Half step constant predictor-corrector method for the solution of second order ordinary differential equation

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Abstract

We consider a half step numerical integrator which is derived by collocating the differential system and interpolating the approximate solution to generate a continuous hybrid linear multistep method which serves as the corrector. The predictors are derived using block method hence a constant order predictors are developed. The properties of the corrector viz; order, consisitency, zero stability and convergence are verified. The new method was tested on some numerical examples and was found to give better approximation than the existing method.

Keyword: half step, collocation, differential system, interpolation, approximate solution, predictor, corrector

A.M.S Subject Classification: 65L05, 65L06, 65D30

1 Introduction

This paper considers the approximate solution to the general second order initial value problems of the form

$$y'' = f(x, y, y') \qquad y^k(x_n) = y_n^k, \quad k = 0, 1 \tag{1}$$

Equation (1) are convectionally solved by reducing to system of first order ordinary differential equation, then any approximate method of solving first order can be adopted to solve the resulting system of first order equation. This method is extensively discussed by Adesanya, Anake and Udoh [5], Awoyemi and Kayode [6], Jator [11] to mention few. These authors suggested that the direct method for solving higher order ordinary differential equations are more efficient since the method of reduction increased the dimension of the resulting system of first order; hence it wastes alot of computer and human efforts.

Scholars have worked on predictor-corrector method for the solution of implicit linear multistep method, among them are Kayode and Adeyeye[12], Adesanya, Anake and Oghoyon [4], Awoyemi [7], Olabode [14]. They individually proposed method in which reducing order predictors are adopted to implement the corrector. The major setback of this method is that the predictors are reducing order of accuracy, therefore it has a great effect on the accuracy of the method. Other setbacks of this method are discussed by Awoyemi [7] and Awoyemi et al. [8].

Scholars later proposed block method to cater for some of the setbacks of predictor-corrector method. Block method has the properties of being self starting and gives evaluation at selected grid points without overlapping. They do not require developing seperate predictors and starting values. moreover it evaluates fewer function per step. Among these authors are Jator [10], Jator and Li [9], Simiak [16], Abbas [1], Adesanya et al. [2], Awoyemi et al. [8], Omar and Suleiman [15] Majid et al. [13].

It was observed that in block method, the number of interpolation points cannot exceed the order of the differential equation, hence this method does not exhaust all possible interpolation points, therefore method of lower order are developed.

In this paper, we developed a method which is implemented in predictor corrector method in which the predictors are constant order of accuracy. This method combines the properties of both predictor-corrector and block method.

2 Methodology

2.1 Development of the corrector

We consider a power series approximate solution of the form

$$y(x) = \sum_{j=0}^{r+s-1} a_j x^j \tag{2}$$

where r and s are the number of interpolation and collocation respectively.

The second derivative of (2) gives

$$y''(x) = \sum_{j=0}^{r+s-1} j(j-1)a_j x^{j-2} \tag{3}$$

substituting (3) into (1) gives

$$f(x, y, y') = \sum_{j=0}^{r+s-1} j(j-1)a_j x^{j-2} \tag{4}$$

Equation (4) is called the differential system. Interpolating (2) at $x_{n+r}, r = 0\left(\frac{1}{8}\right)\frac{3}{8}$ and collocating $x_{n+s}, s = 0\left(\frac{1}{8}\right)\frac{1}{2}$, gives a non linear system of the form

$$AX = U \tag{5}$$

$$A = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \end{bmatrix}^T$$

$$U = \begin{bmatrix} y_n & y_{n+\frac{1}{8}} & y_{n+\frac{1}{4}} & y_{n+\frac{3}{8}} & f_n & f_{n+\frac{1}{8}} & f_{n+\frac{1}{4}} & f_{n+\frac{3}{8}} & f_{n+\frac{1}{2}} \end{bmatrix}^T$$

