Geometric Linear And Nonlinear Analysis Of Beam

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ABSTRACT:-

The beams are structural elements with thickness smaller than other plan dimensions. These structural elements are used in vast varieties of structures. Hence analysis of beams becomes the topic of interest for civil, mechanical, aeronautical and marine engineer. Now a day's steel is an economic and useful material, almost all the structural members are constructed by steel compare to timber and concrete. With the development of construction and manufacturing technology, beams of different shapes and varied sizes are demanded by designers. Analysis of these beams and thin plate attracted attention of many researchers. This paper is addressed to the review of advances, techniques and theoretical background of the non-linear analysis of steel beam. The formulation of beam element in bending has constituted the most exiting area in the development of the solution techniques. If the structure (beams) is made slender along, with bending, membrane action starts coming in picture .The aim of non linear analysis is to predict deflection of beam at various load stages. For present paper two nodes beam element is used for formulation of linear and geometric nonlinear analysis. In the present paper deflection of thin beam is obtained by finite element method in SAP software. The behavior of these flexure members in linear analysis and nonlinear analysis are compared. Some numeric examples are solved.

Keywords: *- Finite element method, Steel beam, SAP2000.*

INTRODUCTION

Structure is a free-standing, immobile outdoor construction. Typical examples include buildings and non-building structures such as bridges, dams, missile launching tower, transmission line towers. Most of structures are permanent though some structures are temporary, built for some events such as launching pads for spacecrafts, trade shows, conferences or theatre, and often dismantled after use. Temporary structures have fewer constraints relating to future use and durability thus these structures may be made slender and thinner. The flexure members of a structure, namely, beams and plates exhibit linear behavior till deflections are small compared to their thicknesses. As deflections increase, membrane forces are introduced and the external transverse load is supported by membrane-bending action. From this paper one can learn about the differences between linear and non-linear analysis and realize that there are optimum times to use one type of analysis versus the other.

Linear Analysis

Linear analysis (first order analysis) is also known as linear elastic analysis. The term of Elastic means that when the structure is unloaded it follows the same deformation path as when loaded. A linear FEA analysis is undertaken when a structure is expected to behave linearly, i.e. obeys Hook's Law. The stress is proportional to the strain, and the structure will return to its original configuration once the load has been removed. A structure is a load bearing member and can normally classified as a bar, beam, column or shaft. In linear elastic analysis, the material is assumed to be unyielding and its properties invariable and the equations of equilibrium are formulated on the geometry of the unloaded structure. It is assumed that the subsequent deflections will be small and will have insignificant effect on the stability and mode of response of the structure. The linear analysis of the beam and thin plate is done using stiffness method. In this approach the primary unknowns are the joint displacements, which are determined first by solving the structure equation of equilibrium. Then the unknown forces can be obtained through compatibility consideration.

Formulation Linear analysis of beam

A beam is a member predominantly supporting applied load by flexural strength of it. Fig.1 (a) shows a typical beam with its discretisation. Here beam is discretised in elements. The beam is discretised in four elements and having five nodes. Take a typical beam element shown in Fig. (b). It has two nodes, for generating formulation slope and deflection at each node is required.

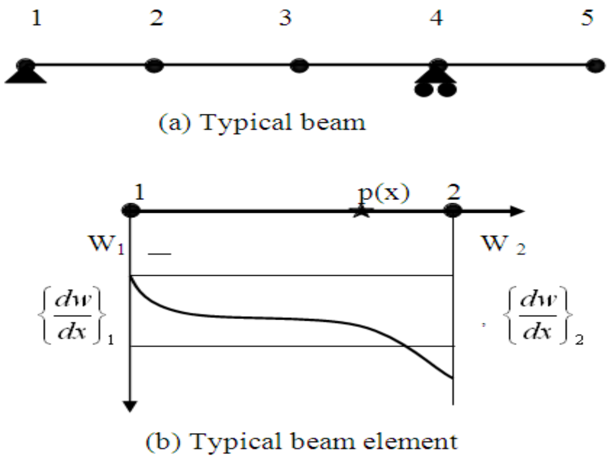


Fig.1:Typical beam and element

In case of two dimensional structures, the displacement at any point can be expressed by its components w, $\frac{dw}{dx}$ which are continues function of x.

