Harmonic Analysis Of Svpwm Techniques For Three-Phase Inverter

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Abstract:
In PWM methods switching instants are chosen so that the desired fundamental component is obtained while acceptable harmonic performance is achieved. Space Vector Modulation (SVM) is the popular PWM method and possibly the best among all the PWM techniques as it generates higher voltages with low total harmonic distortion and works very well with field oriented (vector control) schemes for motor control. High quality output spectra can be obtained by eliminating several lower order harmonics by adopting a suitable harmonic elimination technique. In this paper, a mathematical model of space vector modulated three phase inverter is generated. Further, a method to eliminate the most significant harmonic components of line voltage waveforms of the inverter by SHE is also formulated. The harmonic spectrum analysis of the proposed model shows the effect of eliminating the lower order harmonics in SVM. Simulation results are implemented to confirm the effectiveness of the proposed model. The developed model is used to study the performance of SVM in terms of its harmonic spectrum and associated losses.

Keywords: Harmonic analysis; space-vector modulation; switching losses; simulation; voltage-source converter.

1. INTRODUCTION

The ability of high-power electronic apparatus in controlling the flow of real and reactive power through designated electrical paths, improvement of dynamic and transient performance of the system, and support of voltage profile has led to their widespread use in modern power networks. Among the most employed high-power electronic apparatus are voltage-sourced converters (VSCs), which are able to operate as controlled voltage-sources that convert an essentially constant dc-voltage to an ac voltage of controllable magnitude, frequency, and phase. Pulse-width modulation (PWM) schemes are used to synthesize the output voltage of a VSC [1]. In PWM methods switching instants are chosen so that the desired fundamental component is obtained while acceptable harmonic performance is achieved. The output waveform of a VSC consists of a series of voltage pulses. In a two-level VSC, which is studied in this paper, the voltage levels are +Vdc/2 and −Vdc/2, where Vdc is the dc-link voltage. In this case, the output voltage of each leg (of three legs) of the converter is determined through comparison of a modulating, high-frequency saw-tooth waveform and the desired sinusoidal voltage. This is done independently for each leg. An alternative method to PWM, space-vector modulation (SVM), places the converter in a number of states (that correspond to the so-called space-vectors), which are determined by the ON/OFF state of its controlled switches [2]. Synthesis of the output voltage along with crafting the desired harmonic performance is done by carefully selecting the appropriate states and their time-shares so that the desired voltage is best approximated. The SVM method features a higher level of dc-bus voltage utilization compared to the conventional PWM. It also offers flexibility in its digital implementation by providing several optimization parameters [3], such as enabling different approaches to place space-vectors [4], [5] and the number and arrangement of samples in each cycle [6], [7]. SVM has found numerous applications in...
both power systems and industrial motor drives. This paper presents an implementation of the SVM method in the PSCAD/EMTDC electromagnetic transient’s simulation program [8]. The model is used to assess the harmonic behavior of the synthesized output voltage by studying the impact of such factors as space-vector assignment strategy and reference vector sampling rate. One of the major issues faced in power electronic design is the reduction of harmonic content in inverter circuits. All PWM schemes generate inverter voltage waveforms which contain a rich harmonic spectrum [8].

2. THREE PHASE INVERTER

The structure of a typical 3-phase power inverter is shown in Fig. 1, where $V_A$, $V_B$, $V_C$ are the voltages applied to the star-connected motor windings, and where $V_{DC}$ is the continuous inverter input voltage.

![Figure 1: Basic scheme of 3-phase inverter and AC-motor](image)

The six switches can be power BJT, GTO, IGBT etc. The ON-OFF sequence of all these devices must satisfy the following conditions:

- Three of the switches must always be ON and three always OFF.
- The upper and the lower switches of the same leg are driven with two complementary pulsed signals. In this way no vertical conduction is possible, provided care is taken to ensure that there is no overlap in the power switch transitions [9].

3. OVERVIEW OF SVM METHODS

The Space Vector Pulse Width Modulation (SVPWM) method is an advanced, computation-intensive PWM method and possibly the best among all the PWM techniques for variable frequency drive application [10] [11]. Because of its superior performance characteristics, it has been finding widespread application in recent years [12]. The PWM methods discussed so far have only considered implementation on half bridges operated independently, giving satisfactory PWM performance. With a machine load, the load neutral is normally isolated, which causes interaction among the phases. Space Vector PWM (SVPWM) is one of a number of schemes to increase the output voltage of a PWM drive and limit the number of small pulses. SVPWM does not use carrier-based PWM with modulating signals, but generates switching signals based on look-up tables of possible switching states and instantaneous required output voltages.

In steady state, the output of SVPWM is similar to carrier based PWM with a common mode signal added to all three modulation signals of a three-phase PWM inverter. The common mode signal is made up of third harmonics of the supply frequency (called "triplens"). Since the same common mode signal is added to all three phases, the line-line output will not contain the common mode signal. Consider first the 3-phase inverter shown below (figure 2)

![Figure 2: Space Vector Pulse Width Modulation for 3-phase VSI](image)
The six-transistor combination in the inverter has eight permissible switching states along with the corresponding line to neutral voltage applied to the motor as shown in Fig. 3.

