Generation Of Stationary Pulse Profile For Terahertz (Thz) Radiation In A Nonlinear Uniaxial Medium

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ABSTRACT: Terahertz (THz) radiation is part of the electromagnetic spectrum that lies between the infrared and microwave region. Its generation and theory is based on laser light and nonlinear optics. In this work, we used the Maxwell's equations and nonlinear optical equations to generate a nonlinear Schrödinger equation (NLSE) for fourth order harmonics. The solution of the NLSE is used to generate a stationary THz pulse that has a delta shape with components of sech and Gaussian profiles. These shapes can be used as input pulses capable of generating and propagating dynamic THz pulse profiles in uniaxial medium, which could be employed in optical communications.

Key words: Terahertz Radiation; Nonlinear Schrödinger Equation; Uniaxial Medium; Maxwell's Equation.

1. INTRODUCTION

Electromagnetic wave propagation in nonlinear medium has been given attention in last few decades due to its important application in optical signal transmission [1]. The terahertz (= 10^{12} Hz) lies between the infrared and the microwave regions of the electromagnetic spectrum [2 – 4]. Its science and applications are usually based on laser light. The science also includes the emission, transmission, amplification, detection, modulation and switching of light. Liu *et al*, [5] described the THz radiation as a wave that has the capability of penetrating through many commonly – used non polar materials such as paper, textile, plastic, leather, wood, and ceramic. Many THz and microwave imaging techniques have gained importance as promising tools for various applications, including imaging in pathological diagnosis, dentistry, tomography and material inspection and characterization [6 ,7]. According to Siegel [8], the universe is bathed in terahertz energy. Most of it is going unnoticed and undetected.

According to Baldwin [9], optics recognizes three classes of crystals;

- The cubic crystals, which are optically isotropic, and have their refractive indices the same in all the three axes. i.e $n_x = n_y = n_z = n$.
- The uniaxial crystals, which are optically anisotropic, and have two of the refractive indices to be the same, and are different from the other one. i.e $n_x = n_y = n_o$ and $n_z = n_e$. Examples of such crystals are the trigonal, tetragonal and hexagonal lattice types.

- The biaxial crystals are also optically anisotropic and have all the three refractive indices different from each other. i.e $n_x \neq n_y \neq n_z$. Examples of such crystals are the orthorhombic, monoclinic and triclinic lattice types.

Nonlinear optics is concerned with the response of matter to intense electromagnetic field such as the one obtained from laser light or THz radiation, in which the matter responds in a nonlinear manner to the incident radiation fields. It is a phenomenon that occurs at high optical intensities [10]. In these classes of crystals, the cubic crystal is centrosymmetric in nature, and hence allows for the propagation of laser light. The uniaxial and the biaxial crystals on the other hand are non – centrosymmetric, and allow for the propagation of THz radiation.

The nonlinear equations of mathematical physics are major subjects in physical science. In nonlinear optics, the induced polarization **P** in a medium and the electric field **E** of the electromagnetic wave propagating in the medium are related by [11]

$$\mathbf{P} = \varepsilon_0 \chi \mathbf{E} \tag{1}$$

where χ is the dielectric susceptibility of the medium that depends on the frequency, but independent of the field **E**, and ε_o is the permittivity of free space. Equation (1) is valid for the field strengths of conventional sources. With sufficiently intense laser or THz, equation (1) is no longer adequate, and hence needs to be generalized [12]. The polarization **P** induced in a medium by optical fields can be represented by a power series in the optical fields **E** in the form [13 – 15]

$$\mathbf{P} = \varepsilon_0 (\chi^{(1)}\mathbf{E} + \chi^{(2)}\mathbf{E}^2 + \chi^{(3)}\mathbf{E}^3 + \chi^{(4)}\mathbf{E}^4 + \chi^{(5)}\mathbf{E}^5 + \chi^{(6)}\mathbf{E}^6 \ldots) \tag{2}$$

where $\chi^{(1)}$ is the linear susceptibility, and $\chi^{(2)}$, $\chi^{(3)}$, $\chi^{(4)}$ and so on are the nonlinear susceptibilities.

