

# $g'''$ -Closed and $g'''$ -Open Maps in Fuzzy Topological Spaces

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**Abstract:** A fuzzy set  $A$  in a fuzzy topological space  $(X, \tau)$  is said to be fuzzy  $g'''$ -closed set if  $\text{cl}(A) \leq U$  whenever  $A \leq U$  and  $U$  is fgs-open in  $(X, \tau)$ . In this paper, we introduce fuzzy  $g'''$ -closed map from a fuzzy topological space  $X$  to a fuzzy topological space  $Y$  as the map which maps every fuzzy closed set to a fuzzy  $g'''$ -closed, and notice that the composition of two fuzzy  $g'''$ -closed maps need not be fuzzy  $g'''$ -closed map. We also obtain some properties of fuzzy  $g'''$ -closed maps.

**Key words :** Fuzzy Topological space, fuzzy  $g'''$ -closed map, fuzzy  $g'''$ \*-closed map and fuzzy  $g'''$ -open map, fuzzy  $g'''$ \*-open map.

## I. INTRODUCTION

In the classical paper [19] of 1965, L.A.Zadeh generalized the usual notion of a set by introducing the important and useful notion of fuzzy sets. Subsequently many researchers have worked on various basic concepts from general topology using fuzzy sets and developed the theory of fuzzy topological spaces. The notion of fuzzy sets naturally plays a very significant role in the study of fuzzy topology introduced by C. L. Chang [5].

Fuzzy continuous functions is one of the main topics in fuzzy topology. Various authors introduce various types of fuzzy continuity. The decomposition of fuzzy continuity is one of the many problems in fuzzy topology. Tong [16] obtained a decomposition of fuzzy continuity by introducing two weak notions of fuzzy continuity namely, fuzzy strong semi-continuity and fuzzy precontinuity. Rajamani [9] obtained a decomposition of fuzzy continuity.

In this section, we introduce fuzzy  $g'''$ -closed maps, fuzzy  $g'''$ -open maps, fuzzy  $g'''$ \*-closed maps and

fuzzy  $g'''$ \*-open maps in fuzzy topological spaces and obtain certain characterizations of these maps.

## II. PRELIMINARIES

Throughout this paper,  $(X, \tau)$ ,  $(Y, \sigma)$  and  $(Z, \eta)$  (or  $X$ ,  $Y$  and  $Z$ ) represent fuzzy topological spaces on which no separation axioms are assumed unless otherwise mentioned. For any fuzzy subset  $A$  of a space  $(X, \tau)$ , the closure of  $A$ , the interior of  $A$  and the complement of  $A$  are denoted by  $\text{cl}(A)$ ,  $\text{int}(A)$  and  $A^c$  respectively.

We recall the following definitions which are useful in the sequel.

### Definition 2.1

A fuzzy subset  $A$  of a space  $(X, \tau)$  is called: fuzzy semi-open set [1] if  $A \leq \text{cl}(\text{int}(A))$

The complement of fuzzy semi-open set is fuzzy semi-closed.

The fuzzy semi-closure [18] of a fuzzy subset  $A$  of  $X$ , denoted by  $\text{scl}(A)$ , is defined to be the intersection of all fuzzy semi-closed sets of  $(X, \tau)$  containing  $A$ . It is known that  $\text{scl}(A)$  is a fuzzy semi-closed set. For any fuzzy subset  $A$  of an arbitrarily chosen fuzzy topological space, the fuzzy semi-interior [18] of  $A$ , denoted by  $\text{sint}(A)$ , is defined to be the union of all fuzzy semi-open sets of  $(X, \tau)$  contained in  $A$ .