Therefore, degree of freedom per joint = 2 (i.e. w, $\frac{dw}{dx}$)

The displacement within element at any point p(x) can be as follows

The variation of in plane displacement w, $\left\{\frac{dw}{dx}\right\}$ are prescribe using shape functions –

$$w_p = N_1 w_1 + \overline{N}_1 \left(\frac{\partial w}{\partial x}\right)_1 + N_2 w_2 + \overline{N}_2 \left(\frac{\partial w}{\partial x}\right)_2$$

N and $\overline{N}$ are Hermition shape function

$$w_p = \left[N_1 \quad \overline{N}_1 \quad N_2 \quad \overline{N}_2\right]$$

$$w = [N] \quad \{\delta e\}$$

The linear strains resultant within element can be written as

$$\chi_X = -\frac{\partial^2 w}{\partial x^2}$$

$$= -\left\{\frac{\partial^2 N_1}{\partial x^2} w_1 + \frac{\partial^2 \overline{N}_1}{\partial x^2}\left(\frac{\partial w}{\partial x}\right)_1 + \frac{\partial^2 N_2}{\partial x^2} w_2 + \frac{\partial^2 \overline{N}_2}{\partial x^2}\left(\frac{\partial w}{\partial x}\right)_2\right\}$$

$$= \left[\frac{\partial^2 N_1}{\partial x^2} - \frac{\partial^2 \overline{N}_1}{\partial x^2} - \frac{\partial^2 N_2}{\partial x^2} - \frac{\partial^2 \overline{N}_2}{\partial x^2}\right] \quad \delta e$$

$$\chi_X = [B] \quad \{\delta e\}$$

The linear stress resultant i.e. Moment within element can be express as

$$M = [EI] \quad \{X\}$$

$$M = [D] \quad \{X\}$$

Strain energy within element is calculated using strain energy over tiny length within element.

$$dU = \frac{1}{2} \quad [X]^T \quad \{M\}$$

Then it is integrated over entire element. Thus the strain energy over entire length is

$$U = \int_0^L \frac{1}{2} \{X\}^T \{M\} \, dx$$

$$U = \int_0^L \frac{1}{2} \{\delta e\}^T [B]^T [D] [B] \{\delta e\} \, dx$$

$$U = \frac{1}{2} \delta e^T \left[\int_0^L [B]^T [D] [B] \{\delta e\} \right] dx$$

From the above the element stiffness matrix can be extract as follows

$$\therefore \; Se = \int_0^L [B]^T [D] [B] \, dx$$

Non-Linear Analysis

Typical geometric nonlinearity arises from mid plane stretching of a thin structure coupled with transverse vibrations or loading. This stretching leads to a nonlinear relationship between the strain and the displacement. In mathematics, non-linear systems represent systems whose behavior is not expressible as a sum of the behaviors of its descriptors. In particular, the behavior of non-linear systems is not subjected to the principle of superposition, as linear systems. Crudely, a non-linear system is one whose behavior is not simply the sum of its parts or their multiples.

Formulation of non-linear analysis of beam

The non-linear analysis of beam is due to the bending of beam, and due to thin thickness of beam the neutral axis of beam is stretched due to this additional axial force is induced in the beam Fig.2 Shows a typical element having length L. it has same thickness throughout its length. It has two nodes and also shows the displacement in x direction and y direction are u and w respectively.