In this paper, we are using the Maxwell's equation which provide the most fundamental description of electric and magnetic fields. The propagation of THz radiation in a nonlinear medium is governed by the wave equation derived from Maxwell's equation as

$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{\varepsilon_o c^2}\frac{\partial^2 \mathbf{P}}{\partial t^2} \tag{3}$$

The solution of equation (3) is of the form

$$E(x,y,z,t) = \Gamma F(x,y) A(z,t)\; e^{-i(\omega t - \beta z)} \tag{4}$$

where *A(z,t)* is the amplitude of the pulse envelope, Γ is a normalization constant, and *F(x,y)* is the field distribution in the (*x,y*) plane and corresponds to the mode structure.

A stationary solitary wave solution for the fourth order nonlinear Schrödinger equation (NLSE) for equation (4) has been worked out to generate a stationary pulse profile for THz radiation. The components of this shape can be used as an input pulse to generate the dynamic THz pulse profiles, which can be employed in communications.

2. GENERATION OF DIMENSIONLESS NLSE FOR THE FOURTH ORDER

We Used equations (3) and (4) to obtain two equations; one for the field distribution function F(x,y) given as

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \left(\eta^2 - \beta^2\right)F = 0 \tag{5}$$

and the other one, the perturbed propagation equation for the amplitude of the pulse envelope A(z,t) given as

$$i\left\{\frac{\partial A}{\partial z} - \frac{1}{v_g}\frac{\partial A}{\partial t}\right\} - \frac{\delta}{2}|\beta_2|\frac{\partial^2 A}{\partial t^2} - a_o A + \alpha_2|A|A + \alpha_4|A|^3 A =$$

$$i\frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3} - ia_2\frac{\partial}{\partial t}\left(|A|A\right) - ia_4\frac{\partial}{\partial t}\left(|A|^3 A\right) \tag{6}$$

Equation (5) is in rectangular or Cartesian coordinate whose solutions are consistent with LP_{01} ($\equiv HE_{11}$) mode theory [16], and is of little or no interest in this work. The corresponding dimensionless field amplitude of equation (6) is defined as

$$U(\xi,\tau) \equiv \frac{A(\xi,\tau)}{A_o} = \frac{\tau_o^2}{N}\left(\frac{\alpha_2}{|\beta_2|}\right)^{1/2} A(\xi,\tau) \tag{7}$$

where A_o is the maximum field amplitude, τ_o is the dimensionless pulsewidth, N is the solitary order, α_2 is a nonlinear parameter, while β_2 is the magnitude of the group velocity. From equations (6) and (7), we generated the dimensionless NLSE for the fourth order as thus

$$i\frac{\partial U}{\partial \xi} - \frac{\delta}{2}\frac{\partial^2 U}{\partial \tau^2} - a_o U + |U|U + v_{NL}|U|^3 U = ib_1\frac{\partial^3 U}{\partial \tau^3} - ib_2\frac{\partial}{\partial \tau}\left(|U|U\right) - ib_4\frac{\partial}{\partial \tau}\left(|U|^3 U\right) \tag{8}$$

The terms on the R.H.S of equations (8) are the perturbation terms, and if they are neglected, equation (8) becomes

$$i\frac{\partial U}{\partial \xi} - \frac{\delta}{2}\frac{\partial^2 U}{\partial \tau^2} - \alpha_o U + \delta_o|U|U + \delta_1 v|U|^3 U = 0 \tag{9}$$

where U is the dimensionless amplitude (height), ξ is the dimensionless pulse distance, v is the nonlinear coefficient, and τ is the pulsewidth. $\delta = \pm 1$ defines normal and anomalous dispersion propagation. For $\delta_o = \pm$