### Definition 2.2

A fuzzy subset  $A$  of a space  $(X, \tau)$  is called:

- (i) a fuzzy generalized closed (briefly fg-closed) set [2] if  $\text{cl}(A) \leq U$  whenever  $A \leq U$  and  $U$  is fuzzy open in  $(X, \tau)$ . The complement of fg-closed set is called fg-open set;
- (ii) a fuzzy  $\omega$ -closed set (= f $\omega$ -closed set) [13] if  $\text{cl}(A) \leq U$  whenever  $A \leq U$  and  $U$  is fuzzy semi-

- open in  $(X, \tau)$ . The complement of fuzzy  $\omega$ -closed set is called fuzzy  $\omega$ -open set;
- (iii) a fuzzy semi-generalized closed (briefly fsg-closed) set [3] if  $scl(A) \leq U$  whenever  $A \leq U$  and  $U$  is fuzzy semi-open in  $(X, \tau)$ . The complement of fsg-closed set is called fsg-open set;
- (iv) a generalized fuzzy semi-closed (briefly fgs-closed) set [11] if  $scl(A) \leq U$  whenever  $A \leq U$  and  $U$  is fuzzy open in  $(X, \tau)$ . The complement of fgs-closed set is called fgs-open set;
- (v) a fuzzy  $g'''$ -closed set [7] if  $cl(A) \leq U$  whenever  $A \leq U$  and  $U$  is fgs-open in  $(X, \tau)$ . The complement of fuzzy  $g'''$ -closed set is called fuzzy  $g'''$ -open set;
- (vi) a fuzzy  $g^*s$ -closed set [7] if  $scl(A) \leq U$  whenever  $A \leq U$  and  $U$  is fgs-open in  $(X, \tau)$ . The complement of  $fg^*s$ -closed set is called  $fg^*s$ -open set.

The collection of all fuzzy  $g'''$ -closed sets is denoted by  $G'''C(X)$ .

**Remark 2.3[7]**

Every fuzzy closed set is fuzzy  $g'''$ -closed set but not conversely.

**Example 2.4**

Let  $X = \{a, b\}$  with  $\tau = \{0_x, A, 1_x\}$  where  $A$  is fuzzy set in  $X$  defined by  $A(a)=1, A(b)=0$ . Then  $(X, \tau)$  is a fuzzy topological space. Clearly  $B$  defined by  $B(a)=0.5, B(b)=1$  is fuzzy  $g'''$ -closed set but not fuzzy closed set.

**Lemma 2.5[5]**

Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be a fuzzy function. For fuzzy sets  $A$  and  $B$  of  $X$  and  $Y$  respectively, the following statements hold:

- (i)  $ff^{-1}(B) \leq B$ ;
- (ii)  $f^{-1}f(A) \geq A$ ;
- (iii)  $f(A') \geq (f(A))'$ ;
- (iv)  $f^{-1}(B') = (f^{-1}(B))'$ ;
- (v) if  $f$  is injective, then  $f^{-1}(f(A)) = A$ ;
- (vi) if  $f$  is surjective, then  $f f^{-1}(B) = B$ ;
- (vii) if  $f$  is bijective, then  $f(A') = (f(A))'$ .

**Definition 2.6**

A fuzzy function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called

- (i) fuzzy closed [5] if the image of every fuzzy closed set of  $X$  is a fuzzy closed in  $Y$ .
- (ii) fuzzy open [5] if the image of every fuzzy open set of  $X$  is a fuzzy open in  $Y$ .
- (iii) fuzzy continuous [5] if the inverse image of every fuzzy open set in  $(Y, \sigma)$  is a fuzzy open set in  $(X, \tau)$ .
- (iv) fuzzy  $\omega$ -continuous [13] if the inverse image of every fuzzy closed Set in  $(Y, \sigma)$  is a  $f\omega$ -closed set in  $(X, \tau)$ .

III. FUZZY  $g'''$ -INTERIOR AND FUZZY  $g'''$ -CLOSURE

**Definition 3.1**

(i) For any fuzzy subset  $A$  of  $X$ , fuzzy  $g'''$ -int( $A$ ) is defined as the union of all fuzzy  $g'''$ -open sets contained in  $A$ .

i.e.,  $f g'''$ -int( $A$ ) =  $\bigvee \{G : G \leq A \text{ and } G \text{ is fuzzy } g'''$ -open\}.

(ii) For every fuzzy subset  $A$  of  $X$ , we define the fuzzy  $g'''$ -closure of  $A$  to be the intersection of all fuzzy  $g'''$ -closed sets containing  $A$ .

In symbols,  $f g'''$ -cl( $A$ ) =  $\bigwedge \{F : A \leq F \in F G'''C(X)\}$ .

