Gracefull Ness Of $P_k \circ 2_{C_k}$

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Abstract: In this paper, we obtained that the connected graph $P_k \Delta 2C_4$ is graceful.

Introduction:

Most graph labeling methods trace their origin to one introduced by Rosa [2] or one given Graham and Sloane [1]. Rosa defined a function f, a β-valuation of a graph with q edges if f is an injective map from the vertices of G to the set {0, 1, 2 ,…,q} such that when each edge xy is assigned the label $|f(x)-f(y)|$, the resulting edge labels are distinct.

A. Solairaju and K. Chitra [3] first introduced the concept of edge-odd graceful labeling of graphs, and edge-odd graceful graphs.

A. Solairaju and others [5,6,7] proved the results that(1) the Gracefulness of a spanning tree of the graph of Cartesian product of P_m and C_n,was obtained (2) the Gracefulness of a spanning tree of the graph of cartesian product of S_m and S_n, was obtained (3) edge-odd Gracefulness of a spanning tree of Cartesian product of P_2 and C_n was obtained (4) Even -edge Gracefulness of the Graphs was obtained (5) ladder P_2 x P_n is even-edge graceful, and (6) the even-edge gracefulness of $P_n \circ nC_5$ is obtained.

Section I : Preliminaries

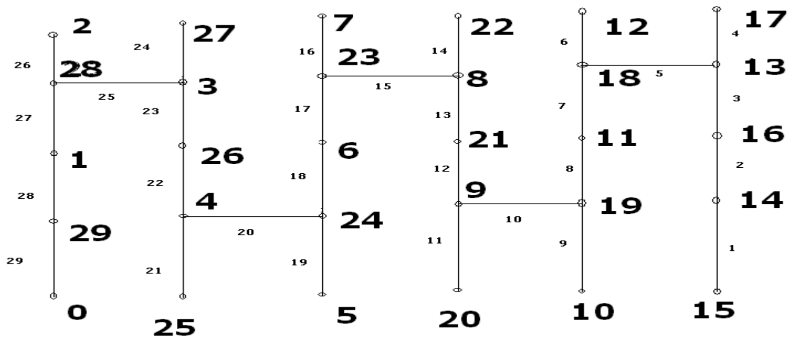
Definition 1.1: Let G = (V,E) be a simple graph with p vertices and q edges.

A map f :V(G) → {0,1,2,…,q} is called a graceful labeling if

(i) f is one – to – one
(ii) The edges receive all the labels (numbers) from 1 to q where the label of an edge is the absolute value of the difference between the vertex labels at its ends.

A graph having a graceful labeling is called a graceful graph.

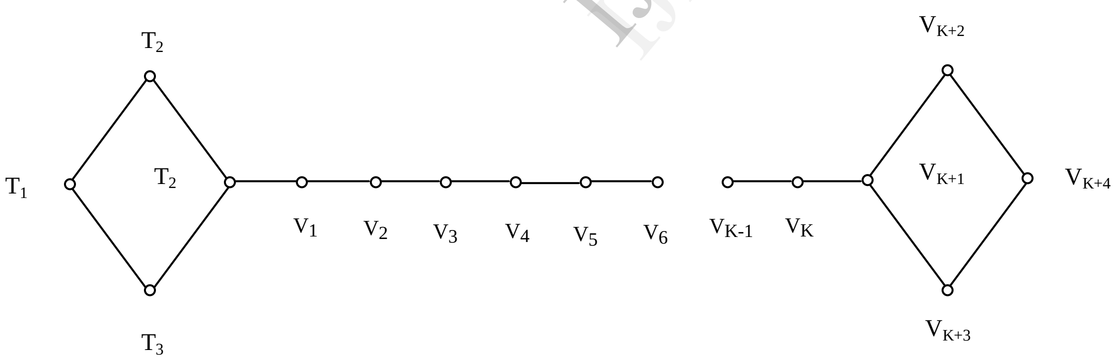
Example 1.1: The graph $6 \Delta P_5$ is a graceful graph.