1 implies $n_1 > 0$ or $n_1 < 0$, and $\delta_1 = \pm 1$ implies $n_3 > 0$ or $n_3 < 0$. For most nonlinear applications, it is the anomalous dispersion that is applicable [16]. This means that in this work $\delta = -1$. But however in equation (9), the fourth and the fifth terms represent the second and the fourth nonlinearities for $\chi^{(2)}$ and $\chi^{(4)}$ respectively, and consequently, the values of $\chi^{(2)}$ and $\chi^{(4)}$ generated by [17] for the uniaxial crystals are positive. This implies that δ_o and δ_1 that respectively represent the coefficients for $\chi^{(2)}$ and $\chi^{(4)}$ should take positive value. And hence $\delta_o = +1$ and so also is $\delta_1 = +1$.

3. STATIONARY SOLITARY WAVE SOLUTION FOR FOURTH ORDER NLSE.

Many problems in natural and engineering sciences are modelled by partial differential equations [18, 19]. The dimensionless NLSE for the fourth order is already defined in equation (9). For a localized and dynamically distortionless profile of the optical pulse, ansatz equation of the following form is required [16] and [20]

$$U(\xi,\tau) = [Q(\xi,\tau)]^{1/2} \exp[i\phi(\xi,\tau)] \tag{10}$$

where $Q(\xi,\tau) = [q(\xi,\tau)]^2$ is the dimensionless field intensity with $q(\xi,\tau)$ having the meaning of dimensionless field amplitude of the modulating function, $\phi(\xi,\tau)$ is the phase function. This solution is also known as the stationary pulse solution equivalent to an incident pulse. Using equation (10) in (9) produces an equation that has two parts, one imaginary, and the other real, which are identically equal to zero. The real part produces an equation of the form

$$\frac{\delta}{8Q^2}\left(\frac{\partial Q}{\partial \tau}\right)^2 - \frac{\delta}{4Q}\frac{\partial^2 Q}{\partial \tau^2} - \alpha_o + \delta_o |Q|^{1/2} + \delta_1 v |Q|^{3/2} = \frac{\partial \phi}{\partial \xi} - \frac{\delta}{2}\left(\frac{\partial \phi}{\partial \tau}\right)^2 \equiv \beta \tag{11}$$

while the imaginary part is given as

$$\frac{\partial Q}{\partial \xi} - \frac{\partial}{\partial \tau}\left(\delta Q \frac{\partial \phi}{\partial \tau}\right) \cong 0 \tag{12}$$

In equation (11), β is the wave number shift. But, since waveforms take only real values [21], equation (12) is ignored, and from equation (11), one obtains

$$\frac{d}{d\tau}\left\{-\frac{\delta}{8Q}\left(\frac{dQ}{d\tau}\right)^2\right\} - \frac{d}{d\tau}(\alpha_o Q) + \frac{2}{3}\delta_o \frac{d}{d\tau}\left(Q^{3/2}\right) + \frac{2}{5}\delta_1 v \frac{d}{d\tau}\left(Q^{5/2}\right) = \frac{d}{d\tau}(\beta Q) \tag{13}$$

By simplifying equation (13) and using the relation

$$\lim_{\tau \to \pm\infty} Q = \lim_{\tau \to \pm\infty} \frac{dQ}{d\tau} = 0 \quad \text{at} \quad Q(\equiv Q_o) \tag{14}$$

From equation (13), β can be written as

$$\beta = -\alpha_o + \frac{2}{3}\delta_o Q^{1/2} + \frac{2}{5}\delta_1 v Q^{3/2} \tag{15}$$

Further simplification of equation (13), using δ = -1 and with the substitution of equation (15) in (13) yields

$$\left(\frac{\partial Q}{\partial \tau}\right) = \pm 2Q\left[\frac{4}{3}\delta_o Q^{1/2} + \frac{4}{5}\delta_1 v Q^{3/2} + \left(\frac{4}{3}\delta_o Q_o^{1/2} + \frac{4}{5}\delta_1 Q_o^{3/2}\right)\right]^{1/2} \tag{16}$$