**Definition 3.2**

Let  $(X, \tau)$  be a fuzzy topological space. Let  $G$  be a fuzzy subset of  $X$ . Then  $G$  is called an fuzzy  $g'''$ -neighborhood of  $A$  (briefly,  $f g'''$ -nbhd of  $A$ ) iff there exists an  $f g'''$ -open set  $U$  of  $X$  such that  $A < U < G$ .

**Definition 3.3**

A fuzzy topological space  $(X, \tau)$  is called a

- (i)  $T f\omega$ -space if every  $f\omega$ -closed set in it is fuzzy closed.
- (ii)  $T f g'''$ -space if every  $f g'''$ -closed set in it is fuzzy closed.

**Example 3.4**

Let  $X = \{a, b\}$  with  $\tau = \{0_x, \lambda, 1_x\}$  where  $\lambda$  is fuzzy set in  $X$  defined by  $\lambda(a)=0.4, \lambda(b)=0.5$ . Then  $(X, \tau)$  is a fuzzy topological space. Clearly  $(X, \tau)$  is a  $T f\omega$ -space.

**Example 3.5**

Let  $X = \{a, b\}$  with  $\tau = \{0_x, \lambda, 1_x\}$  where  $\lambda$  is fuzzy set in  $X$  defined by  $\lambda(a)=0.5, \lambda(b)=0.5$ . Then  $(X, \tau)$  is a fuzzy topological space. Clearly  $(X, \tau)$  is a  $T f g'''$ -space.

**Definition 3.6**

A fuzzy map  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called

- (i) fuzzy  $g'''$ -continuous [8] if the inverse image of every fuzzy closed set in  $(Y, \sigma)$  is  $f g'''$ -closed in  $(X, \tau)$ .
- (ii) fuzzy  $g'''$ -irresolute if the inverse image of every  $f g'''$ -closed set in  $(Y, \sigma)$  is  $f g'''$ -closed in  $(X, \tau)$ .
- (iii) strongly fuzzy  $g'''$ -continuous if the inverse image of every  $f g'''$ -open set in  $(Y, \sigma)$  is fuzzy open in  $(X, \tau)$ .
- (iv) fuzzy  $g'''$ -irresolute if  $f^{-1}(V)$  is fgs-open in  $(X, \tau)$  for every fgs-open subset  $V$  in  $(Y, \sigma)$ .

**Proposition 3.7**

If  $A$  is fuzzy  $g'''$ -open, then fuzzy  $g'''$ -int( $A$ ) =  $A$ . But the converse is not true.

**Proposition 3.8**

If  $A$  is fuzzy  $g'''$ -closed, then fuzzy  $g'''$ -cl( $A$ ) =  $A$ . But the converse is not true.

**Proposition 3.9**

For any two fuzzy subsets A and B of  $(X, \tau)$ , the following hold:

- (i) If  $A \leq B$ , then  $f g''' - \text{cl}(A) \leq f g''' - \text{cl}(B)$ .
- (ii)  $f g''' - \text{cl}(A \wedge B) \leq f g''' - \text{cl}(A) \wedge f g''' - \text{cl}(B)$ .

**Definition 3.10**

A fuzzy map  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called:

- (i) fg-closed if  $f(V)$  is fg-closed in  $(Y, \sigma)$  for every fuzzy closed set  $V$  of  $(X, \tau)$ .
- (ii) fsg-closed if  $f(V)$  is fsg-closed in  $(Y, \sigma)$  for every fuzzy closed set  $V$  of  $(X, \tau)$ .
- (iii) fgs-closed if  $f(V)$  is fgs-closed in  $(Y, \sigma)$  for every fuzzy closed set  $V$  of  $(X, \tau)$ .
- (iv) fg\*s-closed if  $f(V)$  is fg\*s-closed in  $(Y, \sigma)$  for every fuzzy closed set  $V$  of  $(X, \tau)$ .

IV. FUZZY  $g'''$ -CLOSED MAPS

We introduce the following definition:

**Definition 4.1**

A fuzzy map  $f : (X, \tau) \rightarrow (Y, \sigma)$  is said to be fuzzy  $g'''$ -closed if the image of every fuzzy closed set in  $(X, \tau)$  is  $f g'''$ -closed in  $(Y, \sigma)$ .

