Head and Efficiency Prediction Methods for Centrifugal Pumps

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Abstract—An effective prediction method of centrifugal pump characteristic lines – head and efficiency - suited for engineering applications is presented. Various formulas are successfully used to determine both head and efficiency as functions of volume flow. The head distribution in particular is calculated by introducing correlations and a shut-off coefficient. Validation is accomplished by comparing the method's numerical outcome with existing experimental data, for different centrifugal pumps having from one up to seven blades. The results obtained for both pump head and efficiency distributions are more than satisfactory.

Keywords—Centrifugal pump; head, efficincy; characteristic curve; numerical prediction; correlation

I. INTRODUCTION

Flow fields inside centrifugal pumps are remarkably complex, incorporating curved impeller geometry, flow separation phenomena with recirculation regions, secondary and swirling flows. Pumps should have high efficiency to minimize power consumption to enable high performance and minimum net positive suction head (NPSH).

Modern design practices require a thorough investigation of the internal flow at design point and off-design operating conditions. To reduce the design cost of pumps it is crucial to predict their efficiency prior to manufacturing and placement in installation. Today's computer capabilities as well as the progress of numerical methods' accuracy make possible the use of Computational Fluid Dynamics (CFD) in the research and development process of hydraulic machinery. The advantages of using CFD are considerable reduction of time needed for the development of pumps, as well as, the possibility of bypassing time and cost-intensive experimental investigations according to [1].

Alternatively, empirical methods can be used as basic research tools for the comprehension of fundamental fluid phenomena dominating the flow field, but they cannot provide detailed results on the flow pattern inside the pump. These methods, originating from model testing and engineering experience, are rapid and accurate tools for the pump overall efficiency prediction prior to the detailed 3D flow analysis and laboratory testing, [2]. Panoutsopoulou Angeliki Hellenic Defense Systems S.A. 1, Ilioupoleos Avenue, Hymettus, Greece

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II. LITERATURE SURVEY

An overview of methods for calculating the pump head based on one-dimensional design is presented in [3]. An accurate method for centrifugal pump efficiency prediction is proposed in [4]. It is based on the Euler and energy equation, including various types of losses. The predicted hydraulic pump efficiency was found consistent with experimental data in [5]. For the single test case examined, the predicted efficiency was overestimated at low mass flow rates. This indicates the difficulty of such methods to provide an accurate prediction of pump efficiency over the whole range of volume flows.

The losses associated to the pump impeller were effectively modeled in [6]. Based on this model, the blade angle influence on pump head and efficiency was examined in [7]. Calculations showed that when the blade exit angle is increasing, the efficiency is also increasing, although, in some cases, depending on the value of the specific speed, when the blade exit angle is decreasing, the pump efficiency is still increasing.

An optimization method for the impeller outlet using a polynomial formula for the pump head and an exponential formula for the pump efficiency, based on correlations derived from experimental data was introduced in [8].

A generalized approximate formula of pump efficiency by means of an exponential function was proposed in [9]. This function represents the pump efficiency in terms of the impeller rotational speed and the pump volume flow.

A method to design centrifugal pump impellers employing empirical correlations by means of polynomial functions of specific speed and geometrical parameters was developed in [10]. Contributions for centrifugal pump characteristic curves predictions based on exact solutions of the Navier-Stokes equations aiming to be used as engineering tools for fast assessment of centrifugal pumps were introduced in [11], [12], [13].

A remarkable review paper [14] summarizes CFD efforts on centrifugal pump flow field calculations. Calculations were performed in [15], [16], [17], [18], [19], [20], [21], [22], [23] using the FLUENT package. The code uses the finite volume method and solves the fully 3D incompressible Navier-Stokes equations. The pressure-velocity coupling is performed using the SIMPLE algorithm.

The fully 3D incompressible Navier-Stokes equations are performed with the ANSYS CFX package in [24], [25], [26]. Turbulence is simulated with the shear stress transport (SST) k- ω turbulence model.

