International Journal of Engineering Research & Technology (IJERT)
ISSN: 2278-0181
Vol. 3 Issue 4, April - 2014

GPS/INS Integreted Using Adaptive Kalman Filter

Ashim Pradhananga
Information Technology and Engineering
Tianjin University of Technology and Education (TUTE)
Tianjin, China

Prof. Zhang Xing Hui/ Asst. Prof. Li Shi Xin
Information Technology and Engineering / School of Electronic Engineering
Tianjin University of Technology and Education (TUTE)
Tianjin, China

***Abstract* --Inertial navigation is a self-contained navigation which are used to track the position and orientation of an object relative to a known starting point, orientation and velocity, like GPS. The INS can calculate the position of the aircraft without any help of other sources. Due, to a large numbers of error occurred by the sensors which leads to an unacceptable results. Therefore, GPS has been used to in INS using a Kalman filter. Where INS, Kalman Filter are widely used to increase the accuracy and reliability of the navigation solution. Usually, Direct Kalman filter is used for among quantities like velocity,position and attitude are among the state variable of the filter, which allows them to be estimate directly.**

Keywords—GPS/INS; Kalman Filter; Adaptive Kalman Filter; GPS; INS

I. INTRODUCTION

Most of these navigation systems use no longer only the Global Positioning System (GPS) but also an inertial navigation system (INS).Together the two systems complement each other and permit improved navigation accuracy and reliability especially when GPS is degraded or interrupted for example because of buildings or tunnels and for this application the Kalman filter provides the basis.
Most INS errors are attributed to the inertial sensors (instrument errors). These are the residual errors exhibited by the installed gyros and accelerometers following calibration of the INS such error leads to error in the velocity and position of an object. So unaided INS leads to error grow with time which is undesirable this is the reason we usually integrate INS with GPS. The primary role of GPS is to provide highly accurate position and velocity world-wide, based on range and range-rate measurements. GPS can be implemented in navigation as a fixing aid by being a part of an integrated navigation system, for example GPS/INS.

It constitutes a tool for correcting the predicted INS trajectory with GPS measurements. Also the determination of a reference orbit for these GPS satellites and correcting it with the data from the GPS control stations is a very important application of the Kalman filter. But these are only two examples of the wide variety of fields where Kalman filtering plays an important role. The application areas span from aerospace, to marine navigation, demographic modeling, weather science, manufacturing and many others. Because the Kalman filter is very effective and useful for such a large class of problems, it has been subject of extensive research.
The performance of the Kalman filter is crucial to overall system performance, especially when low-cost sensors are integrated. This paper focus on integrated GPS and Inertial Navigation System.

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II. NAVIGATION EQUATIONS

The navigation equation is important for Kalman filter and the algorithm. Where INS consists of the 3-axis gyroscope from where we can have pitch and yaw rates about the body axes. It also has 3-axis accelerometers which give the accelerations along the three body axes. There are two basic inertial mechanisms which are used to derive the Euler angles from the rate gyros, viz. stable platform and strap-down INS. The acceleration values from the accelerometers are then corrected for rotation of the earth and gravity to give the velocity and position of the aircraft.
The orientation of an aircraft with respect to a fixed inertial frame of axes is given by three Euler angles. The relationship between the angular rates of roll, pitch and yaw, p, q, r (measured by the body mounted gyro), the Euler angles, **ψ, θ, φ** and their relation are shown in figure:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (1)$$

To determine the attitude at a given time we can derive the Euler angles using initial condition from equation (1) considering for the pitch angles around ±90 the error becomes unbounded as tan θ tends to infinity. Where it depends on the parameters (e_0, e_1, e_2, e_3). Once we have calculated the time history of the four parameters, we can calculate the Euler angles using the following equations:

$$\theta = \sin^{-1}\left[-2(e_1e_3 - e_0e_2)\right] \quad (2)$$

$$\phi = \cos^{-1}\left[\frac{e_0^2 - e_1^2 - e_2^2 + e_3^2}{\sqrt{1-4(e_1e_3 - e_0e_2)^2}}\right] sign[2(e_2e_3 + e_0e_1)] \quad (3)$$

$$\psi = \cos^{-1}\left[\frac{e_0^2 + e_1^2 - e_2^2 - e_3^2}{\sqrt{1-4(e_1e_3 - e_0e_2)^2}}\right] sign[2(e_1e_2 + e_0e_3)] \quad (4)$$

We now have with us the attitude of the aircraft. To calculate the position we use the accelerations given by the accelerometers.

$$\dot{U} = a_x + Vr - Wq + g\sin\theta \quad (5)$$

$$\dot{V} = a_y - Ur + Wp - g\cos\theta\sin\phi \quad (6)$$

$$\dot{W} = a_z + Uq - Vp - g\cos\theta\cos\phi \quad (7)$$

Now the actual angular rate is given by

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}_m - DCM[\Omega + \omega^{-1}] \quad (8)$$