**Example 4.2**

Let  $X = Y = \{a, b\}$  with  $\tau = \{0_x, \alpha, 1_x\}$  where  $\alpha(a) = 0.5, \alpha(b) = 0$  and  $\sigma = \{0_y, \beta, 1_y\}$  where  $\beta(a) = 1, \beta(b) = 0$ . Then  $(X, \tau)$  and  $(Y, \sigma)$  are fuzzy topological spaces. Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be the identity fuzzy map. Clearly  $f$  is a fuzzy  $g'''$ -closed map.

**Proposition 4.3**

A fuzzy map  $f : (X, \tau) \rightarrow (Y, \sigma)$  is fuzzy  $g'''$ -closed if and only if  $f g''' - \text{cl}(f(A)) \leq f(\text{cl}(A))$  for every fuzzy subset  $A$  of  $(X, \tau)$ .

**Proof**

Suppose that  $f$  is fuzzy  $g'''$ -closed and  $A$  is a fuzzy subset of  $X$ . Then  $\text{cl}(A)$  is fuzzy closed in  $X$  and so  $f(\text{cl}(A))$  is fuzzy  $g'''$ -closed in  $(Y, \sigma)$ . We have  $f(A) \leq f(\text{cl}(A))$  and by Propositions 3.8 and 3.9,  $f g''' - \text{cl}(f(A)) \leq f g''' - \text{cl}(f(\text{cl}(A))) = f(\text{cl}(A))$ .

Conversely, let  $A$  be any fuzzy closed set in  $(X, \tau)$ . Then  $A = \text{cl}(A)$  and so  $f(A) = f(\text{cl}(A)) \geq f g''' - \text{cl}(f(A))$ , by hypothesis. We have  $f(A) \leq f g''' - \text{cl}(f(A))$  by Proposition 3.8. Therefore  $f(A) = f g''' - \text{cl}(f(A))$ , i.e.,  $f(A)$  is  $f g'''$ -closed by Proposition 3.8 and hence  $f$  is  $f g'''$ -closed.

**Proposition 4.4**

Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be a fuzzy map such that  $f g''' - \text{cl}(f(A)) \leq f(\text{cl}(A))$  for any fuzzy closed subset  $A$  of  $X$ . Then the image  $f(A)$  is  $f g'''$ -closed in  $(Y, \sigma)$ .

**Proof**

Let  $A$  be a fuzzy closed set in  $(X, \tau)$ . Then by hypothesis  $f g''' - \text{cl}(f(A)) \leq f(\text{cl}(A)) = f(A)$  and so  $f g''' - \text{cl}(f(A)) = f(A)$ . Therefore  $f(A)$  is  $f g'''$ -closed in  $(Y, \sigma)$ .

**Theorem 4.5**

A fuzzy map  $f : (X, \tau) \rightarrow (Y, \sigma)$  is fuzzy  $g'''$ -closed if and only if for each fuzzy subset  $S$  of  $(Y, \sigma)$  and each fuzzy open set  $U$  containing  $f^{-1}(S)$  there is a fuzzy  $g'''$ -open set  $V$  of  $(Y, \sigma)$  such that  $S \leq V$  and  $f^{-1}(V) \leq U$ .

**Proof**

Suppose  $f$  is fuzzy  $g'''$ -closed. Let  $S$  be a fuzzy subset of  $Y$  and  $U$  be a fuzzy open set of  $(X, \tau)$  such that  $f^{-1}(S) \leq U$ . Then  $V = (f(U))^c$  is a fuzzy  $g'''$ -open set containing  $S$  such that  $f^{-1}(V) \leq U$ .

For the converse, let  $F$  be a fuzzy closed set of  $(X, \tau)$ . Then  $f^{-1}((f(F))^c) \leq F^c$  and  $F^c$  is fuzzy open. By assumption, there exists a fuzzy  $g'''$ -open set  $V$  in  $(Y, \sigma)$  such that  $(f(F))^c \leq V$  and  $f^{-1}(V) \leq F^c$  and so  $F \leq (f^{-1}(V))^c$ . Hence  $V^c \leq f(F) \leq f((f^{-1}(V))^c) \leq V^c$  which implies  $f(F) = V^c$ . Since  $V^c$  is fuzzy  $g'''$ -closed,  $f(F)$  is  $f g'''$ -closed and therefore  $f$  is  $f g'''$ -closed.