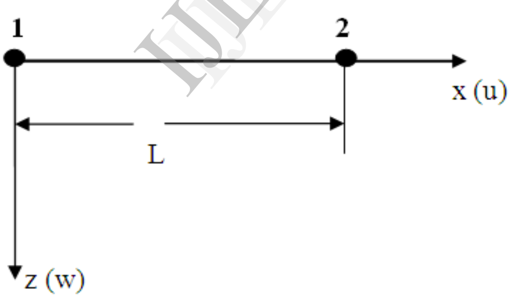


Fig. 2: Typical nonlinear beam element

In nonlinear analysis each node of a beam element has three degree of freedom in x direction and z direction

Degree of freedom per joint = u, w, $\frac{dw}{dx}$

u displacement due to axial force

w, $\frac{dw}{dx}$ displacement due to bending

$$\delta e = \left\{ u_1, w_1, \frac{dw_1}{dx}, u_2, w_2, \frac{dw_2}{dx} \right\}^T$$

The displacement within element at any point p(x) can be as follows.

The variations of in plane displacement u, w, dw/dx are prescribing using shape functions. Fig. 3 shows u_1 and u_2 displacements in x direction

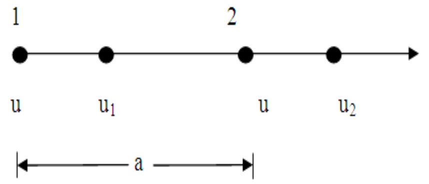


Fig. 3: Typical Non-Linear beam element

$$U_p = l_1 u_1 + l_2 u_2$$

$$w_p = N_1 w_1 + \overline{N}_1 \left(\frac{\partial w}{\partial x}\right)_1 + N_2 w_2 + \overline{N}_2 \left(\frac{\partial w}{\partial x}\right)_2$$

Where,

l_1 and l_2 are Lagrangian shape function in x direction

$l_1 = [1 - x/a]$ and $l_2 = {}^{x}/_{a}$

The Non-Linear strains resultant within element can be written as

$$\text{€}_p = \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2$$

The strains resultant is taken from the theory of elasticity, due to stretching of neutral axis the point p is also displace in z direction. The displacement of a point at distance z from the middle plane can be as fig.4 shows the displacement of point p in u and z direction.

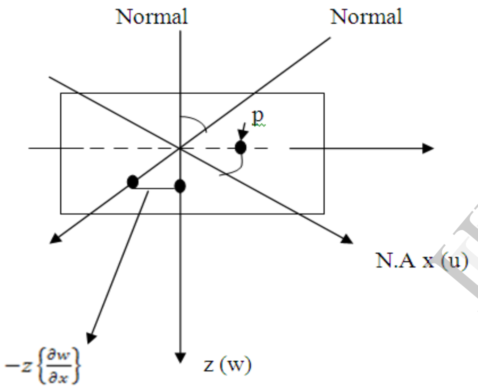


Fig.4: Displacement of point p in x and z direction

$$u(z) = u - z\left\{\frac{\partial w}{\partial x}\right\}$$

$$w(z) = w$$

Now substitute the nodal displacements in strain resultant

$$\text{€}_p = \frac{\partial u}{\partial x} - z\frac{d^2 w}{dx^2} + \frac{1}{2}\left\{\frac{\partial w}{\partial x}\right\}^2$$

Where, $\frac{\partial u}{\partial x}$ is constant across the thickness of beam, $z\frac{d^2 w}{dx^2}$ is varies with Z distance

Strain resultant due to axial force within element

$$\text{€}_p = \frac{\partial u}{\partial x} + \frac{1}{2}\left\{\frac{\partial w}{\partial x}\right\}^2$$

Strain resultant due to bending within element $\text{€}_p = -z\frac{d^2 w}{dx^2}$