Experimental data and numerical simulation of the threedimensional unsteady pump flow taking into account the impeller-volute interaction with different outlet blade angles was introduced in [27]. Results obtained using impellers with different outlet angles showed that when the blade outlet angle increases, the centrifugal pump efficiency improves.

Analysis of the flow field inside a centrifugal pump, solving the 3D Unsteady Reynolds Averaged Navier Stokes (URANS) equations using three different turbulence models was performed in [28] and in [29]. Results revealed the unsteady flow pattern at higher and lower volume flows.

A 1D method for the design of pump impeller and volute was presented in [30]. Having designed the pump, the flow field was calculated using a 3D CFD package. A fair agreement with experimental data was observed. The interaction of impeller and volute was also studied in [31] and in [32].

All CFD methods require the construction of the computational grid based on the detailed geometry of the impeller and volute, therefore, due to this laborious effort, their use is not cost and time efficient. Moreover, at off-design conditions, a realistic prediction should take into account the interaction between impeller and volute, requiring extra computer resources.

At the same time up to now, the empirical methods referred above were either applied to a limited number of similar centrifugal pump impellers, or were used to study the effect of a specific parameter of the pump in question.

The present paper, on the contrary, proposes a fast engineering method applied to a variety of pumps with impellers having one up to seven blades, covering a wide range of industrial applications that a centrifugal pump may have. This method is based on previous correlations, as well as experimental data obtained from in-house testing of centrifugal pumps. For all these different cases, predictions of head and efficiency are compared to existing experimental data found in the literature. It can be seen that the present method can be successfully applied to predict performances of centrifugal pumps.

III. PERFORMANCE PREDICTION METHOD

A. Correlations for Pump Manometric Head

Experience gained from numerous centrifugal pump's laboratory testing with different geometrical characteristics, fig. 1, provided the opportunity to derive correlations for head and efficiency, applicable up to 3000 rpm rotational speed.

The present method's originality relies on the fact that the pump's head prediction is based on the estimation of the shutoff manometric head corresponding to the closed position of the pump's discharge. This is accomplished by defining the so-called shut-off head coefficient, ξ

$$H_{shut off} = \xi \left(n D_2 \right)^2 \tag{1}$$



Fig. 1. Test facility for centrifugal pump testing

where n is the rotational speed of the impeller in rps, D_2 is the outlet impeller diameter. The shut-off coefficient is defined as:

$$\xi = 0.9 \frac{\sigma}{e^{\left(\frac{D_1}{D_2}\right)}} \left(\frac{z}{3}\right)^{0.1}$$
(2)

where z is the number of the impeller blades and D_1 is the inlet pump impeller diameter. The slip factor σ used here, resembles to the empirical definition given in [33] and was modified in [10]. It is valid for a wide range of blade angles and number of blades:

$$\sigma = 1 - \frac{\sin \beta_2}{z^{0.7}} \tag{3}$$

where β_2 is the exit blade angle.

Other correlations for the slip factor, such as the one proposed in [34], or in [35] can be obtained as well.

The maximum pump volume flow corresponding to the fully opened discharge, is evaluated using the proposed formula:

$$\dot{V}_{\text{max}} = \pi A_2 c_{2m} \left(\frac{b_2}{k}\right) \left(\frac{z}{5.5}\right)^{1/2}$$
 (4)

where $A_2 = D_2 b_2$

The empirical constant k appearing in the denominator of the formula depends on the thickness and height of the impeller

blades. Thus the factor $\frac{b_2}{k}$ is somehow the non-dimensional

blade height at the impeller trailing edge.

The constant k, was found to depend on the number of impeller blades. From experimental studies in the lab, it is proposed to be as follows:

$$k = \begin{cases} 1.4 & 3 < z < 7 \\ 1.5 & z \le 3 \end{cases}$$
(5)

The influence of the blade number is taken into account also in the ratio $\frac{z}{z}$ that is in a way the non-dimensional number

in the ratio $\frac{z}{5.5}$ that is in a way the non-dimensional number of impeller blades.