**Proposition 4.6**

If  $f : (X, \tau) \rightarrow (Y, \sigma)$  is fuzzy gs-irresolute fuzzy  $g'''$ -closed and  $A$  is a fuzzy  $g'''$ -closed subset of  $(X, \tau)$ , then  $f(A)$  is fuzzy  $g'''$ -closed in  $(Y, \sigma)$ .

**Proof**

Let  $U$  be a fgs-open set in  $(Y, \sigma)$  such that  $f(A) \leq U$ . Since  $f$  is fgs-irresolute,  $f^{-1}(U)$  is a fgs-open set containing  $A$ . Hence  $\text{cl}(A) \leq f^{-1}(U)$  as  $A$  is fuzzy  $g'''$ -closed in  $(X, \tau)$ . Since  $f$  is fuzzy  $g'''$ -closed,  $f(\text{cl}(A))$  is a fuzzy  $g'''$ -closed set contained in the fgs-open set  $U$ , which implies that  $\text{cl}(f(\text{cl}(A))) \leq U$  and hence  $\text{cl}(f(A)) \leq U$ . Therefore,  $f(A)$  is a fuzzy  $g'''$ -closed set in  $(Y, \sigma)$ .

The following example shows that the composition of two fuzzy  $g'''$ -closed maps need not be fuzzy  $g'''$ -closed.

**Example 4.7**

Let  $X = Y = Z = \{a, b\}$  with  $\tau = \{0_x, \alpha, 1_x\}$  where  $\alpha(a) = 0.4, \alpha(b) = 0$ ,  $\sigma = \{0_y, \beta, 1_y\}$  where  $\beta(a) = 1, \beta(b) = 0$  and  $\eta = \{0_z, \gamma, \beta, 1_z\}$  where  $\gamma(a) = 0.5, \gamma(b) = 0$ . Then  $(X, \tau), (Y, \sigma)$  and  $(Z, \eta)$  are fuzzy topological spaces. Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be the identity map and  $g : (Y, \sigma) \rightarrow (Z, \eta)$  be the identity fuzzy map. Clearly both  $f$  and  $g$  are fuzzy  $g'''$ -closed maps but their composition  $g \circ f : (X, \tau) \rightarrow (Z, \eta)$  is not a fuzzy  $g'''$ -closed map.

**Corollary 4.8**

Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be fuzzy  $g'''$ -closed and  $g : (Y, \sigma) \rightarrow (Z, \eta)$  be fuzzy  $g'''$ -closed and fuzzy gs-irresolute, then their composition  $g \circ f : (X, \tau) \rightarrow (Z, \eta)$  is fuzzy  $g'''$ -closed.

**Proof**

Let  $A$  be a fuzzy closed set of  $(X, \tau)$ . Then by hypothesis  $f(A)$  is a fuzzy  $g'''$ -closed set in  $(Y, \sigma)$ . Since  $g$  is both fuzzy  $g'''$ -closed and fgs-fuzzy irresolute by Proposition 4.6,  $g(f(A)) = (g \circ f)(A)$  is fuzzy  $g'''$ -closed in  $(Z, \eta)$  and therefore  $g \circ f$  is fuzzy  $g'''$ -closed.

**Proposition 4.9**

Let  $f : (X, \tau) \rightarrow (Y, \sigma)$ ,  $g : (Y, \sigma) \rightarrow (Z, \eta)$  be fuzzy  $g'''$ -closed maps and  $(Y, \sigma)$  be a  $Tfg'''$ -space. Then their composition  $g \circ f : (X, \tau) \rightarrow (Z, \eta)$  is fuzzy  $g'''$ -closed.

**Proof**

Let  $A$  be a fuzzy closed set of  $(X, \tau)$ . Then by assumption  $f(A)$  is fuzzy  $g'''$ -closed in  $(Y, \sigma)$ . Since  $(Y, \sigma)$  is a  $Tfg'''$ -space,  $f(A)$  is fuzzy closed in  $(Y, \sigma)$  and again by assumption  $g(f(A))$  is fuzzy  $g'''$ -closed in  $(Z, \eta)$ . i.e.,  $(g \circ f)(A)$  is fuzzy  $g'''$ -closed in  $(Z, \eta)$  and so  $g \circ f$  is fuzzy  $g'''$ -closed.