Therefore $\chi_b = (-)\frac{d^2 w}{dx^2}$

The linear stress resultant i.e. Moment within element can be express as

Axial force within element

$$\{N\} = [EA] \{\text{€}_p\}$$

Moment within Element

$$\{M\} = [EI]\{\chi_b\}$$

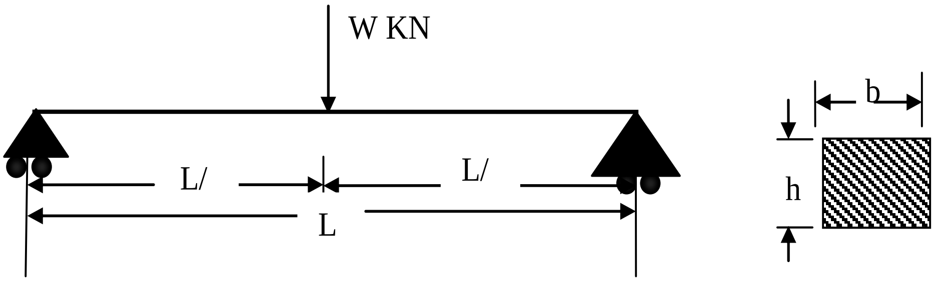
Therefore stress resultant is written in the matrix form is as follows.

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} EA & 0 \\ 0 & EI \end{bmatrix} \begin{Bmatrix} €_P \\ \chi_b \end{Bmatrix}$$

NUMERICAL EXAMPLES

To check the validity of the present formulation, some examples are solved by using computer program i.e. Simply-supported beam subjected to a center point load, the beam has a length (L) 700mm, width (b)50mm and depth (h) 10mm,05mm.

Behavior of geometric nonlinear Example-



(a)Simply supported beam (b) cross section of beam

Fig.5: Simply supported beam and its cross section

The simply supported beam over uniformed distributed load is increase gradually in this cases non-linearity so very large it shows following cases

The deflection of beam by theoretical calculated formula is given below

$$\Delta = \frac{wL^3}{48EI}$$

The above formula is most suitable for calculating deflections of linear analysis for thick beam. For thin beams this formula is not suitable. Hence we use SAP software for calculated nonlinear analysis.

Case 1- *Thickness of beam = t = 10 mm*

Table 1: Loading and deflection for thickness 10 mm

Load in KN	Deflection Δ (mm)					% of variation SAP Linear and Nonlinear
	Theoretical Calculation	Linear Analysis	Nonlinear Analysis	Difference SAP Linear and Theoretical	Difference Linear and Nonlinear	
0.25	2.143748285	2.1448	2.144725	7.5E-05	0.001051715	0.00349683
0.50	4.28749657	4.2896	4.28899	0.00061	0.00210343	0.01422044
0.75	6.431244855	6.4344	6.432341	0.002059	0.003155145	0.031999876
1.00	8.57499314	8.5792	8.574322	0.004878	0.00420686	0.056858448
1.25	10.71874143	10.724	10.714478	0.009522	0.005258575	0.088791496
1.50	12.86248971	12.8688	12.852357	0.016443	0.00631029	0.127774151
1.75	15.006238	15.0136	14.987513	0.026087	0.007362005	0.173755795
2.00	17.14998628	17.1584	17.119498	0.038902	0.00841372	0.226722771

2.25	19.29373457	19.3032	19.247873	0.055327	0.009465435	0.286620871
2.50	21.43748285	21.448	21.372202	0.075798	0.01051715	0.353403581
2.75	23.58123114	23.5928	23.492049	0.100751	0.011568865	0.427041301
3.00	25.72497942	25.7376	25.606997	0.130603	0.01262058	0.507440476
3.25	27.86872771	27.8824	27.716623	0.165777	0.013672295	0.594557857
3.50	30.01247599	30.0272	29.820515	0.206685	0.01472401	0.688325918
3.75	32.15622428	32.172	31.918269	0.253731	0.015775725	0.788670272
4.00	34.29997256	34.3168	34.009487	0.307313	0.01682744	0.895517647
4.25	36.44372085	36.4616	36.10506233	0.356537667	0.017879155	0.977844271
4.50	38.58746913	38.6064	38.19954833	0.406851667	0.01893087	1.053845131
4.75	40.73121742	40.7512	40.29403433	0.457165667	0.019982585	1.121845901
5.00	42.8749657	42.896	42.301354	0.594646	0.0210343	1.386250466
6.00	51.44995884	51.4752	50.459134	1.016066	0.02524116	1.973894225
7.00	60.02495198	60.0544	58.461846	1.592554	0.02944802	2.651852321
8.00	68.59994512	68.6336	66.29217	2.34143	0.03365488	3.41149233
9.00	77.17493826	77.2128	73.9318	3.281	0.03786174	4.249295454
10.00	85.7499314	85.792	81.370995	4.421005	0.0420686	5.153166962
11	94.32492454	94.3712	88.598738	5.772462	0.04627546	6.116762317
12	102.8999177	102.9504	95.608059	7.342341	0.05048232	7.131920808
13	111.4749108	111.5296	102.39405	9.13555	0.05468918	8.191143876
14	120.049904	120.1088	108.954105	11.154695	0.05889604	9.287158809
15	128.6248971	128.688	115.287572	13.400428	0.0631029	10.41311389
16	137.1998902	137.2672	121.395496	15.871704	0.06730976	11.56263405
17	145.7748834	145.8464	127.280368	18.566032	0.07151662	12.72985278
18	154.3498765	154.4256	132.945884	21.479716	0.07572348	13.90942693
19	162.9248697	163.0048	138.396568	24.608232	0.07993034	15.09663028
20	171.4998628	171.584	143.63108	27.95292	0.0841372	16.2910994