Section II – Path merging with circulits of length four

Definition 2.1: $P_k \Delta 2C_4$ is a connected graph obtained by merging a circuit of length 4 with isolated vertex of a path of length k.

Theorem 2.1: The connected graph $P_k \Delta 2C_4$ is graceful.



Case (i): **k is even.**

Define f: V {1,…, q} by

$f(T_1) = 0;$ $f(T_2) = q,$ $f(T_3) = q-1,$ $f(T_4) = 2$

$$f(V_i) = \begin{cases} (q-2) - \left(\frac{i-1}{2}\right), \text{ i is odd, i} = 1,3, \dots, k+1 \\ \end{cases}$$

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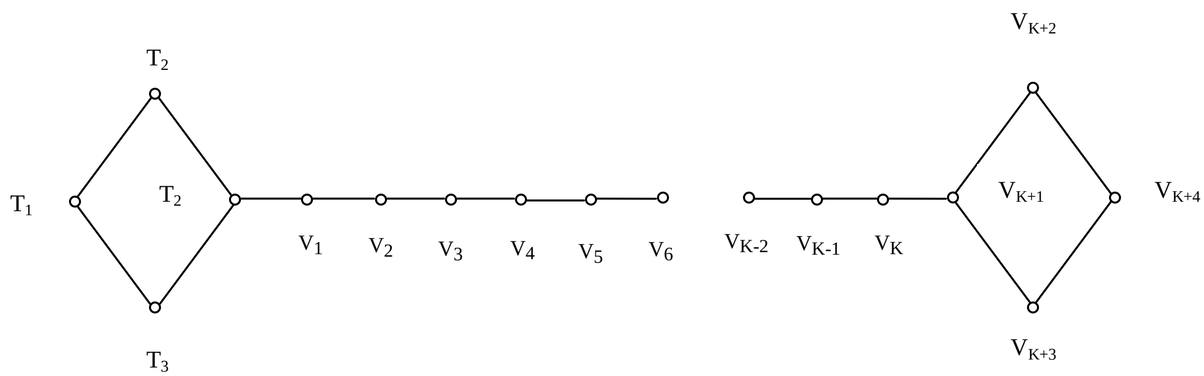
$(2+\frac{i}{2})$, i is even, i = 2,4,…, k+2

$f(V_{k+3}) = f(V_{k+2}) + 1$

$f(V_{k+4}) = f(V_{k+3}) + 1$

Case (ii): k is odd.



Define f: V {1,…, q}by

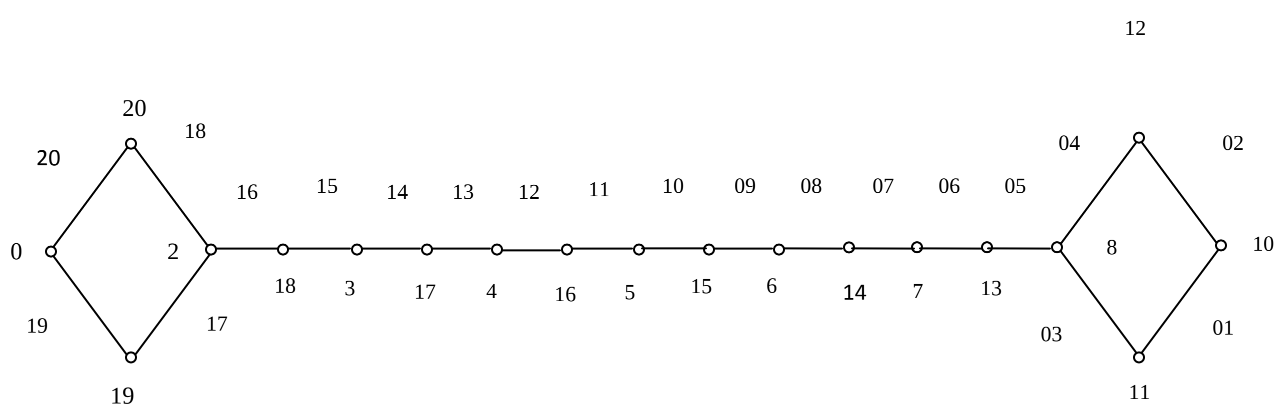
$f(T_1) = 0$; $f(T_2) = q$, $f(T_3) = q-1$, $f(T_4) = 2$

$$f(V_i) = \begin{cases} (q-2) - (\frac{i-1}{2}), & \text{i is odd, } i = 1,3, \ldots, k, k+2 \\ (2+\frac{i}{2}), & \text{i is even, } i = 2,4,\ldots, k+1 \end{cases}$$