From equation (16)

$$\pm\tau = \int \frac{dQ}{2Q\left[\frac{4}{5}\delta_1 v Q^{3/2} + \frac{4}{3}\delta_o Q^{1/2} + \left(\frac{4}{5}\delta_1 v Q_o^{3/2} + \frac{4}{3}\delta_o Q_o^{1/2}\right)\right]^{1/2}} \tag{17}$$

which was evaluated as

$$\pm\tau = \int \frac{dx}{x\Phi^{p+1}} = \frac{2}{9d^2}\left\{3d\Phi^{1/2} + 3bx\Phi^{1/2} + \Phi^{3/2}\right\} \tag{18}$$

Equation (18) is the stationary solitary wave solution for fourth order nonlinear Schrödinger equation for THz radiation.

4. GENERATION OF RESULTANT THz STATIONARY PULSE SHAPE

To generate the stationary THz pulse shape, the following expressions were used in equation (18)

$\Phi = ax^3 + bx + d$; $\delta_1 = +1$; $\delta_o = +1$; $v = 0.044$, $Q_o = 1$; $a = \frac{4}{5}\delta_1 v$; $b = \frac{4}{3}\delta_o$ and

$$d = -\left[\frac{4}{5}\delta_1 v Q_o^{3/2} + \frac{4}{3}\delta_o Q_o^{1/2}\right]$$

From equation (18), a plot of the dimensionless field amplitude $q(\tau)$ against the pulsewidth τ_o provides the resultant THz stationary pulse shape, and is shown in Fig. 1.

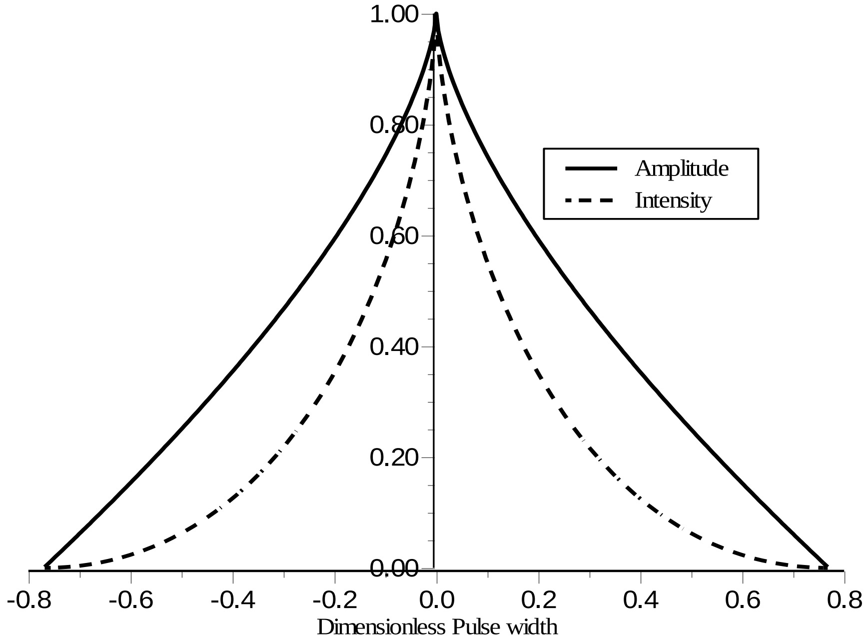

Fig.1: The resultant THz stationary pulse profile

Fig. 1 is the resultant stationary pulse profile for THz radiation that is of delta shape, with both components of sech and Gaussian shapes in it.

5. CONCLUSION

In this paper, we used Maxwell's equations to develop a nonlinear Schrödinger equation (NLSE) for fourth order harmonics. The solution of the NLSE was used to generate a stationary THz pulse that has delta shape with sech and Gaussian components. These components can be used as input pulses to generate dynamic THz pulse profiles that can be propagated in nonlinear uniaxial media as carrier waves.

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