***Case 1-** Thickness of beam = t = 5 mm*

Table 2: Loading and deflection for thickness 5 mm

Load in KN	Deflection Δ (mm)					% of variation SAP Linear and Nonlinear
	Theoretical Calculation	Linear Analysis	Nonlinear Analysis	Difference SAP Linear and Theoretical	Difference Linear and Nonlinear	
00.25	17.15010985	17.1521	17.113226	0.038874	0.00199015	0.226642802
00.50	34.3002197	34.3042	33.997099	0.307101	0.0039803	0.895228573
00.75	51.45032955	51.4563	50.440925	1.015375	0.00597045	1.973276353
01.00	68.6004394	68.6084	66.267603	2.340797	0.0079606	3.411822751
01.25	85.75054925	85.7605	81.342251	4.418249	0.00995075	5.15184613
01.50	102.9006591	102.9126	95.514867	7.397733	0.0119409	7.18836469
01.75	120.050769	120.0647	108.916793	11.147907	0.01393105	9.284916383
02.00	137.2008788	137.2168	121.354369	15.862431	0.0159212	11.5601231
02.25	154.3509887	154.3689	132.901244	21.467656	0.01791135	13.90672344
02.50	171.5010985	171.521	143.590376	27.930624	0.0199015	16.28408416
02.75	188.6512084	188.6731	153.467391	35.205709	0.02189165	18.65963351
03.00	205.8013182	205.8252	162.585132	43.240068	0.0238818	21.00815061
03.25	222.9514281	222.9773	170.999589	51.977711	0.02587195	23.31076347
03.50	240.1015379	240.1294	178.167026	61.962374	0.0278621	25.80374332
03.75	257.2516478	257.2815	185.941889	71.339611	0.02985225	27.72823192
04.00	274.4017576	274.4336	192.576189	81.857411	0.0318424	29.82776562
04.25	291.5518675	291.5857	198.718131	92.867569	0.03383255	31.8491507
04.50	308.7019773	308.7378	204.412236	104.325564	0.0358227	33.79099158
04.75	325.8520872	325.8899	209.699207	116.190693	0.03781285	35.65335808
05.00	343.002197	343.042	214.616032	128.425968	0.039803	37.43738901
06.00	411.6026364	411.6504	231.203951	180.446449	0.0477636	43.8348776
07.00	480.2030758	480.2588	244.024701	236.234099	0.0557242	49.18891627
08.00	548.8035152	548.8672	254.187124	294.680076	0.0636848	53.68877499
09.00	617.4039546	617.4756	262.427523	355.048077	0.0716454	57.49993635
10.00	686.004394	686.084	269.24404	416.83996	0.079606	60.75640301
11.00	754.6048334	754.6924	274.983056	479.709344	0.0875666	63.56355835
12.00	823.2052728	823.3008	279.888113	543.412687	0.0955272	66.00414903
13.00	891.8057122	891.9092	284.136646	607.772554	0.1034878	68.14287306
14.00	960.4061516	960.5176	287.859374	672.658226	0.1114484	70.03080693
15.00	1029.006591	1029.126	291.154627	737.971373	0.119409	71.70855396
16.00	1097.60703	1097.7344	294.09753	803.63687	0.1273696	73.20868053
17.00	1166.20747	1166.3428	296.746422	869.596378	0.1353302	74.55752957
18.00	1234.807909	1234.9512	299.147179	935.804021	0.1432908	75.77659919
19.00	1303.408349	1303.5596	301.336401	1002.223199	0.1512514	76.88357318
20.00	1372.008788	1372.168	303.3436	1068.8244	0.159212	77.89311513