$$X = \begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & x_n^8 \\ 1 & x_{n+\frac{1}{8}} & x_{n+\frac{1}{8}}^2 & x_{n+\frac{1}{8}}^3 & x_{n+\frac{1}{8}}^4 & x_{n+\frac{1}{8}}^5 & x_{n+\frac{1}{8}}^6 & x_{n+\frac{1}{8}}^7 & x_{n+\frac{1}{8}}^8 \\ 1 & x_{n+\frac{1}{4}} & x_{n+\frac{1}{4}}^2 & x_{n+\frac{1}{4}}^3 & x_{n+\frac{1}{4}}^4 & x_{n+\frac{1}{4}}^5 & x_{n+\frac{1}{4}}^6 & x_{n+\frac{1}{4}}^7 & x_{n+\frac{1}{4}}^8 \\ 1 & x_{n+\frac{3}{8}} & x_{n+\frac{3}{8}}^2 & x_{n+\frac{3}{8}}^3 & x_{n+\frac{3}{8}}^4 & x_{n+\frac{3}{8}}^5 & x_{n+\frac{3}{8}}^6 & x_{n+\frac{3}{8}}^7 & x_{n+\frac{3}{8}}^8 \\ 0 & 0 & 2 & 6x_n & 12x_n^2 & 20x_n^3 & 30x_n^4 & 42x_n^5 & 56x_n^6 \\ 0 & 0 & 2 & 6x_{n+\frac{1}{8}} & 12x_{n+\frac{1}{8}}^2 & 20x_{n+\frac{1}{8}}^3 & 30x_{n+\frac{1}{8}}^4 & 42x_{n+\frac{1}{8}}^5 & 56x_{n+\frac{1}{8}}^6 \\ 0 & 0 & 2 & 6x_{n+\frac{1}{4}} & 12x_{n+\frac{1}{4}}^2 & 20x_{n+\frac{1}{4}}^3 & 30x_{n+\frac{1}{4}}^4 & 42x_{n+\frac{1}{4}}^5 & 56x_{n+\frac{1}{4}}^6 \\ 0 & 0 & 2 & 6x_{n+\frac{3}{8}} & 12x_{n+\frac{3}{8}}^2 & 20x_{n+\frac{3}{8}}^3 & 30x_{n+\frac{3}{8}}^4 & 42x_{n+\frac{3}{8}}^5 & 56x_{n+\frac{3}{8}}^6 \\ 0 & 0 & 2 & 6x_{n+\frac{1}{2}} & 12x_{n+\frac{1}{2}}^2 & 20x_{n+\frac{1}{2}}^3 & 30x_{n+\frac{1}{2}}^4 & 42x_{n+\frac{1}{2}}^5 & 56x_{n+\frac{1}{2}}^6 \end{bmatrix}$$

Solving (5) using Guassian elinination method and substituting into (2) gives a continuous hybrid linear multistep method of the form

$$y(x) = \alpha_0 y_n + \alpha_{\frac{1}{8}} y_{\frac{1}{8}} + \alpha_{\frac{1}{4}} y_{\frac{1}{4}} + \alpha_{\frac{3}{8}} y_{\frac{3}{8}} + h^2 \left(\begin{array}{c} \beta_0 f_n + \beta_{\frac{1}{8}} f_{n+\frac{1}{8}} + \beta_{\frac{1}{4}} f_{n+\frac{1}{4}} \\ +\beta_{n+\frac{3}{8}} f_{\frac{3}{8}} + \beta_{\frac{1}{2}} f_{n+\frac{1}{2}} \end{array} \right) \quad (6)$$

where $y_{n+j} = y\left(x_n + jh\right), \ \ f\left((x_n + jh), y\left(x_n + jh\right) y'\left(x_n + jh\right)\right)$

$$\alpha_0 = \frac{1}{21}\left(2097152t^7 - 3670016t^6 + 2408448t^5 - 716800t^4 + 86016t^3 - 596t + 21\right)$$

$$\alpha_{\frac{1}{8}} = \frac{1}{217}\left(\begin{array}{c} 176160768t^8 - 329252864t^7 + 24221056t^6 - 89112576t^5 \\ +17002496t^4 - 1462272t^3 + 6912t \end{array} \right)$$

