Graphical Simulations of a Mathematical Model of HIV/AIDS Dynamics with Three Compactments

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***Abstract* - This work presents computer simulations of a mathematical model of HIV/AIDS dynamics which is a system of three ordinary differential equations using Mathcad software. The model assumes the interactions of three major players of the susceptible, latent and infected populations and the effects of public campaign and anti-retroviral drug administration aimed at slowing down the death of the victims considered. The susceptibles are virus free but are prone to infection through interaction with the latent and infected classes. The latent class refers to members of the population that contracted the HIV virus but have no symptom of AIDS. Members of the infected class are already manifested with AIDS. Parameter values were used for the simulations and meaningful conclusion derived. The research work sees public enlightenment campaign as an important factor in ensuring safe population.**

1. INTRODUCTION

According to Akinwande (2014), modeling is a simplified or idealized descriptions or conception of a particular system, situation or process. Sowunmi (1993), described this as experimenting on paper which is Safer than using human or animal lives. Benyah (2009), sees mathematical modeling as "the process of creating a mathematical representation of some phenomena in order to gain better understanding of that phenomena". This study presents graphical simulations of three dimensional mathematical model of the HIV/AIDS disease dynamics which is a system of ordinary differential equations. The population is partitioned into three interacting classes of the susceptible S(t), latent L(t) and infected I(t). The susceptible class is the class in which members are virus free but are prone to infection by interacting with the latent and infected classes. The latent class is the class of those that have contracted the virus but have no symptom of the AIDS disease. The members of this class are still very active in the population both sexually and economically and as such form a mainstream for the spread of HIV. The last class of the infected is a class of those that have the manifestation of the AIDS symptoms of infection, this class is generally weak and inactive. Mathcad computer program is used to generate computer simulation for the model.

2. THE MODEL EQUATIONS

The S-L-I model is adapted which gives a system of three ordinary differential equations as shown below

At time t, let

S(t) = Susceptible class
L(t) = Latent class
I(t) = Infected class
P(t) = Total population

Thus

P(t) = S(t) + L(t) + I(t)

And the model equations are given by:

$$\frac{dS(t)}{dt} = (\beta - \mu)S(t) + \theta\beta L(t) - \alpha S(t)[L(t) + I(t)]$$

$$\frac{dL(t)}{dt} = ((1-\theta)\beta - \mu - \sigma)L(t) + \alpha S(t)[L(t) + I(t)]$$

$$\frac{dI(t)}{dt} = (\beta - \mu - \delta)I(t) + \sigma L(t)$$

With parameters defined by:

β = Natural birth rate for the population
μ = Natural death rate for the population
α = Rate of contracting the HIV-Virus when S(t) interacts with L(t) and I(t)
θ = The proportion of the off-springs of the latent class L(t) which are virus free at birth
σ = Rate of flow of members from the latent class L(t) into the infected class I(t)
δ = Additional death rate due to infection.

3. GRAPHICAL SIMULATIONS

There are problems keeping adequate records. HIV/AIDS case reporting has been characterized by under-recognition, under-reporting and delayed reporting particularly in the developing countries. No adequate data bank and so the data varied hypothetically. Hence, the parameter values used for the simulations are:

$P_0 = 10$, $S_0 = 9.4$ $L_0 = 0.5$, $I_0 = 0.1$

$\beta = 0.0523$, $\mu = 0.0118$, $\alpha = 0.053$, $\delta = 0.5$

$\sigma = 0.132$, $o = 0.812$,

The simulations are meant to study the profile of the population in respect of the distinct classes of the susceptible (y0), latent (y1) and the effected (y2) and to consider the effect of varying some parameter values on the population.