**Proposition 4.10**

If  $f : (X, \tau) \rightarrow (Y, \sigma)$  is fuzzy  $g'''$ -closed,  $g : (Y, \sigma) \rightarrow (Z, \eta)$  is fuzzy  $g'''$ -closed (resp. fg-closed, fg\*s-closed, fsg-closed and fgs-closed) and  $(Y, \sigma)$  is a  $Tfg'''$ -space, then their composition  $g \circ f : (X, \tau) \rightarrow (Z, \eta)$  is fuzzy  $g'''$ -closed (resp. fg-closed, fg\*s-closed, fsg-closed and fgs-closed).

**Proof**

Similar to Proposition 4.9.

**Proposition 4.11**

Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be a fuzzy closed map and  $g : (Y, \sigma) \rightarrow (Z, \eta)$  be an fuzzy  $g'''$ -closed map, then their composition  $g \circ f : (X, \tau) \rightarrow (Z, \eta)$  is fuzzy  $g'''$ -closed.

**Proof**

Similar to Proposition 4.9.

**Remark 4.12**

If  $f : (X, \tau) \rightarrow (Y, \sigma)$  is a fuzzy  $g'''$ -closed and  $g : (Y, \sigma) \rightarrow (Z, \eta)$  is fuzzy closed, then their composition need not be a fuzzy  $g'''$ -closed map as seen from the following example.

**Example 4.13**

Let  $X = Y = Z = \{a, b\}$  with  $\tau = \{0_x, \alpha, 1_x\}$  where  $\alpha(a) = 0.4, \alpha(b) = 0$ ,  $\sigma = \{0_y, \beta, 1_y\}$  where  $\beta(a) = 1, \beta(b) = 0$  and  $\eta = \{0_z, \gamma, \beta, 1_z\}$  where  $\gamma(a) = 0.5, \gamma(b) = 0$ . Then  $(X, \tau)$ ,  $(Y, \sigma)$  and  $(Z, \eta)$  are fuzzy topological spaces. Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be the identity fuzzy map and  $g : (Y, \sigma) \rightarrow (Z, \eta)$  be the identity fuzzy map. Clearly both  $f$  is a fuzzy  $g'''$ -closed map and  $g$  is fuzzy closed map but their composition

$g \circ f : (X, \tau) \rightarrow (Z, \eta)$  is not an fuzzy  $g'''$ -closed map.

Analogous to a fuzzy  $g'''$ -closed map, we define a fuzzy  $g'''$ -open map as follows:

**Definition 4.14**

A fuzzy map  $f : (X, \tau) \rightarrow (Y, \sigma)$  is said to be a fuzzy  $g'''$ -open map if the image  $f(A)$  is fuzzy  $g'''$ -open in  $(Y, \sigma)$  for each fuzzy open set  $A$  in  $(X, \tau)$ .

**Proposition 4.15**

For any fuzzy bijection  $f : (X, \tau) \rightarrow (Y, \sigma)$ , the following statements are equivalent:

- (i)  $f^{-1} : (Y, \sigma) \rightarrow (X, \tau)$  is fuzzy  $g'''$ -continuous.
- (ii)  $f$  is fuzzy  $g'''$ -open map.
- (iii)  $f$  is fuzzy  $g'''$ -closed map.

**Proof**

(i)  $\Rightarrow$  (ii). Let  $U$  be a fuzzy open set of  $(X, \tau)$ . By assumption,  $(f^{-1})^{-1}(U) = f(U)$  is fuzzy  $g'''$ -open in  $(Y, \sigma)$  and so  $f$  is fuzzy  $g'''$ -open.

(ii)  $\Rightarrow$  (iii). Let  $F$  be a fuzzy closed set of  $(X, \tau)$ . Then  $F^c$  is fuzzy open set in  $(X, \tau)$ . By assumption,  $f(F^c)$  is fuzzy  $g'''$ -open in  $(Y, \sigma)$ . That is  $f(F^c) = (f(F))^c$  is fuzzy  $g'''$ -open in  $(Y, \sigma)$  and therefore  $f(F)$  is fuzzy  $g'''$ -closed in  $(Y, \sigma)$ . Hence  $f$  is fuzzy  $g'''$ -closed.

