	0.550000	0.550000	0.00E+00
0.2	1.846416	1.846416	0.00E+00	0.400000	0.400000	0.00E+00
0.5	1.208910	1.208910	0.00E+00	0.250000	0.250000	0.00E+00
0.8	0	0	0.00E+00	0.100000	0.100000	0.00E+00
0.85	0.019536	0.019536	0.00E+00	0.075000	0.075000	0.00E+00
0.9	0.025820	0.025820	0.00E+00	0.050000	0.050000	0.00E+00
0.95	0.019225	0.019225	0.00E+00	0.025000	0.025000	0.00E+00
1	0	0	0.00E+00	0	0	0.00E+00

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Table 4b. Comparison of velocity and temperature with $Br = 0.01$, $GR_T = 5$, $GR_C = 5$, $p = -5$, $\sigma = 2$, $\alpha = 0.5$ **and** $y^* = 0.8$.

	Velocity			Temperature		
y	DTM	PM	Error	DTM	PM	Error
-1	0	0	0.00E+00	1.000000	1.000000	0.00E+00
-0.7	1.413848	1.412013	1.84E-03	0.896275	0.893685	2.59E-03
-0.4	2.033294	2.030110	3.18E-03	0.774717	0.770439	4.28E-03
-0.1	2.158789	2.155026	3.76E-03	0.637104	0.632050	5.05E-03
0.2	1.905465	1.902031	3.43E-03	0.482902	0.478051	4.85E-03
0.5	1.246039	1.243875	2.16E-03	0.313674	0.309943	3.73E-03
0.8	0	0	0.00E+00	0.129530	0.127829	1.70E-03
0.85	0.019855	0.019837	1.80E-05	0.097150	0.095884	1.27E-03
0.9	0.026183	0.026162	2.10E-05	0.064767	0.063936	8.31E-04
0.95	0.019452	0.019439	1.30E-05	0.032385	0.031978	4.07E-04
1	0	0	0.00E+00	0	0	0.00E+00

Table 4c. Comparison of velocity and temperature with $Br = 0.08$, $GR_T = 5$, $GR_C = 5$, $p = -5$, $\sigma = 2$, $\alpha = 0.5$ **and** $y^* = 0.8$.

	Velocity			Temperature		
y	DTM	PM	Error	DTM	PM	Error
-1	0	0	0.00E+00	1.000000	1.000000	0.00E+00
-0.7	1.967939	1.623945	3.44E-01	1.681644	1.199480	4.82E-01
-0.4	2.993084	2.395032	5.98E-01	2.064952	1.263514	8.01E-01
-0.1	3.292486	2.583697	7.09E-01	2.157408	1.206403	9.51E-01
0.2	2.939830	2.291334	6.48E-01	1.941974	1.024408	9.18E-01
0.5	1.898517	1.488634	4.10E-01	1.440481	0.729547	7.11E-01
0.8	0	0	0.00E+00	0.654813	0.322629	3.32E-01
0.85	0.025656	0.021940	3.72E-03	0.493150	0.242075	2.51E-01
0.9	0.032869	0.028560	4.31E-03	0.330969	0.161490	1.69E-01
0.95	0.023701	0.020936	2.77E-03	0.167381	0.080825	8.66E-02
1	0	0	0.00E+00	0	0	0.00E+00

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NOMENCLATURE

Br : Brinkman number $\left(\dfrac{\overline{u_1^2}\mu_1}{K\Delta T}\right)$

C_p : specific heat at constant pressure

C_1 : the concentration in stream-I

C_0 : reference concentration

D : diffusion coefficients

g : acceleration due to gravity

Gr : Grashoff number $\left(\dfrac{g\,\beta_T\,\Delta T\,h^3}{\gamma^2}\right)$

Gc : modified Grashoff Number $\left(\dfrac{g\,\beta c\,\Delta C\,h^3}{\gamma^2}\right)$

GR_T : thermal Grashoff number

GR_C : mass Grashoff number

h : channel width

h^* : width of passage

K : thermal conductivity of fluid

p : non-dimensional Pressure Gradient $(\frac{h^2}{\overline{U_1}\mu}\frac{\partial p}{\partial X})$

Re : Reynolds number $(\frac{\overline{U_1}h}{\gamma})$

$\overline{U_1}$: reference velocity

T_1, T_2 : dimensional temperature distributions

T_{w_1}, T_{w_2} : temperatures of the boundaries

U_1, U_2 : dimensional velocity distributions

u_1, u_2 : non dimensional velocities in stream-I, stream-II

y^* : baffle position

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Greek Symbols

α : chemical reaction parameters

β_T : coefficients of thermal expansion

β_C : coefficients of concentration expansion

$\Delta T, \Delta C$: difference in temperatures and concentration

σ : porous parameter $\left(\frac{h}{\sqrt{\kappa}}\right)$

κ : permeability of the porous medium

θ_i : non-dimensional temperature $\left(\frac{T_i - T_{W_2}}{T_{W_1} - T_{W_2}}\right)$

γ : kinematics viscosity

ϕ_1 : non-dimensional concentrations

ρ : density

μ : viscosity

Subscripts

i : refer 1 and 2 quantities for the fluids in stream-I and stream-II, respectively.

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