$$\alpha_{\frac{1}{4}} = -\frac{1}{217}\left(\begin{array}{c} 352321536t^8 - 593494016t^7 + 370671616t^6 - 103563264t^5 \\ +11784192t^4 - 258048t^3 - 2916t \end{array} \right)$$

$$\alpha_{\frac{3}{8}} = \frac{1}{651}\left(\begin{array}{c} 528482304t^8 - 857735168t^7 + 499122176t^6 - 118013952t^5 \\ +6565888t^4 + 946176t^3 - 11008t \end{array} \right)$$

$$\beta_0 = \frac{1}{312480}\left(\begin{array}{c} 16515072t^8 - 65404928t^7 + 84926464t^6 - 51351552t^5 \\ +15829184t^4 - 2421664t^3 - 2421t \end{array} \right)$$

$$\beta_{\frac{1}{8}} = -\frac{1}{19530}\left(\begin{array}{c} 24772608t^8 - 14303232t^7 - 21489664t^6 + 23466240t^5 \\ -8094464t^4 + 1002624t^3 - 4887t \end{array} \right)$$

$$\beta_{\frac{1}{4}} = -\frac{1}{17360}\left(\begin{array}{c} 177995776t^8 - 287309824t^7 + 164749312t^6 - 37044224t^5 \\ +1231552t^4 + 452928t^3 - 4455t \end{array} \right)$$

$$\beta_{\frac{3}{8}} = -\frac{1}{19530}\left(\begin{array}{c} 24772608t^8 - 39698432t^7 + 22951936t^6 - 5531904t^5 \\ +377216t^4 + 30464t^3 - 423t \end{array}\right)$$

$$\beta_{\frac{1}{2}} = \frac{1}{44640}\left(\begin{array}{c} 2359296t^8 - 3538944t^7 + 1974272t^6 - 479232t^5 \\ +39232t^4 + 1248t^3 - 27t \end{array}\right)$$

$t = \frac{x-x_n}{h}$

Evaluating (6) at $t = \frac{1}{2}$, gives a discrete scheme

$$y_{n+\frac{1}{2}} + \frac{128}{31}y_{n+\frac{3}{8}} - \frac{318}{31}y_{n+\frac{1}{4}} + \frac{128}{31}y_{n+\frac{1}{8}} + y_n = \frac{h^2}{29760}\left(\begin{array}{c} 23f_{n+\frac{1}{2}} + 688f_{n+\frac{3}{8}} + \\ 2358f_{n+\frac{1}{4}} + 688f_{n+\frac{1}{8}} + 23f_n \end{array}\right) \quad (7)$$

2.2 Development of predictors

In developing the predictor, we interpolate equation (2) at $x_{n+r}, r = \frac{1}{4}, \frac{3}{8}$ and collocating (4) at $x_{n+s}, s = 0\left(\frac{1}{8}\right)\frac{1}{2}$ to generate a system of non linear equation in the form (5) where

$$A = \left[\begin{array}{ccccccc} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \end{array}\right]^T$$

$$U = \left[\begin{array}{ccccccc} y_{n+\frac{1}{4}} & y_{n+\frac{3}{8}} & f_n & f_{n+\frac{1}{8}} & f_{n+\frac{1}{4}} & f_{n+\frac{3}{8}} & f_{n+\frac{1}{2}} \end{array}\right]^T$$

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$$X = \begin{bmatrix} 1 & x_{n+\frac{1}{4}} & x^2_{n+\frac{1}{4}} & x^3_{n+\frac{1}{4}} & x^4_{n+\frac{1}{4}} & x^5_{n+\frac{1}{4}} & x^6_{n+\frac{1}{4}} \\ 1 & x_{n+\frac{3}{8}} & x^2_{n+\frac{3}{8}} & x^3_{n+\frac{3}{8}} & x^4_{n+\frac{3}{8}} & x^5_{n+\frac{3}{8}} & x^6_{n+\frac{3}{8}} \\ 0 & 0 & 2 & 6x_n & 12x_n^2 & 20x_n^3 & 30x_n^4 \\ 0 & 0 & 2 & 6x_{n+\frac{1}{8}} & 12x^2_{n+\frac{1}{8}} & 20x^3_{n+\frac{1}{8}} & 30x^4_{n+\frac{1}{8}} \\ 0 & 0 & 2 & 6x_{n+\frac{1}{4}} & 12x^2_{n+\frac{1}{4}} & 20x^3_{n+\frac{1}{4}} & 30x^4_{n+\frac{1}{4}} \\ 0 & 0 & 2 & 6x_{n+\frac{3}{8}} & 12x^2_{n+\frac{3}{8}} & 20x^3_{n+\frac{3}{8}} & 30x^4_{n+\frac{3}{8}} \\ 0 & 0 & 2 & 6x_{n+\frac{1}{2}} & 12x^2_{n+\frac{1}{2}} & 20x^3_{n+\frac{1}{2}} & 30x^4_{n+\frac{1}{2}} \end{bmatrix}$$