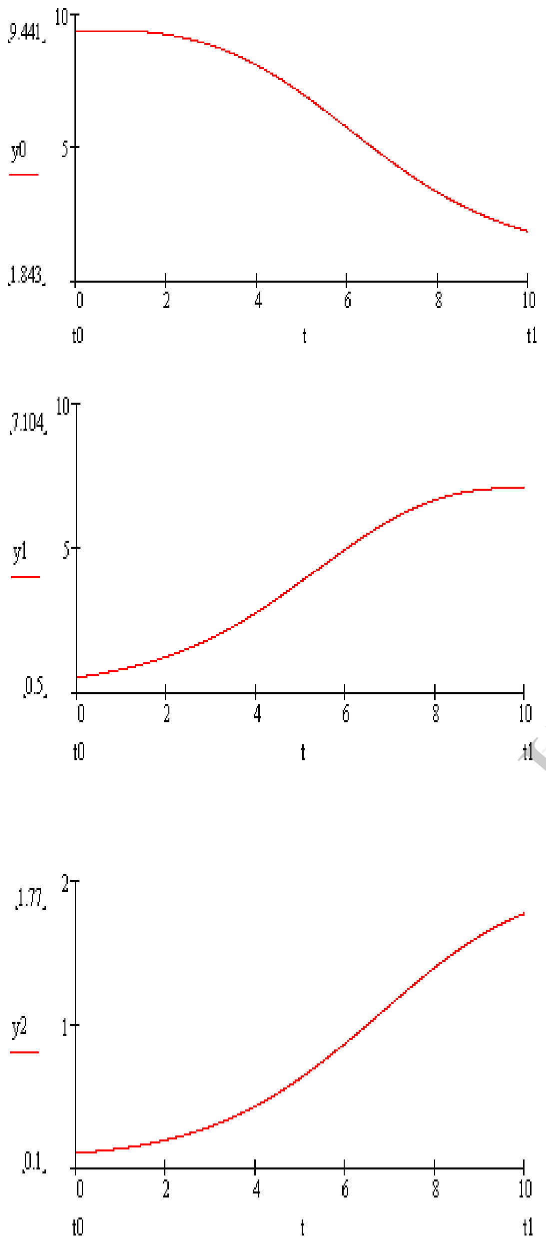


Fig 4.1 Graphical Profile of the Classes α = 0.053, σ = 0.132.

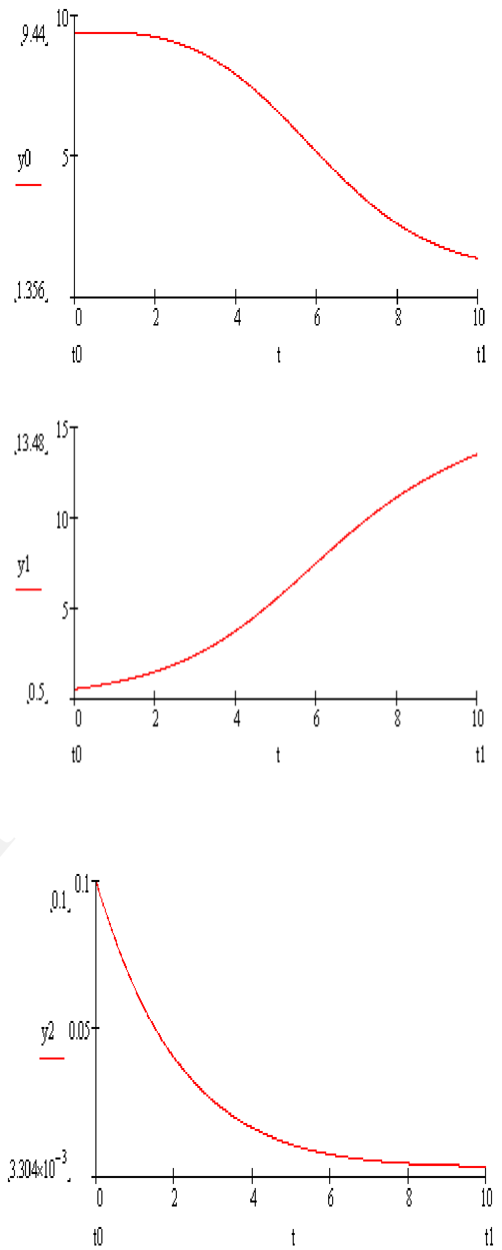
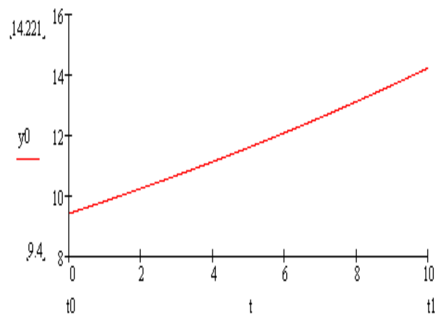


Fig. 4.2 Graphical profile for α = 0.053, σ = 0.0001 (Effective anti-retroviral drugs)



