# Geometric Approach For Inverse Kinematics Solution: 3-PSU Parallel Kinematic Manipulator 

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#### Abstract

The parallel kinematic manipulators have shown their capabilities like high structural rigidity, high dynamic performance, and high positional accuracy. The main aim of this paper is to explore a simplified geometrical approach for the solution of Inverse Kinematics problem of a 3-PSU Parallel Kinematic Manipulator. The geometric data is obtained through a preliminary CAD model and the methodology of solution is verified in MATLAB to obtain solutions; with the solution data, toolpath simulation has been carried out using Pro/Mechanism.


## 1. Introduction

The Parallel Kinematic Machines (or Manipulators) have attracted a lot of attention to researchers and academicians from research, academics and industrial organizations. The main reason, behind this, is the high dynamic capabilities, high structural rigidity, noncumulative error propagation and high positional accuracies of the parallel kinematic manipulator (PKM) [1] [2] [3]. The closed loop connections of linkages establish a rigid structure of PKM. The payload to self weight ratio is also reduced because of multiple linkages support the moving link, which further results into good dynamic characteristics [4]. The PKMs generally have different theoretically possible configurations of joints and actuators, notations, different kinematic and dynamic characteristics, control schemes [5] and workspaces [6].
The main aim of this paper focuses on to study and to verify a novel, simplified geometrical approach to solve inverse kinematic solution of 3PUU PKM configurations using advanced CAD software and Simulation software like Pro/E mechanism and MATLAB.

## 2. Notations Scheme

In literature, uniform notation scheme of mechanical link arrangement of PKM has not been defined. Because of lack in standard scheme, no sophisticated software has been designed with proper modular library facility [2]. Although, the configuration (also known as 3-PSU configuration)
of Parallel Manipulator can be elaborated as following; ' 3 ' denotes three Degree of Freedom (DoF), ' P ' stands for actuated prismatic joint, ' S ' refers to ball \& socket (spherical) joint \& ' $U$ ' means Universal joint. Many serial P-S-U links are connected in parallel manner in spatial space.

## 3. Degree of Freedom

In a manipulator, a moving platform with an endeffector is moved with respect to fixed base in different manners. A platform changes its position and orientation considering fixed base co-ordinates as an origin. The total numbers of the possible motion is limited to three rotational and three translatory for any rigid link in space.
These three translatory motions are considered along mutually orthogonal axes, and three rotary motions around these axes. Degree of Freedom (DoF) does not answer the question if the PKM is Planar or Spatial. The total numbers of variable required for actuator parameter (either prismatic or rotary) can be related with total DoF.
For example, DoF of manipulator of Stewart platform is six [3], means all three rotational and all three translatory motions are available with this PKM. The PKM suggested by [7] has 3-DoF (actuated by three stepper motors) though, it is an example of planar robot manipulator.


Figure 1 3-DoF Planar Robot [7]

Depending upon the DoF the range of applications of application ranging from spatial applications like pick and place, material handling, MEMS, medical application, wire robots, CMM,
astronomy, vibrations, sun-tracking, biomechanics, simulators, machine tools to Planar applications. [2] [5] [8] [9]
According to [10];
If, $n \quad=$ number of rigid bodies

$$
j \quad=\text { number of joints }
$$

$\mathrm{f}_{\mathrm{i}} \quad=$ DoF of the $\mathrm{i}^{\text {th }}$ joint

$$
D o F=6(n-j-1)+\sum_{i=1}^{j} f_{i}
$$

$$
D o F=3
$$

## 4. Mathematical Modelling

3-PSU configuration of PKM can be modeled as per figure, Where;
$A_{i} B_{i}=i^{\text {th }}$ rail on which spherical joint moves (prismatic joint)

$$
\begin{array}{ll}
\mathrm{A}_{\mathrm{i}} \mathrm{P}_{\mathrm{i}}=\mathrm{d}_{\mathrm{i}} & =\text { Distance of actuator from } \mathrm{A}_{\mathrm{i}} \\
\mathrm{O}_{\mathrm{xyz}} & \\
\mathrm{M}_{\mathrm{xyz}} & =\text { Co-ordinate of fixed frame } \\
\mathrm{T}_{\mathrm{xyz}} & =\text { Co-ordinate of moving frame } \\
\mathrm{P}_{\mathrm{i}} \mathrm{Q}_{\mathrm{i}} & =l_{\mathrm{i}} \\
& =\text { Co-ordinate of Machine Tool } \\
& =\text { Length of } \mathrm{i}^{\text {th }} \text { link }
\end{array}
$$

With the condition that two consecutive pairs are always kept parallel, resulting into $1\|2,3\| 4,5 \| 6$.
Mathematically; $\mathrm{d}_{1}=\mathrm{d}_{2} ; \mathrm{d}_{3}=\mathrm{d}_{4} ; \mathrm{d}_{5}=\mathrm{d}_{6}$.