Solving this equation using Guassian elinimation method and substituting into (2) gives a continuous hybrid linear multistep method of the form

$$y(x) = \alpha_{\frac{1}{4}} y_{\frac{1}{4}} + \alpha_{\frac{3}{8}} y_{\frac{3}{8}} + h^2 \left(\beta_0 f_n + \beta_{\frac{1}{8}} f_{n+\frac{1}{8}} + \beta_{\frac{1}{4}} f_{n+\frac{1}{4}} + \beta_{n+\frac{3}{8}} f_{\frac{3}{8}} + \beta_{\frac{1}{2}} f_{n+\frac{1}{2}} \right) \quad (8)$$

$$\alpha_{\frac{1}{4}} = 3 - 8t \qquad \alpha_{\frac{3}{8}} = 8t - 2$$

$$\beta_0 = \frac{1}{46080} \left(262144t^6 - 491520t^5 + 358400t^4 - 128000t^3 + 23040t^2 - 1900t + 51 \right)$$

$$\beta_{\frac{1}{8}} = -\frac{1}{11520} \left(262144t^6 - 442368t^5 + 266240t^4 - 61440t^3 + 1908t - 189 \right)$$

$$\beta_{\frac{1}{4}} = \frac{1}{7680} \left(262144t^6 - 393216t^5 + 194560t^4 - 30720t^3 - 644t + 20t \right)$$

$$\beta_{\frac{3}{8}} = -\frac{1}{11520} \left(262144t^6 - 344064t^5 + 143360t^4 - 20480^3 + 284t - 39 \right)$$

$$\beta_{\frac{1}{2}} = \frac{1}{46080} \left(262144t^6 - 294912t^5 + 112640t^4 - 15360t^3 + 132t - 9 \right)$$

Solving for the independent solution $y_{n+s}, s = \frac{1}{8} \left(\frac{1}{8} \right) \frac{1}{2}$, gives a continuous hybrid

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block formula of the form

$$y(x) = \sum_{j=0}^{1} \frac{(jh)^m}{m!} y_n^{(m)} + h^2 \left(\Psi_0 f_n + \Psi_{\frac{1}{8}} f_{n+\frac{1}{8}} + \Psi_{\frac{1}{4}} f_{n+\frac{1}{4}} + \Psi_{n+\frac{3}{8}} f_{\frac{3}{8}} + \Psi_{\frac{1}{2}} f_{n+\frac{1}{2}} \right) \quad (9)$$

Where

$$\Psi_0 = \frac{1}{90} \left(512t^6 - 960t^5 + 700t^4 - 250t^3 + 45t^2\right)$$

$$\Psi_{\frac{1}{8}} = -\frac{1}{45} \left(1024t^6 - 1728t^5 + 1040t^4 - 240t^3\right)$$

$$\Psi_{\frac{1}{4}} = \frac{1}{15} \left(512t^6 - 768t^5 + 380t^4 - 60t^3\right)$$

$$\Psi_{\frac{3}{8}} = -\frac{1}{45} \left(1024t^6 - 1344t^5 + 560t^4 - 80t^3\right)$$

$$\Psi_{\frac{1}{2}} = \frac{1}{45} \left(256t^6 - 288t^5 + 110t^4 - 15t^3\right)$$