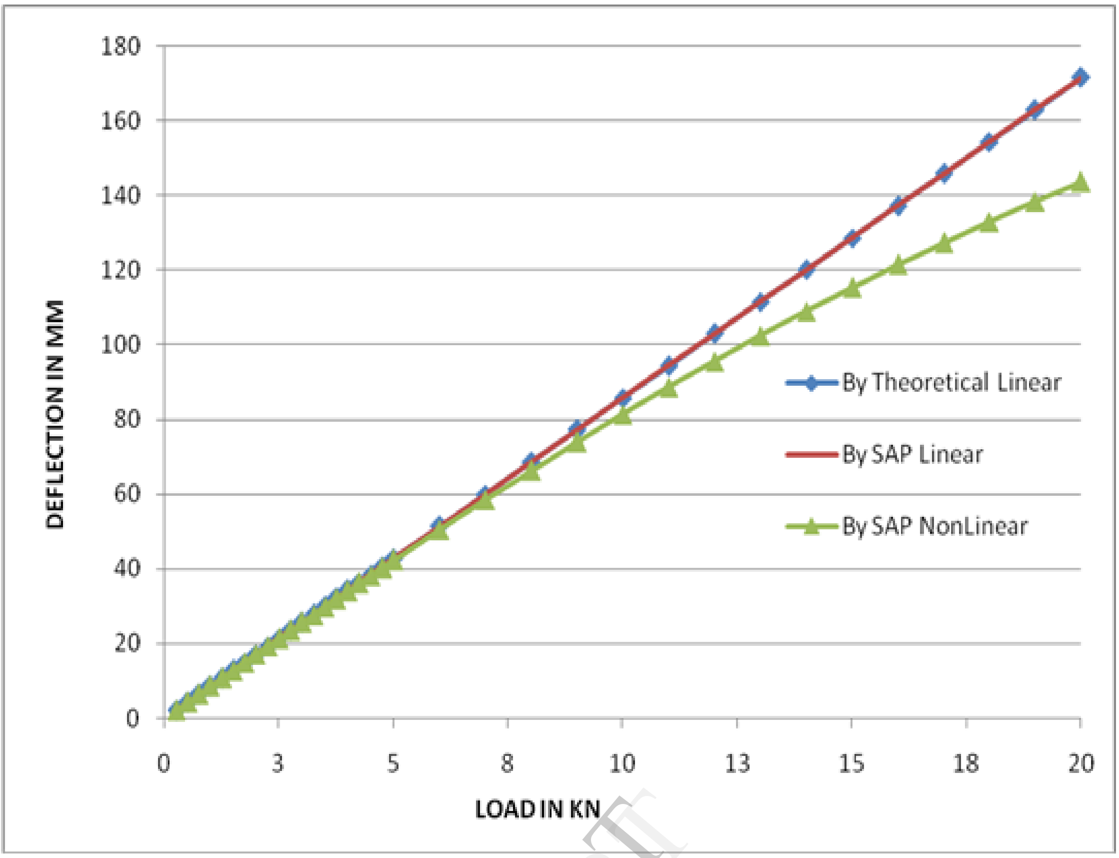


Fig. 6 Load Vs Deflection curve theoretical, SAP and nonlinear (10mm)