![Space Vector Diagram – Line to Neutral Voltages](image)

**Figure 3:** Space Vector Diagram – Line to Neutral Voltages

Each stationary vector corresponds to a particular fundamental angular position as shown in Fig. 4.

![Inverter phasor angular positions in fundamental cycle](image)

**Figure 4:** Inverter phasor angular positions in fundamental cycle

The inverter has six states when a voltage is applied to the motor and two states (0 and 7) when the motor is shorted through the upper or lower transistors resulting in zero volts being applied to the motor. The six vectors including the zero voltage vectors can be expressed geometrically as shown in Fig. 3. SVPWM seeks to average out the adjacent vectors for each sector.

**Tab. 1: Inverter Switching States**

<table>
<thead>
<tr>
<th>State</th>
<th>On Devices</th>
<th>Van</th>
<th>Vbn</th>
<th>Vcn</th>
<th>Space Voltage Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>T2,T4,T6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>V_0(000)</td>
</tr>
<tr>
<td>1</td>
<td>T1,T4,T6</td>
<td>2/3 Vdc</td>
<td>-1/3 Vdc</td>
<td>-2/3 Vdc</td>
<td>V_1(100)</td>
</tr>
<tr>
<td>2</td>
<td>T1,T3,T6</td>
<td>1/3 Vdc</td>
<td>1/3 Vdc</td>
<td>-1/3 Vdc</td>
<td>V_2(110)</td>
</tr>
<tr>
<td>3</td>
<td>T3,T2,T6</td>
<td>-1/3 Vdc</td>
<td>2/3 Vdc</td>
<td>-1/3 Vdc</td>
<td>V_3(010)</td>
</tr>
<tr>
<td>4</td>
<td>T2,T3,T5</td>
<td>-2/3 Vdc</td>
<td>1/3 Vdc</td>
<td>1/3 Vdc</td>
<td>V_4(011)</td>
</tr>
<tr>
<td>5</td>
<td>T2,T4,T5</td>
<td>-1/3 Vdc</td>
<td>-1/3 Vdc</td>
<td>2/3 Vdc</td>
<td>V_5(001)</td>
</tr>
<tr>
<td>6</td>
<td>T1,T4,T5</td>
<td>1/3 Vdc</td>
<td>-2/3 Vdc</td>
<td>1/3 Vdc</td>
<td>V_6(101)</td>
</tr>
<tr>
<td>7</td>
<td>T1,T3,T5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>V_7(111)</td>
</tr>
</tbody>
</table>

Using the appropriate PWM signals a vector is produced that transitions smoothly between sectors and thus provide sinusoidal line to line voltages to the motor. In order to generate the PWM signals that produce the rotating vector, formulae must be derived to determine the PWM time intervals for each sector. [14], [15].
4. SELECTIVE HARMONIC ELIMINATION

The undesirable lower order harmonics can be eliminated by the popular selective harmonic elimination method, which is based on the harmonic elimination theory [16, 17] as shown in Fig. 5 shows the gating signals and output voltage of an inverter, in which S1 is turned on at various switching angles \( \alpha_1, \alpha_3, \ldots, \alpha_{N-1} \) and S4 is turned off at \( \alpha_2, \alpha_4, \ldots, \alpha_N \) per quarter cycle. At each instant, three required firing pulses are generated for upper switches. These three switches can be simply inverted to obtain the other three pulses for bottom switches. The firing commands are programmed in such a way that they provide three-phase symmetry, half-wave symmetry and quarter-wave symmetry. As shown in Fig. 6, the quarter-wave symmetry is preserved which will eliminate the even harmonics. The fundamental component can be controlled and a selected low order harmonics can be eliminated by the proper choice of SVM switching angles. By applying Fourier series analysis, the output voltage can be expressed as,

\[
V(t) = \sum_{n=1,3,5} \frac{4}{n\pi} \left[ V_1 \cos(n\alpha_1) \pm V_2 \cos(n\alpha_2) \pm \ldots \pm V_N \cos(n\alpha_N) \right] \sin(n\omega t) \tag{1}
\]

where, \( N \) is the number of switching angles per quarter cycle, and \( V_1, V_2, \ldots, V_N \) are the level of DC voltages. However, if the switching angles do not satisfy the condition, this method no longer exists. From (2), it can be seen that the output voltage has no even harmonics because the output voltage waveform is odd quarter-wave symmetry. It can also be seen that the peak values of these odd harmonics are expressed in terms of the switching angles \( \alpha_1, \alpha_2, \ldots, \alpha_N \). Based on the harmonic elimination theory, for eliminating the \( n \)th harmonics,

\[
\cos(n\alpha_1) \pm \cos(n\alpha_2) \pm \ldots \pm \cos(n\alpha_N) = 0 \tag{3}
\]