Evaluating (9) at $t = \frac{1}{8} \left(\frac{1}{8}\right) \frac{1}{2}$, gives a discrete block formula in the form

$$A^{(0)} \mathbf{Y}_m = \mathbf{e} y_n + h^2 \mathbf{d} f\left(y_n\right) + h^2 \mathbf{b} \mathbf{F}\left(Y_m\right) \quad (10)$$

$$\mathbf{Y}_m = \begin{bmatrix} y_{n+\frac{1}{8}} & y_{n+\frac{1}{4}} & y_{n+\frac{3}{8}} & y_{n+\frac{1}{2}} \end{bmatrix}^T \quad f(y_n) = \begin{bmatrix} y_{n-1} & y_{n-2} & y_{n-3} & y_n \end{bmatrix}^T$$

$$\mathbf{F}\left(Y_m\right) = \begin{bmatrix} f_{n+\frac{1}{8}} & f_{n+\frac{1}{4}} & f_{n+\frac{3}{8}} & f_{n+\frac{1}{2}} \end{bmatrix}^T \quad \mathbf{e} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$$\mathbf{b} = \begin{bmatrix} \frac{367}{92160} & \frac{144}{5760} & \frac{468}{10240} & \frac{24}{360} \\ \frac{-282}{92160} & \frac{-30}{5760} & \frac{54}{10240} & \frac{6}{360} \\ \frac{116}{92160} & \frac{16}{5760} & \frac{60}{10240} & \frac{8}{360} \\ \frac{-21}{92160} & \frac{-3}{5760} & \frac{-9}{10240} & 0 \end{bmatrix}$$

$$\mathbf{d} = \begin{bmatrix} \frac{367}{92160} & \frac{53}{5760} & \frac{147}{10240} & \frac{7}{360} \end{bmatrix}^T$$

Evaluating the first derivative of (9) at $t = \frac{1}{8}\left(\frac{1}{8}\right)\frac{1}{2}$ and substituting in (10) gives

$$y'_{n+\frac{1}{8}} = y'_n + \frac{h}{5760}\left(251f_n + 646f_{n+\frac{1}{8}} - 264f_{n+\frac{1}{4}} + 10f_{n+\frac{3}{8}} - 19f_{n+\frac{1}{2}}\right)$$

$$y'_{n+\frac{1}{4}} = y'_n + \frac{h}{720}\left(29f_n + 124f_{n+\frac{1}{8}} + 24f_{n+\frac{1}{4}} + 4f_{n+\frac{3}{8}} - f_{n+\frac{1}{2}}\right)$$

$$y'_{n+\frac{3}{8}} = y'_n + \frac{h}{640}\left(27f_n + 102f_{n+\frac{1}{8}} + 72f_{n+\frac{1}{4}} + 42f_{n+\frac{3}{8}} - 3f_{n+\frac{1}{2}}\right)$$

$$y'_{n+\frac{1}{2}} = y'_n + \frac{h}{180}\left(7f_n + 32f_{n+\frac{1}{8}} + 12f_{n+\frac{1}{4}} + 32f_{n+\frac{3}{8}} + 7f_{n+\frac{1}{2}}\right)$$

3 Analysis of the basic properties of the block

3.1 Order of the method

We defined a linear operator on (7) to give

$$\mathcal{L}\{y(x) \quad : \quad h\} = y(x) - y_{n+\frac{1}{2}} + \frac{128}{31}y_{n+\frac{3}{8}} - \frac{318}{31}y_{n+\frac{1}{4}} + \frac{128}{31}y_{n+\frac{1}{8}} + y_n - \frac{h^2}{29760}\left(23f_{n+\frac{1}{2}} + 688f_{n+\frac{3}{8}} + 2358f_{n+\frac{1}{4}} + 688f_{n+\frac{1}{8}} + 23f_n\right) \quad (11)$$

Expanding y_{n+j} and f_{n+j} in Taylor series and comparing the coefficient of h

gives

$$\mathcal{L}\{y(x) \ : \ h\} = C_0 y(x) + C_1 h y'(x) + ... + C_p h^p y^p(x) + C_{p+1} h^{p+1} y^{p+1}(x)$$
$$+ C_{p+2} h^{p+2} y^{p+2}(x) + ... \qquad (12)$$