Where,
DCM is the direction cosine matrix or the transformation matrix which is given by

$$DCM = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\theta\sin\phi\cos\psi - \sin\psi\cos\phi & \sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi & \sin\phi\cos\theta \\ \sin\theta\cos\phi\cos\psi + \sin\psi\sin\phi & \sin\phi\sin\theta\cos\phi - \cos\psi\sin\theta & \cos\phi\cos\theta \end{bmatrix}$$

(9)

To Calculate the Velocities along with north, east and the downward velocity is given by the equation (9)

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix} = DCM^T \begin{bmatrix} U \\ V \\ W \end{bmatrix} \quad (10)$$

Whereas, $\begin{bmatrix} V_N = NorthVelocity \\ V_E = EastVelocity \\ V_D = DownwardVelocity \end{bmatrix}$

Now on this way we can calculate the rate of latitude λ, longitude μ and height H

$$\dot{\lambda} = \frac{V_N}{R_e} \qquad \dot{\mu} = \frac{V_E}{R_e\cos\lambda} \qquad \dot{H} = -V_D \quad (11)$$

Now,
The INS program now takes 6 states from this time history.After we obtain the row, pitch, yaw (viz. p, q, r, a_x, a_y, a_z) from the gyros and accelerometers. Then the program integrates and calculates the Euler angles are calculated using equations 2-4 Now the accelerations from the accelerometers are used to calculate $U^{\cdot}$,$V^{\cdot}$, $W^{\cdot}$ given by equations 5-7, which are then integrated to get the values of U , V , W. Now to calculate the velocity components in the body frame we have to convert the components into the navigation frame where we use the DCM matrix as in equation 8 and calculate velocities in equation 10. These velocities are then integrated to get the position (X, Y, and Z) along the axes. Hence, we can finally calculate the latitude, longitude and the height can be calculated using equation 11.
Similarly, the GPS give the latitude, longitude and altitude of the current location of the receiver through which we can calculate the F_{long} and F_{lat} in equation 12-13. Considering that earth has a semi major axis equatorial radius (a=6,378,137m) and with polar radius of (b= 6,356,752.3142m) where the distance corresponding to a 1^0 change in longitude F_{long} and latitude F_{lat} for a specific latitude λ and height h.

$$F_{long} = \frac{\pi}{180^0}\left(\frac{a^2}{\sqrt{a^2\cos^2\lambda + b^2\sin^2\lambda}} + h\right)\cos\lambda \quad (12)$$

$$F_{lat} = \frac{\pi}{180^0}\left(\frac{a^2b^2}{\sqrt{a^2\cos^2\lambda + b^2\sin^2\lambda}} + h\right) \quad (13)$$

Hence, the latitude and longitude at the current location (λ2, μ2) can be calculated from the latitude and longitude from the previous location (λ1, μ1) in the following manner

$$\lambda_2 = \frac{\delta X}{F_{lat}} + \lambda_1 \quad (14)$$

$$\mu_2 = \frac{\delta Y}{F_{lon}} + \mu_1 \quad (15)$$

Where, δX and δY are the changes in position along North direction and East direction on the earth, respectively.

III. KALMAN FILTER MOUDLE

Kalman filter is the best procedure for combining noisy sensor output to estimate the state of a system with uncertain dynamics. The noisy sensor output include output from the

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GPS and INS which include the position, velocity, attitude and attitude rate of the object and uncertain dynamics caused by a human error or any medium. The Kalman filter used to estimate the error introduce into the unaided INS system due to the gyro and accelerometer. In this model the error from the INS state vector $\hat{x}^{-}_{k}$ and the measured value of the state vector from the GPS forms a measurement vector Z. The GPS/INS integrate system configuration and the error are wither compensated by the two mechanism as feedback mechanism or feedback mechanism as shown in figure I and II.

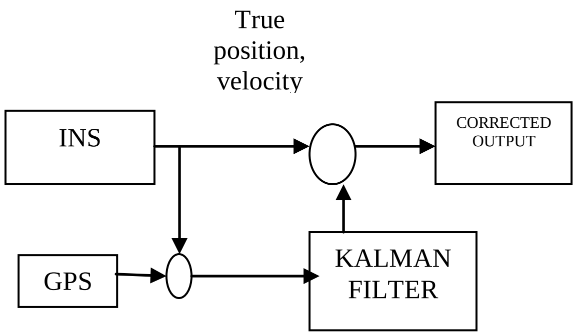

Fig I: Feed forward aided INS

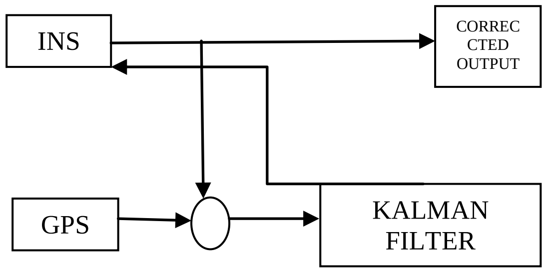

Fig II: Feed backward aided INS

IV. ADAPTIVEKALMANFILTERING ALGORITHMS

A number of adaptive Kalman filtering techniques exist to achieve the criteria described in adapting the stochastic properties of the Kalman Filter. Most techniques use the innovation sequence as the basis for adapting the measurement noise covariance, Rk, and the process noise covariance, Qk. These methods use a windowing function on the most recent innovations. Correct identification of the window size also needs to be identified to obtain the correct balance between filter adaptivity and stability. Multiple Model Kalman filtering is also described as a potential method for filter adaptation.