Therefore, an equation with \( N \) switching angles will be used to control the \( N \) different harmonic values. Generally, an equation with \( N \) switching angles is used to determine the fundamental frequency value, and to eliminate \( N-1 \) low order harmonics [18]. The set of equations given by (2) is nonlinear since they are trigonometric functions of the variables \( \alpha_1, \alpha_2, \ldots, \alpha_N \). Newton’s iteration method is the convenient way to calculate the angles. In this, \( N \) equations can be written in vector form as,

\[
f_1(\alpha_1, \alpha_2, \ldots, \alpha_N) = \begin{bmatrix} f_1(\alpha_1, \alpha_2, \ldots, \alpha_N) \\ f_2(\alpha_1, \alpha_2, \ldots, \alpha_N) \\ \vdots \\ f_M(\alpha_1, \alpha_2, \ldots, \alpha_N) \end{bmatrix} \tag{4}
\]

These can be achieved in off line and the values of switching angles for particular modulation index are stored as look up tables. These tables are stored in a programmable memory and for example, microcomputer or microcontroller board which has been programmed to accept the value of modulation index and generate the corresponding switching angles. The Weighted Total Harmonic Distortion (WTHD) method is used to indicate the quantity of harmonics contents in the output waveforms. It is calculated in the same way as THD, but each harmonic component is divided by its order, so that higher order harmonics receive lower weight and contribute less in this figure of merit [19]. An upper limit of 50 is often recommended for the calculation of the WTHD [20]. The definition of WTHD in an inverter circuit is given as,
which can be simplified by computing for \( n = 5, 7, 11, 13, \ldots, 49 \) due to the reasons that the even and the triplen harmonics do not appear in the line voltages.

5. SIMULATION MODEL AND RESULTS

In this section, an implementation of space-vector modulation is presented. Generation of high-order voltage and current harmonics as a result of rapid switching of a converter necessitate the use of a transient simulation tool. As a platform able to handle extensive network studies in power systems, digital implementation and testing of SVM is carried out using the electromagnetic transients simulation using MATLAB

Figure 7: MATLAB/Simulink model of three phase inverter based on SVM

Fig. 7 shows the developed model of three phase inverter. For motor drive applications, it is necessary to eliminate low order harmonics up to 19. Hence, the 5th, 7th, 11th, 13th, 17th and 19th harmonics need to be theoretically eliminated from line

Simulation Results: Sector corresponds to the location of voltage in the circular locus traced by it and is divided into six sectors of 60 degree each. Figure 8 and Figure 9 shows the generated line voltage and phase voltage respectively. In a two level VSC the voltage levels are \( \pm \frac{V_{DC}}{2} \), where \( V_{DC} \) is the DC link voltage. The line voltage alternates between \( +V_{DC} \) and \( -V_{DC} \) while the phase voltage can assume the values of \( \frac{2V_{DC}}{3} \) and \( \frac{V_{DC}}{3} \). Fig. 10 and Fig. 11 shows the line current, because of the inductive nature of the load, higher order harmonics have been filtered out and the current waveform is sinusoidal in nature.

Figure 8: Response of line voltage versus time
Fig. 10 shows the response of pole voltage for m=0.8 and Fig. 11 shows the line current, because of the inductive nature of the load, higher order harmonics have been filtered out and the current waveform is sinusoidal nature.

Number of switching angles Vs switching frequency:
Table II shows the relationship between the number of switching angles per quarter cycles (N) of the output waveform and the switching frequency (f_s) from the simulation result. For eliminating harmonics up to 19, N must be not less than 7. From table II the suitable value of switching frequency corresponds to N=7 as 800Hz.

<table>
<thead>
<tr>
<th>N</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>13</th>
<th>17</th>
<th>31</th>
</tr>
</thead>
<tbody>
<tr>
<td>f_s (Hz)</td>
<td>600</td>
<td>800</td>
<td>1000</td>
<td>1500</td>
<td>2000</td>
<td>3000</td>
</tr>
</tbody>
</table>

For eliminating harmonics up to 19, N must be not less than 7. From table II the suitable value of switching frequency corresponds to N=7 as 800Hz.
6. CONCLUSION

Selective harmonic elimination method is systematically applied to the generated waveform and the harmonic performance of the waveform is studied. Simulation results reported in this paper confirm that the developed model generates waveforms with complete symmetry. Amplitude of line to line voltage is as high as DC bus voltage in SVM technique. With the increased output voltage, the user can design the motor control system with reduced current rating, which helps to reduce inherent conduction loss of the voltage source inverter. It is found that by adopting the selective harmonic elimination technique, least harmonic distortion (using WTHD) is achievable with modulation index ranging from 0.5 to 0.9. This has considerable practical use for variable-speed AC motor drive applications, where harmonics in the output of the inverter pose serious problems in the motor performance. The developed model allows the user to select parameters according to their design requirements. The model can be adapted for applications like UPS with fixed frequency or in application where the inverter has to track the grid.

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[15] PROGRAMMABLE LABORATORY INVERTER AND SPACE VECTOR PWM Ing. Pavel Gajdůšek, Doctoral Degree Programme (2) Dept. of Electrical Power Engineering, FECE, VUT mail: xgajdu02@stud.feec.vutbr.cz