The meridional velocity is approximated in a similar way to [36] as:

$$c_{2m} = 0.98 c_0 \tag{6}$$

where c_0 is the suction velocity, defined in the present study as:

$$c_0 = 0.2 (g H_{shutoff})^{1/2}$$
 (7)

The best efficiency point (BEP) of the pump is defined by approximating the optimum volume flow corresponding to the maximum pump efficiency at the optimum manometric head. For the prediction of the head and volume flow at BEP conditions are determined using the shut-off head.

$$\dot{V}_{opt} = 1.7 \left(\frac{1 - H_{opt}}{H_{shut off}}\right)^{1/2} \dot{V}_{max}$$
(8)

where \dot{V}_{opt} , \dot{V}_{max} , are expressed in m^3 / h and

$$H_{opt} = 0.85 \,\mathrm{H}_{shut\,off} \tag{9}$$

In the present article several original functions are proposed to predict the pump head-volume flow characteristic curve:

1st function, named parabola

$$H_{1} = H_{shut off} \left[1 - 0.3 \left(\frac{\dot{V}}{\dot{V}_{opt}} \right)^{2} \right]$$
(10)

2nd function, named bernstein

$$H_{2} = \frac{\mathrm{H}_{shut off}}{1 + 0.16 \left(\frac{\dot{V}}{\dot{V}_{opt}}\right)^{2}} \tag{11}$$

3rd function, named cosinus1

$$H_{3} = H_{shut off} \cos\left(\frac{\pi}{4} \cdot \frac{\dot{V}}{\dot{V}_{opt}}\right)$$
(12)

4^{rth} function, named machine

$$H_4 = \frac{1}{z} \left(\frac{\dot{V}}{b_2 D_2}\right)^2 + H_{shutoff}$$
(13)

5th function, named cosinus2

$$H_5 = \mathbf{H}_{shut off} \left[\cos \left(\frac{\dot{V}}{\dot{V}_{opt}} \right) \right]^{0.6}$$
(14)

6th function, named tangent hyperbolic

$$H_{6} = H_{shut off} \left(1 - 2 \left[\tanh\left(\frac{\dot{V}}{2\dot{V}_{max}}\right) \right]^{2.4} \right)$$
(15)

7th function, named cosinus hyperbolic

$$H_{7} = \mathrm{H}_{shut off} \left(2 - \left[\cosh\left(\frac{\dot{V}}{2\dot{V}_{\mathrm{max}}}\right) \right]^{2.2} \right) (17)$$

8th function, named Bessel

$$\boldsymbol{H}_{7} = \boldsymbol{\mathrm{H}}_{shut \ off} \begin{bmatrix} \boldsymbol{J}_{0, \left(\dot{\boldsymbol{V}}_{\dot{\boldsymbol{V}}_{\mathrm{max}}} \right)} \end{bmatrix}^{1.2} \tag{18}$$

where J_0 is the Bessel function on zero order.

B. Correlations for Pump Efficiency

The pump specific speed is defined as [6]:

$$n_q = \frac{nV_{opt}^{1/2}}{H_{opt}^{0.75}}$$
(19)

where *n* is expressed in *rpm* and \dot{V}_{opt} in m^3/s .

The hydraulic pump efficiency of the impeller is defined as:

The maximum efficiency is defined as in analogy to [37]:

$$n_{\max} = 0.01 \left[-32 + 145 \log n_q - 41 \left(n_q \right)^2 \right] \quad (20)$$

Empirical correlations expressing the pump efficiency using exponential functions were also introduced in the past by [9] who used the ratio of the volume flow divided by the volume flow at design point. Other researchers, such as [38] used polynomial functions in terms of the impeller specific speed. Two original functions are proposed in this article to predict the pump efficiency in terms of the volume flow:

1st function, named polynomial

$$n_{1} = 3.85n_{\max} \left[1.4 - 2.1 \left(\frac{\dot{V}}{\dot{V}_{\max}} \right) \right] \left(\frac{\dot{V}}{\dot{V}_{\max}} \right) \left(\frac{D_{2}}{355} \right)^{0.115}$$

$$4 < z < 7$$

$$n_{1} = 3.35n_{\max} \left[1.4 - 2.1 \left(\frac{\dot{V}}{\dot{V}_{\max}} \right) \right] \left(\frac{\dot{V}}{\dot{V}_{\max}} \right) \left(\frac{D_{2}}{355} \right)^{0.115}$$

$$2 \le z \le 4$$

(21b)

$$n_{1} = 3.2n_{\max} \left[1.4 - 2.1 \left(\frac{\dot{V}}{\dot{V}_{\max}} \right) \right] \left(\frac{\dot{V}}{\dot{V}_{\max}} \right) \left(\frac{D_{2}}{355} \right)^{0.115} z = 1$$
(21c)

For centrifugal pump impellers with 4 or more thin blades (21a) is applied with the multiplier 3.85.

For centrifugal pump impellers from 2 to 4 thick blades (21b) is applied with the multiplier 3.35.

For centrifugal pump impellers with 1 blade (21c) is applied with the multiplier 3.2.

2nd function, named sinus

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$$n_{2} = 0.9 n_{\max} \sin\left(\frac{\frac{\pi}{2}\dot{V}}{0.98\dot{V}_{opt}}\right) \left(\frac{D_{2}}{355}\right)^{0.115} 4 < z < 7$$
(22a)

$$n_2 = 0.75 n_{\max} \sin\left(\frac{\frac{\pi}{2}\dot{V}}{0.98\dot{V}_{opt}}\right) \left(\frac{D_2}{355}\right)^{0.115} 2 \le z \le 4$$

$$n_2 = 0.7 n_{\text{max}} \sin\left(\frac{\frac{\pi}{2}\dot{V}}{0.98\dot{V}_{opt}}\right) \left(\frac{D_2}{355}\right)^{0.115} z = 1$$

For centrifugal pump impellers with 4 or more thin blades (22a) is applied with the multiplier 0.9.

For centrifugal pump impellers from 2 to 4 thick blades (22b) is applied with the multiplier 0.75.

For centrifugal pump impellers with 1 blade (22c) is applied with the multiplier 0.7.

IV. RESULTS

In order to evaluate the accuracy of the prediction formulas, a series of centrifugal pumps of different geometrical and operational characteristics, having well-established experimental data in the literature are analyzed. In particular, for the head prediction, results are given only using the functions (10), (11), (12). The results using the rest of the functions are not presented, although verified, due to the fact that the output they produce is almost the same and their illustration cannot be clearly distinguished on the graph.

A. Single-bladed centrifugal pump

A method for the design of a single-bladed centrifugal sewage pump impeller was developed in [23]. Experimental data was used to validate the numerical data obtained. The results of the present method are compared to the available experimental data in [23].

Figure 2 shows the comparison between predicted head using (10), (11), (12) and experimental data. As it can be observed, there is an exceptional good agreement between experimental data and numerical predictions for the whole range of pump operation.



Fig. 2. Comparison between predicted head by the present method and experimental data by [23]

(22b)

(22c)

Figure 3 illustrates the comparison between predicted efficiency using (21), (22) and experimental data. The agreement between predicted efficiency using is very good for the whole range of volume flows.



Fig. 3. Comparison between predicted efficiency by the present method and experimental data by [23]

B. Two-bladed centrifugal pump

A 2-bladed centrifugal pump for groundwater applications was analyzed in [39]. Experimental data and numerical predictions based on one-dimensional empirical correlations performed in [39] demonstrate the head and efficiency variation in terms of the volume flow.

Comparison between head prediction using various methods proposed in the present article and experimental data are shown in fig. 4. It can be seen that numerical predictions almost coincide to experimental data for whole range of volume flows.



Figure 5 illustrates the comparison between predicted efficiency using (21), (22) and experimental data. The agreement is very good for the whole range of volume flows, capturing accurately the maximum and the minimum values of the pump efficiency.



Fig. 5. Comparison between predicted efficiency by the present method and experimental data in [39]

C. Three-bladed centrifugal pump

A 3-bladed slurry pump was studied experimentally and numerically in [22] by solving the 3D RANS using the FLUENT code.