(iii)  $\Rightarrow$  (i). Let  $F$  be a fuzzy closed set of  $(X, \tau)$ . By assumption,  $f(F)$  is fuzzy  $g'''$ -closed in  $(Y, \sigma)$ . But  $f(F) = (f^{-1})^{-1}(F)$  and therefore  $f^{-1}$  is fuzzy  $g'''$ -continuous.

**Theorem 4.16**

A fuzzy map  $f : (X, \tau) \rightarrow (Y, \sigma)$  is  $f g'''$ -open if and only if for any fuzzy subset  $S$  of  $(Y, \sigma)$  and for any fuzzy closed set  $F$  containing  $f^{-1}(S)$ , there exists an  $f g'''$ -closed set  $K$  of  $(Y, \sigma)$  containing  $S$  such that  $f^{-1}(K) \leq F$ .

**Proof**

Similar to Theorem 4.5.

**Corollary 4.17**

A fuzzy map  $f : (X, \tau) \rightarrow (Y, \sigma)$  is fuzzy  $g'''$ -open if and only if  $f^{-1}(g'''\text{-cl}(B)) \leq \text{cl}(f^{-1}(B))$  for each fuzzy subset  $B$  of  $(Y, \sigma)$ .

**Proof**

Suppose that  $f$  is fuzzy  $g'''$ -open. Then for any fuzzy subset  $B$  of  $Y$ ,  $f^{-1}(B) \leq \text{cl}(f^{-1}(B))$ . By Theorem 4.16, there exists a fuzzy  $g'''$ -closed set  $K$  of  $(Y, \sigma)$  such that  $B \leq K$  and  $f^{-1}(K) \leq \text{cl}(f^{-1}(B))$ . Therefore,  $f^{-1}(g'''\text{-cl}(B)) \leq (f^{-1}(K)) \leq \text{cl}(f^{-1}(B))$ , since  $K$  is a fuzzy  $g'''$ -closed set in  $(Y, \sigma)$ .

Conversely, let  $S$  be any fuzzy subset of  $(Y, \sigma)$  and  $F$  be any fuzzy closed set containing  $f^{-1}(S)$ . Put  $K = g'''\text{-cl}(S)$ . Then  $K$  is a fuzzy  $g'''$ -closed set and  $S \leq K$ . By assumption,  $f^{-1}(K) = f^{-1}(g'''\text{-cl}(S)) \leq \text{cl}(f^{-1}(S)) \leq F$  and therefore by Theorem 4.16,  $f$  is fuzzy  $g'''$ -open.

Finally in this section, we define another new class of fuzzy maps called  $f g'''$ \*-closed maps which are stronger than  $f g'''$ -closed maps.

**Definition 4.18**

A fuzzy map  $f : (X, \tau) \rightarrow (Y, \sigma)$  is said to be  $f g'''$ -closed if the image  $f(A)$  is fuzzy  $g'''$ -closed in  $(Y, \sigma)$  for every fuzzy  $g'''$ -closed set  $A$  in  $(X, \tau)$ .

For example the fuzzy map  $f$  in Example 4.2 is an  $f g'''$ -closed map.

**Remark 4.19**

Since every fuzzy closed set is a fuzzy  $g'''$ -closed set we have  $f g'''$ -closed map is a  $f g'''$ -closed map. The converse is not true in general as seen from the following example.

**Example 4.20**

Let  $X = Y = \{a, b\}$  with  $\tau = \{0_x, \alpha, 1_x\}$  where  $\alpha(a) = 1, \alpha(b) = 0$  and  $\sigma = \{0_y, \beta, \alpha, 1_y\}$  where  $\beta(a) = 0.5, \beta(b) = 0$ . Then  $(X, \tau)$  and  $(Y, \sigma)$  are fuzzy topological spaces. Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be the identity fuzzy map. Then  $f$  is a  $f g'''$ -closed but it is not  $f g'''$ -closed map.