Definition 1 *Order*

The difference operator $\mathcal{L}$ and the associated continuous linear multistep method (15) are said to be of order p if $C_0 = C_1 = ... = C_p = C_{p+1} = 0$ and C_{p+2} is called the error constant and implies that the local truncation error is given by $t_{n+k} = C_{p+2} h^{(p+2)} y^{(p+2)}(x) + O\left(h^{p+3}\right)$

The order of our discrete scheme is 8, with error constant $C_{p+2} = \frac{-79}{599961600}$

3.2 Consistency

A linear multistep method (7) is said to be consistent if it has order $p \geq 1$ and if $\rho(1) = \rho'(1) = 0$ and $\rho''(1) = 2!\sigma(1)$ where $\rho(r)$ is the first characteristic polynomial and $\sigma(r)$ is the second characteristic polynomial.

For our method,

$\rho(r) = r^2 + \frac{128}{31} r^{\frac{3}{2}} - \frac{318}{31} r + \frac{128}{31} r^{\frac{1}{2}} + 1$

and $\sigma(r) = \frac{1}{465}\left(23r^2 + 688r^{\frac{3}{2}} + 2358r + 688r^{\frac{1}{2}} + 23\right)$.

Clearly $\rho(1) = \rho'(1) = 0$ and $\rho''(1) = 2!\sigma(1)$.

Hence our method is consistent

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3.3 Zero stability

A linear multistep method is said to be zero stable, if the zeros of the first characteristic polynomial $\rho(r)$ satisfies $\mid r \mid \leq 1$ and for $\mid r \mid = 1$ is simple

Our method was found to be zero stable.

3.4 Region of absolute stability

The method (7) is said to be absolute stable if for a given h, all roots z_s of the characteristic polynomial $\pi(z, h) = \rho(z) + h^2\sigma(z) = 0$, satisfies $\mid z_s \mid < 1, s = 1, 2, ..., n.$ where $h = -\lambda^2 h^2$ and $\lambda = \frac{\partial f}{\partial y.}$.

The boundary locus method is adopted to determine the region of absolute stability. Substituting the test equation $y'' = -\ \lambda^2 h^2$ into (7) and writing $r = \cos\theta + i\sin\theta$ gives the stability region as shown in fig. (1), plotted using Scientific workplace software.

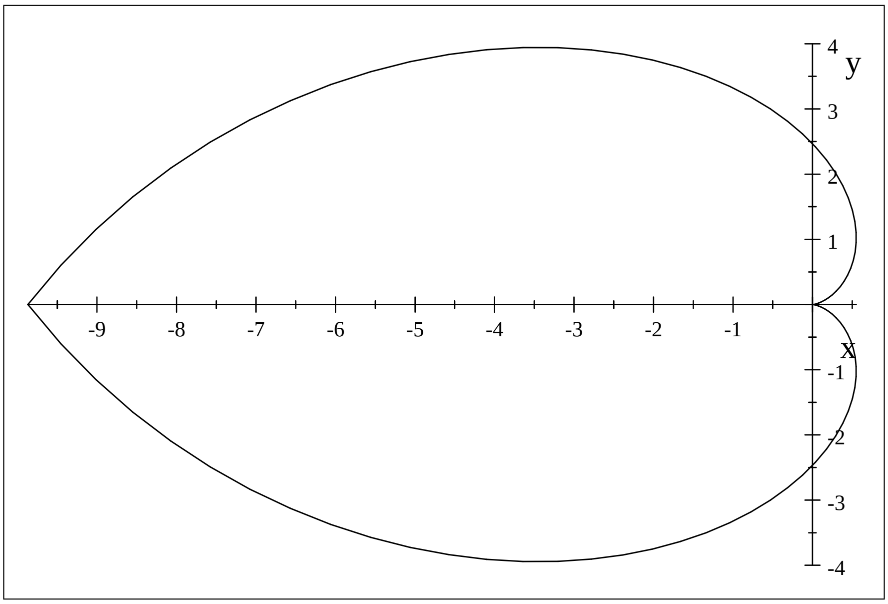

fig (1)