Numerical predictions of the head using (10), (11), (12) and experimental data are shown in fig. 6. Once more, one can see that numerical predictions are almost identical to experimental data for the whole range of volume flows.



experimental data in [22]

It is verified that the predicted efficiency using (21), (22) in fig. 7 agrees fairly well to the available experimental data for the whole range of volume flows.



Fig. 7. Comparison between predicted efficiency by the present method and experimental data in [22]

D. Four-bladed centrifugal pump

A four-bladed centrifugal pump for groundwater applications was analyzed in [39]. Experimental data and numerical predictions based on one-dimensional empirical correlations performed in [39] show the head and efficiency variation in terms of the volume flow.

Comparisons between predicted head using (10), (11), (12) and experimental data are shown in fig. 8. Very good agreement between experimental data and numerical predictions can be observed for the whole range of pump operation.



Fig. 8. Comparison between predicted head by the present method and experimental data in [39]

Figure 9 shows the comparison between predicted efficiency using (21), (22) and experimental data. The agreement between predicted efficiency using is very good for the whole range of volume flows.



Fig. 9. Comparison between predicted efficiency by the present method and experimental data in [39]

E. Five-bladed centrifugal pump

A five-bladed centrifugal pump with 2D curvature blade geometry was used in [40].

The numerical results of the flow field prediction were obtained by the commercial code FINE/TURBOTM, a threedimensional structured mesh code that solves the time dependent Reynolds-Averaged Navier-Stokes equations.

Figure 10 shows the comparison between predicted head using (10), (11), (12) and experimental data. Very good agreement between experimental data and numerical predictions can be observed for the whole range of pump operation.



Fig. 10. Comparison between predicted head by the present method and experimental data in [40]

Figure 11 shows the comparison between predicted efficiency using (21), (22) and experimental data. The agreement between predicted efficiency using is very good for the whole range of volume flows.



Fig. 11. Comparison between predicted efficiency by the present method and experimental data in [40]

F. Six-bladed centrifugal pump

The flow filed of a commercial 6-bladed centrifugal impeller was analyzed in [16] and numerical predictions were compared with existing experimental data.

Figure 12 shows the comparison between predicted head using (10), (11), (12) and experimental data. Experimental data are very close to numerical predictions as it can be observed for the whole range of pump operation.



Figure 13 shows the comparison between predicted efficiency using (21), (22) and experimental data. The agreement between predicted efficiency using is very good for the whole range of volume flows



Fig. 13. Comparison between predicted efficiency by the present method and experimental data in [16]

G. Seven-bladed centrifugal pump

The performance of seven-bladed deep well centrifugal pumps with four different impeller outlet widths was investigated in [41]. The have also performed measurements to compare the theoretical predictions they made.

Figure 14 shows the comparison between predicted head using (10), (11), (12) and experimental data. Numerical predictions are in very good agreement to experimental data for the whole range of pump operation.

Figure 15 shows the comparison between predicted efficiency using (21), (22) and experimental data. The agreement between predicted efficiency using data is very good for the whole range of volume flows.



Fig. 14. Comparison between predicted head by the present method and experimental data in [41]



Fig. 15. Comparison between predicted efficiency by the present method and experimental data in [41]

V. CONCLUSIONS

In this article, an original method based on correlations for centrifugal pump characteristic curve prediction is discussed. The method is using innovative functions for head and efficiency. The new aspect introduced is the estimation of the shut-off head by means of an empirical coefficient depending on important geometrical pump parameters, which in his turn estimates the head and volume flow at BEP. The ratio of the volume flow to the volume flow at BEP is used to evaluate the head distribution. Eight original functions are presented for the head prediction and two for the efficiency prediction. The method is validated by comparing predicted centrifugal pump characteristic curves of different number of blades (one to seven) and different specific speeds, to experimental data.

Therefore it can be concluded that this method could be established as a reliable fast track engineering tool for centrifugal pump estimation.

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