## 5. Inverse Kinematic Solution

Inverse Kinematic solution for 3-PSU PKM focuses to establish the relation between ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) variables of vector $\mathrm{T}_{\mathrm{xyz}}$ and actuator lengths $\left(d_{1}, d_{3}, d_{5}\right)$. A single link shown in figure 2 is to be considered for analysis of inverse kinematic solution.
A transformation of vector from $T_{x y z}$ to $\mathrm{M}_{\mathrm{xyz}}$ is found out first. Because of symmetric arrangement of linkages parallelepiped construction is made and no rotational component exists for $\mathrm{M}_{\mathrm{xyz}}$. Further, from $\mathrm{M}_{\mathrm{xyz}}$, different coordinates for $\mathrm{Q}_{\mathrm{i}}(\mathrm{i}=1$ to 6) is obtained. Due to Spherical Joint(S), $\mathrm{Q}_{\mathrm{i}}\left(\mathrm{x}_{\mathrm{q}}, \mathrm{y}_{\mathrm{q}}, \mathrm{z}_{\mathrm{q}}\right)$ will be a centre of sphere and radius will be equal to $P_{i} Q_{i}\left(=l_{i}\right) . P_{i}\left(x_{p}, y_{p}, z_{p}\right)$ is a point on sphere as well as on line $A_{i} B_{i}$ both, so it satisfies the equations as following. Line $A_{i} B_{i}$ is represented by, for $\mathrm{i}^{\text {th }}$ link and $\mathrm{m} \in[0,1]$.


Figure 2 Modelling of 3-PSU configuration


Figure 3 A Single link analysis of 3-PSU

$$
\begin{array}{ll}
x_{p}=x_{a}+m\left(x_{p}-x_{a}\right) & \text {.. } 2 \\
y_{p}=y_{a}+m\left(y_{p}-y_{a}\right) & \text {.. } 3 \\
z_{p}=z_{a}+m\left(z_{p}-z_{a}\right) & \text {.. } 4
\end{array}
$$

For sphere,

$$
\begin{aligned}
& \left(\mathrm{x}_{\mathrm{p}}-\mathrm{x}_{\mathrm{q}}\right)^{2}+\left(\mathrm{y}_{\mathrm{p}}-\mathrm{y}_{\mathrm{q}}\right)^{2}+\left(\mathrm{z}_{\mathrm{p}}-\mathrm{z}_{\mathrm{q}}\right)^{2}=l_{i}^{2} \\
& \left(\mathrm{x}_{\mathrm{a}}+\mathrm{m}\left(\mathrm{x}_{\mathrm{p}}-\mathrm{x}_{\mathrm{a}}\right)-\mathrm{x}_{\mathrm{q}}\right)^{2}+\left(\mathrm{y}_{\mathrm{a}}+\mathrm{m}\left(\mathrm{y}_{\mathrm{p}}-\right.\right. \\
& \left.\left.\mathrm{y}_{\mathrm{a}}\right)-\mathrm{y}_{\mathrm{q}}\right)^{2}+\left(\mathrm{z}_{\mathrm{a}}+\mathrm{m}\left(\mathrm{z}_{\mathrm{p}}-\mathrm{z}_{\mathrm{a}}\right)-\mathrm{z}_{\mathrm{q}}\right)^{2}=l_{i}^{2}
\end{aligned}
$$

.. 6

Which on expansion forms a quadratic equation of m , by solving it two values of m will be obtained, among them only one value would belong to $[0,1]$. This value of valid $m$ is used to solve the $\mathrm{P}\left(\mathrm{x}_{\mathrm{p}}, \mathrm{y}_{\mathrm{p}}, \mathrm{z}_{\mathrm{p}}\right)$ from equations $2,3 \& 4$.
From
$d_{i}^{2}=\left(\mathrm{x}_{\mathrm{p}}-\mathrm{x}_{\mathrm{a}}\right)^{2}+\left(\mathrm{y}_{\mathrm{p}}-\mathrm{y}_{\mathrm{a}}\right)^{2}+\left(\mathrm{z}_{\mathrm{p}}-\mathrm{z}_{\mathrm{a}}\right)^{2}$
a set of $d_{i}$ is achieved among them $d_{1}=d_{2} ; d_{3}=d_{4}$; $d_{5}=d_{6}$ only three values $\left(d_{1}, d_{3}, d_{5}\right)$ is used to operate prismatic actuator.

## 6. Simulation

The Inverse Kinematic solution of 3-PSU PKM as per the geometric approach discussed above is solved with the use of software MATLAB.


Figure 4 Flow of Inverse Kinematic Solution

The obtained data of $\left(\mathrm{d}_{1}, \mathrm{~d}_{3}, \mathrm{~d}_{5}\right)$ are sent to Pro/E mechanism for preliminary CAD model developed in Pro/E.


Figure 5 Preliminary CAD model in Pro/E

For the following line segment motor data for simulation were found.
Start point:
[27.788946, -136.043956, 11.930000]
End point:
[04.511992, $-122.605000, \quad 22.140000]$


Figure 6 Motor data w.r.t time

## 7. Conclusion

For 3- PSU Configuration PKM, a simplified geometric approach to solve Inverse Kinematic solution has been done with the help of MATLAB software.
The simulation for the solved data has been carried out in Pro/ E mechanism with preliminary CAD model. Same method can be used with prototype to find actuator data.

## 8. References

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