a. *Covariance Scaling.*

The covariance scaling method was used for improving the stochastic modeling of deferential pseudo-range GPS. The predicted covariance, P(k+x)1 , is artificially scaled by the factor Sk>1 to apply more weight to the measurements:

$$P_k^{(+)} = S_k\left(\Phi P_{k-1}^{(-)}\Phi^T + Q_{k-1}\right) \qquad (16)$$

Deferent techniques can be used in estimating Sk. For example, a priori methods can be used in alignment of low cost IMUs, where it is known that the inertial sensor errors will be larger before the system has been aligned.

b. *Adaptive Kalman Filter.*

The principle of the Adaptive Kalman Filter (AKF) is to make the Kalman filter residuals consistent with their theoretical co-variances (Mehra, 1972). An estimate of the covariance of the innovation residual is obtained by averaging the previous residual sequence over a window length N:

$$C_{v_k^{(-)}} = \frac{1}{N}\sum V_j^{(-)}\left(v_j^{(-)}\right)^T \qquad (17)$$

Where: $v_j^{(-1)} = z_k - H_k x_k^{(-)}$ the innovation residual from the Kalman filter. The estimated measurement noise is computed by comparing the theoretical covariance $(H_k P_k^{(-)} H^T_k + R_k)$ with the estimated covariance to give :

$$\hat{R}_k = C_{v_k^{(-)}} - H_k P_k^{(-)} H_k^T \qquad (18)$$

The estimated process noise can also be to give:

$$\hat{Q}_k = \frac{1}{N}\sum_{j=k-N+1}^{k}\Delta x_j \Delta x_j^T + P_k^{(+)} - \Phi P_{k-1}^{(+)}\Phi^T \qquad (19)$$

Where, $\Delta x_k = x_k^{(-)} - x_k^{(+)}$.This is known as a residual based estimate. Equation (19) can be written in terms of the innovation sequence by making the following substitution for the covariance of the state corrections.

$$\hat{C}_{\Delta x_k} \approx \frac{1}{N}\sum_{j=k-N+1}^{k}\Delta x_j \Delta x_j^T \approx K_k \hat{C}_{v_k^{(-)}} K_k^T \qquad (20)$$

V. RESULT

The graphs for attitude computed and corrected by the adaptive Kalman filter are given in Figure III. We cannot expect the Kalman filter to correct the attitude, velocity and positions given by the INS perfectly. This corrected attitude forms a part of the integration loop in the whole system. In figure III, the Red line denotes the output obtained by adaptive kalman filter where as the blue line is the output obtained by kalman filter respectively. As can be seen from the graph, the error of adaptive kalman filter was smaller than the error of kalman filter.

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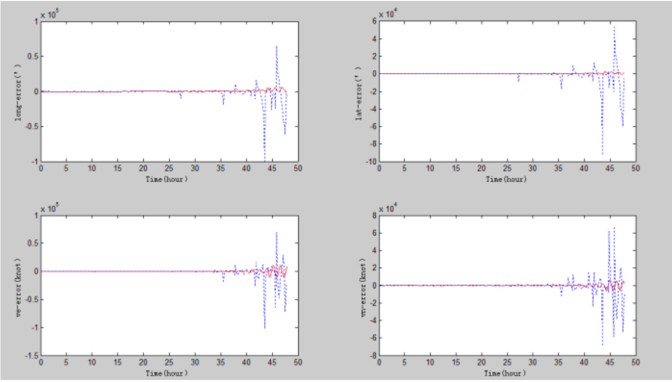

Figure III: longitude, latitude and velocity error obtained by the Kalman Filter and Adaptive Kalman Filter

In the results, the X-axis shows the time period and the Y-axis shows the estimated error of the longitude, the latitude, the eastward velocity and the northward velocity.

VI. CONCLUSION

This paper has shown that how adaptive Kalman filter can be configured. The innovation and residual sequences provide a useful performance indicator that can be used to adaptively tune the stochastic information used in the filter. This paper has shown the use of adaptive filtering techniques to improve the error. The integration of INS and GPS is generally implemented through a adaptive Kalman flter. This filter showed a major improvement over the kalman filter. The performance of the adaptive Kalman filter, for most of the navigation parameters used in this study, is improved by almost 50% or more when compared to that of the kalman filter.

In this paper, an adaptive Kalman flter, based on the flter innovation sequence, is introduced as an alternative for integrating INS/GPS systems.

IJERT

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