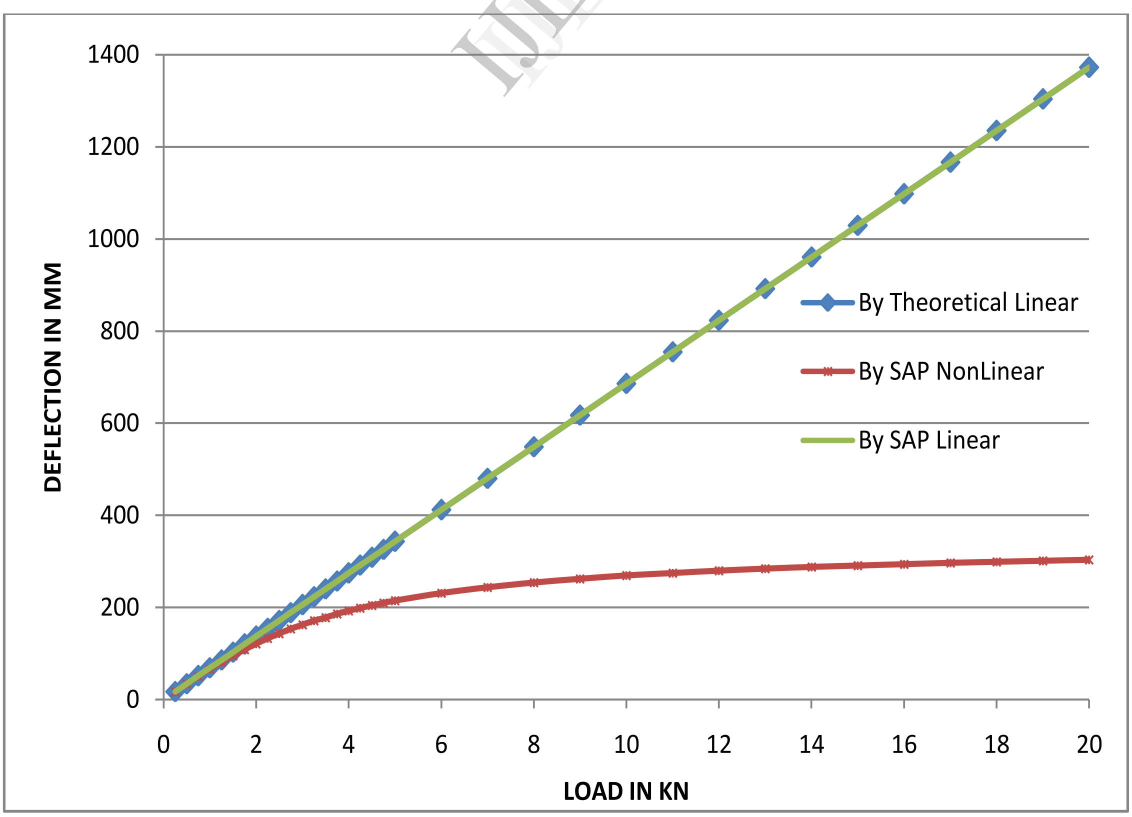


Fig.7: Load Vs Deflection curve theoretical, SAP linear and nonlinear (5mm)

CONCLUSIONS:-

The general purpose finite element software SAP2000 was used to conduct the linear and nonlinear analysis. Certain examples are analysed by SAP2000 and the results are compared with theoretical calculation.The studies on Software and theoretical results associated with them lead to the following conclusions:

1. When loads intensity is small there is a very small (Negligible) variation between theoretical and SAP 2000in linear deflection.
2. P-delta value shows the significant geometric nonlinear analysis as compared to the linear analysis in this study.
3. Geometric nonlinearity is not induced in the thick beam when the load intensity is small and it is induced in the thick beam when the big load intensity goes on increasing.
4. Geometric nonlinearity is induced in the beam because of its lesser thickness (thinner). it is produced more in the thinner beam, when load is increased.
5. As the stretching of middle plane (Neutral axis) starts, the stiffness of structure increases (axial stiffness is added with bending stiffness).Thus the beam becomes stiffer progressively. It is a positive aspect of geometric nonlinearity.

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