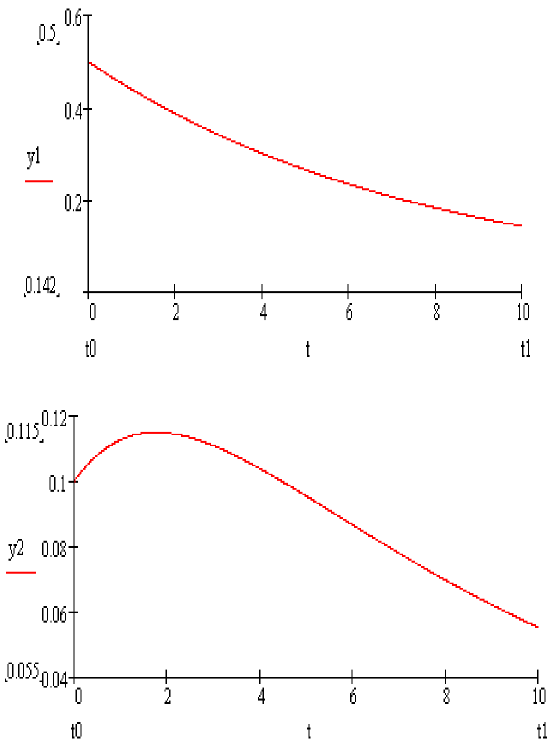
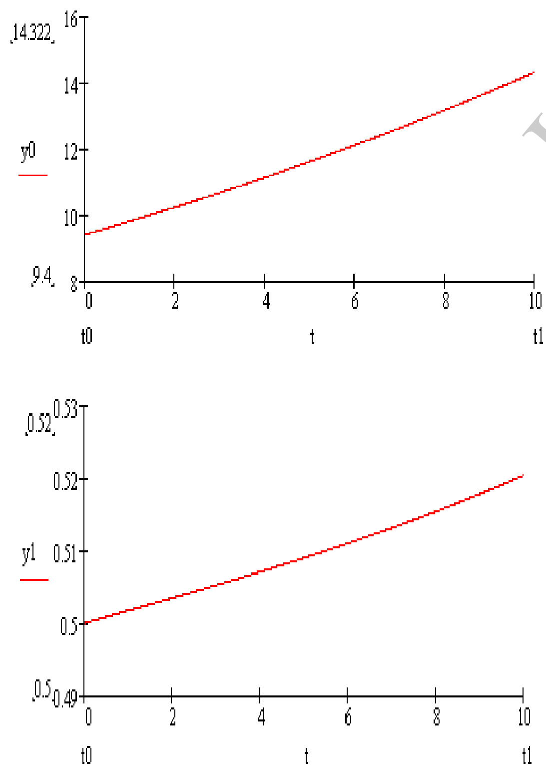


Fig. 4.3 Graphical profile for α = 0.0005, σ = 0.132 (Effective public campaign and education)



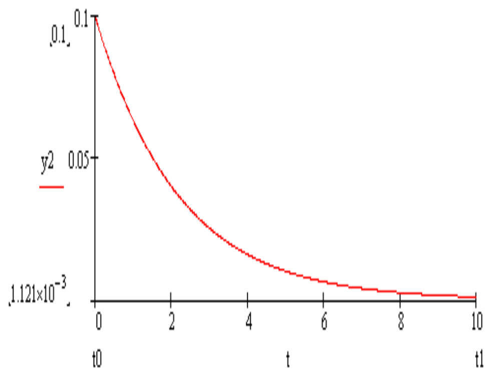


Fig. 4.4 Graphical profile for α = 0.0005, σ = 0.0001 (Effective anti-retroviral drugs, public campaign and education)

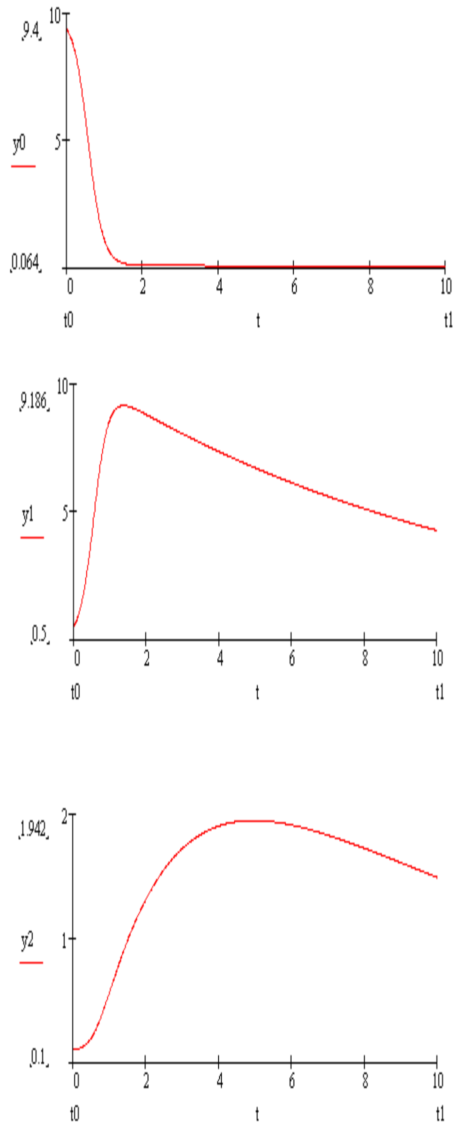


Fig. 4.5 Graphical profile for α = 0.5, σ = 0.132 (Low public campaign and education)