**Proposition 4.21**

A fuzzy map  $f : (X, \tau) \rightarrow (Y, \sigma)$  is  $f g'''$ -open if and only if  $f g'''\text{-cl}(f(A)) \leq f(g'''\text{-cl}(A))$  for every fuzzy subset  $A$  of  $(X, \tau)$ .

**Proof**

Similar to Proposition 4.3.

Analogous to  $f g'''$ -closed map we can also define  $f g'''$ -open map.

**Proposition 4.22**

For any fuzzy bijection  $f : (X, \tau) \rightarrow (Y, \sigma)$ , the following statements are equivalent:

- (i)  $f^{-1} : (Y, \sigma) \rightarrow (X, \tau)$  is fuzzy  $g'''$ -irresolute.
- (ii)  $f$  is  $f g'''$ -open map.
- (iii)  $f$  is  $f g'''$ -closed map.

**Proof**

Similar to Proposition 4.15.

**Proposition 4.23**

If  $f : (X, \tau) \rightarrow (Y, \sigma)$  is fgs-irresolute and  $f g'''$ -closed, then it is a  $f g'''$ -closed map.

**Proof**

The Proof follows from Proposition 4.6.

## REFERENCES

- [1] K. K. Azad, On fuzzy semicontinuity, fuzzy almost continuity and fuzzy weakly continuity, J. Math. Anal. Appl., 82(1) (1981), 14-32.
- [2] G. Balasubramanian and P. Sundaram, On some generalizations of fuzzy continuous functions, Fuzzy Sets and Systems, 86(1) (1997), 93-100.
- [3] G. Balasubramanian, On fuzzy pre-separation axioms, Bull. Calcutta Math. Soc., 90(6) (1998), 427-434.
- [4] A.S. Bin shahna, On fuzzy strong semicontinuity and fuzzy precontinuity, fuzzy precontinuity, fuzzy Sets and Systems, 44(2)(1991), 303-308.
- [5] C.L. Chang, Fuzzytopological spaces, J. Math. Anal. Appl., 24 (1968), 182-190.
- [6] E. Hatir, T. Noiri and S. Yuksel, A decomposition of continuity, Acta Math. Hungar., 70(1-2)(1996), 145-150.
- [7] M. Jeyaraman, J. Rajalakshmi, and O. Ravi, Another generalization of closed sets in fuzzy topological spaces, IJMA-4(8), August-2013.
- [8] M. Jeyaraman, J. Rajalakshmi, and R. Muthuraj, Decomposition of continuity in fuzzy topological spaces, IJMTT, 19-21, August-2014.
- [9] M. Rajamani, On decomposition of fuzzy continuity in fuzzy topological spaces, Acta Ciencia Indica, XXVIIM(4) (2001), 545-547.
- [10] R. K. Saraf, and M. Khanna, Fuzzy generalized semi preclosed sets, J. Tripura Math. Soc., 3(2001) 59-68.
- [11] R. K. Saraf, and M. Khanna, On  $g_s$ -closed sets in fuzzy topology, J. Indian Acad. Math., 25(1) (2003), 133-143.
- [12] R. K. Saraf, M. Caldas and S. Mishra, Results via  $Fg_\alpha$ -closed sets and  $Fag$ -closed sets, preprint.
- [13] M. Sudha, E. Roja and M. k. Uma, Slightly fuzzy  $\omega$ -continuous mappings, Int. Journal of Math. Analysis, 5(16) (2011), 779-787.
- [14] P. Sundaram, Generalized continuity in fuzzy topology. (preprint).
- [15] S. S. Thakur and S. Singh, On fuzzy semi-preopen sets and fuzzy semi-precontinuity, fuzzy Sets and Systems, 98(3) (1998), 383-391.
- [16] J. Tong, A decomposition of fuzzy continuity, Fuzzy Math., 7(1987), 97-98.
- [17] J. Tong, On decomposition of continuity in topological spaces, Acta Math. Hungar., 48(1986), 11-15.
- [18] T. H. Yalvac, Semi-interior and semi-closure of a fuzzy set, J. Math. Anal. Appl. 132(2) (1988), 356-364.
- [19] L. A. Zadeh, Fuzzy sets, Information and Control, 8(1965), 338-353.