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4 Numerical Experiments

4.1 Test Problems

We test our method with second order initial value problems

Problem 1: Consider the non-linear initial value problem (I.V.P)

$y'' - x(y')^2 = 0, y(0) = 1, y'(0) = \frac{1}{2}, h = 0.05$

Exact solution: $y(x) = 1 + \frac{1}{2}\ln\left(\frac{2+x}{2-x}\right)$

Jator [10] solved this problem in block method where a block of order 6 and step-length of 5 is proposed with $h = 0.05$.Adesanya et al. [2] also solve this problem where the adopted constant predictor corrector method , wher a corrector of order 8 is proposed. Though we did not show the result of Jator [9] but Adesanya et al. [2] was better in term of accuracy. We compare our result with this result as shown in table 1

Problem 2: We consider the non-linear initial value problem (I.V.P)

$y'' = \frac{(y')^2}{2y} - 2y, y(\frac{\pi}{6}) = \frac{1}{4}, y'(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}, h = 0.05$

Exact solution:$(\sin x)^2$

Jator [10] solved this problem in block method where a block of order 6 and step-length of 5 is proposed with $h = 0.05$.Adesanya et al. [2] also solve this problem where the adopted constant predictor corrector method , wher a corrector of order 8 is proposed. Though we did not show the result of Jator [9] but Adesanya et al. [2] was better in term of accuracy. we compare our result with this result as shown in table 2

Error=|Exact result-computed result|

table 1 for problem 1

x	Exact result	Computed result	Error	Error in [2]
0.1	1.050041729278	1.050041729278	5.5511(-15)	7.5028(-13)
0.2	1.100335347731	1.100335347731	2.0650(-15)	9.7410(-12)
0.3	1.151140435936	1.151140435936	5.0404(-14)	3.7638(-11)
0.4	1.202732554054	1.202732554054	9.6145(-14)	9.7765(-11)
0.5	1.255412811882	1.255412811882	1.7230(-13)	2.0825(-10)
0.6	1.309519604203	1.309519604203	2.8288(-13)	3.9604(-10)
0.7	1.365443754271	1.365443754271	4.6473(-13)	7.0460(-10)
0.8	1.423648930193	1.423648930192	7.5250(-13)	1.2095(-09)
0.9	1.484700278594	1.484700278593	1.2370(-12)	2.0511(-09)
1.0	1.549306144334	1.549306144332	2.0736(-12)	3.5066(-09)

table 2 for problem 2

x	Exact result	Computed result	Error	Error in [2]
1.1048	0.7981568789707	0.798156789000	3.0250(-12)	1.8811(-10)
1.2048	0.8719546393729	0.8719546393769	4.0059(-12)	2.4539(-10)
1.3048	0.9309237421478	0.9309237421528	5.0252(-12)	3.0306(-10)
1.4048	0.9727132751817	0.9727132751875	6.0277(-12)	3.5819(-10)
1.5048	0.9956572216671	0.9956572216741	6.9687(-12)	4.0838(-10)
1.6048	0.9988408788614	0.9988408788929	7.7953(-12)	4.5128(-10)
1.7048	0.9821373243990	0.9821373244077	8.4637(-12)	4.8473(-10)
1.8048	0.9462124762851	0.9462124762940	8.9351(-12)	5.0696(-10)
1.9048	0.8924985448466	0.8924985448558	9.1801(-12)	5.1697(-10)
2.0048	0.8231369350259	0.8231369350350	9.1735(-12)	5.1381(-10)

5 Conclusion

We have proposed a two steps-four hybrid points method in this paper. Continuous block method which has the properties of evaluation at all points with the interval of integration is adopted to give the independent solution at non overlapping intervals as the predictor to an order eight corrector. This new method forms a bridge between the predictor-corrector method and block method. Hence it shares the properties of both method. the new method evaluate fewer function per step hence makes this performed better than the existing method i.e. block method and the predictor corrector method as shown in the numerical examples.

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