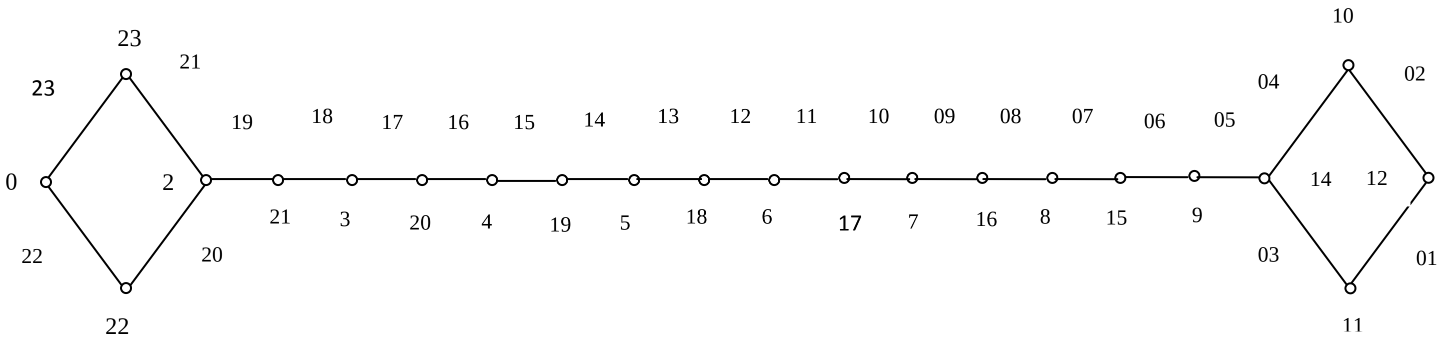
$f(V_{k+3}) = f(V_{k+2}) - 1$

$f(V_{k+4}) = f(V_{k+3}) - 1$

Example 2.1: k = 11 (odd) ; P: V |→ 19; Q: e |→ 20



Example 2.2: k =14 (even) ; P: V |→ 22; Q: e |→ 23



References:

1. R. L. Graham and N. J. A. Sloane, On additive bases and harmonious graph, SIAM J. Alg. Discrete Math., 1 (1980) 382 – 404.
2. A. Rosa, On certain valuation of the vertices of a graph, Theory of graphs (International Synposium,Rome,July 1966),Gordon and Breach, N.Y.and Dunod Paris (1967), 349-355.
3. A.Solairaju and K.Chitra Edge-odd graceful labeling of some graphs, Electronics Notes in Discrete Mathematics Volume 33,April 2009,Pages 1.
4. A. Solairaju and P.Muruganantham, even-edge gracefulness of ladder, The Global Journal of Applied Mathematics & Mathematical Sciences(GJ-AMMS). Vol.1.No.2, (July-December-2008):pp.149-153.
5. A. Solairaju and P.Sarangapani, even-edge gracefulness of P_n O nC_5, Preprint (Accepted for publication in Serials Publishers, New Delhi).
6. A.Solairaju, A.Sasikala, C.Vimala Gracefulness of a spanning tree of the graph of product of P_m and C_n, The **Global Journal** of Pure and Applied Mathematics of Mathematical Sciences, Vol. 1, No-2 (July-Dec 2008): pp 133-136.
7. A. Solairaju, C.Vimala,A.Sasikala Gracefulness of a spanning tree of the graph of Cartesian product of S_m and S_n, The **Global Journal** of Pure and Applied Mathematics of Mathematical Sciences, Vol. 1, No-2 (July-Dec 2008): pp117-120.