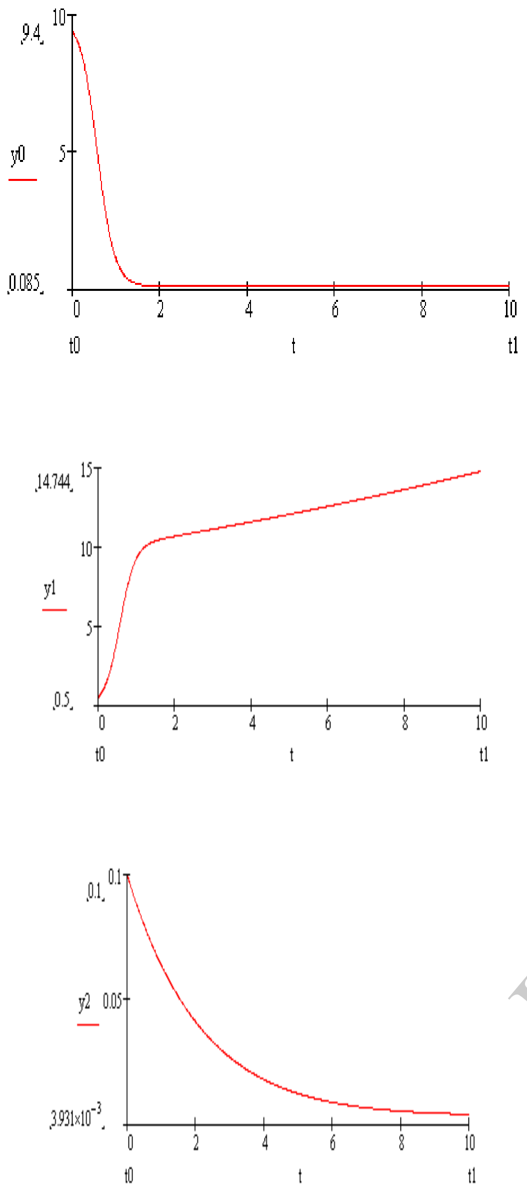


Fig. 4.6 Graphical Profile for α = 0.5, σ = 0.0001 (Effective anti-retroviral drugs, low public campaign and education).

3. ANALYSIS OF RESULT

In all the figures, we started with the initial population of P_0 = 10, S_0 = 9.4, L_0 = 0.5, I_0 = 0.1. Taking initial population in a small community as 3,950.249, this will corresponds to P_0 = 3,950,249, S_0 = 3,713,234, L_0 = 197,512, I_0 = 39,503 with parameter values β = 0.0523, μ = 0.0118, α = 0.053, δ = 0.5, σ = 0.132, θ = 0.812 at initial time t = 0

From fig. 4.1, it is seen that the susceptible population is heading to extinction as sharp reduction is noticed from time t = 4 to a level of 728,031 susceptible members at time t = 10. Initially, this class seems to be growing before time t = 4 but thereafter the contraction rate gets higher and consequently increased the latent population to 2,806, 257 and the infected population to 699,194 at time t = 10.

We now consider the possibilities of sustaining the population with increasing quantity and quality. Varying σ from 0.132 to 0.0001 meaning improving the efficacy of anti-retroviral drugs gives fig. 4.2. In this case, the contraction rate still persists higher gradually sending the susceptible and the infected populations to extinction with 535,654 and 1,303 members respectively, while the latent population is 5,324,936 all at time t = 10. This implies that if only administration of anti-retroviral drugs is emphasized, the population is tending towards risk of latent class domination.

In fig. 4.3, α is reduced from 0.053 to 0.0005 that is the effectiveness of public campaign and education is improved in the generated data. This shows increasing susceptible and reduction in both the latent and infected classes. Maintaining this trend will guarantee a HIV/AIDS free generation. Already, at time t = 10; P = 5,695,469, S = 5,617,649, L = 56,094 and I = 21,726, that is both the latent and the infected formed only 1.4% of the total population as against the initial 6%. In fig. 4.4, at time t = 10; P = 5,863,394, S = 5,657,547, L = 205,413 and I = 434.

Taking α = 0.5 gives fig. 4.5 which indicates a speedy decline in the susceptible population leading to initial sharp increase in both the latent and infected classes. At time t = 10, the susceptible population is 25,282 forming only 0.6% of the entire population. This is not a healthy population and will be too bad for any nation as there will be sudden extinction of the population. However, fig. 4.6 maintained α = 0.5 and make σ more efficacious at 0.0001, this still ensure sudden extinction of the susceptible as well as the infected classes and more population retained in the latent class.

4. CONCLUSION

The interplay in the parameters used indicated the need for strategic and comprehensive plan for HIV/AIDS prevention, care and support services for qualitative and quantitative population sustenance. This can be facilitated through adequate records kept and made assessable at designated centers and medium to help in researches to forecast the future trend of the pandemic and how to overcome it. It can be concluded therefore that to ensure safe population, emphasis should be on intensive and effective enlightenment campaign.

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