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# Generation of Dynamic Terahertz (THz) Pulse Profiles for Fourth Order Harmonics in A Fiber – Like Nonlinear Uniaxial Crystal.

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***Abstract* — In This work, the Maxwell's equations were used to obtain a transverse wave equation. A trial solution of the wave equation that has both the amplitude and the field distribution function of the terahertz (THz) radiation were used to generate a dimensionless nonlinear Schrödinger equation (NLSE) for the fourth order harmonics. The numerical simulation of the NLSE for the THz radiation was tested on a fiber – like nonlinear uniaxial crystal based on three parameters. The nonlinear coefficient, the dimensionless pulsewidth, and the order of the solitary wave. The dynamic pulse profiles show that THz could have an application in optical communications, just like Laser in optical fiber communication.**

***keywords — terahertz, uniaxial crystal, Maxwell's equations, nonlinear Schrödinger equation, pulse profiles***

## 1. INTRODUCTION

The THz radiation (~ 1 x $10^{12}$ Hz) is broadly applied to sub millimeter wave energy that fills the wavelength between 0.001 m to 0.0001 m, corresponding to frequency range between 0.3 THz – 3 THz [1]. Spectroscopists had coined the term for emission frequencies that fall below the far infra red (FIR) and the microwave regions of the electromagnetic spectrum. The ability of THz radiation to penetrate packaging, similar to X – rays but without ionizing the contents makes THz potentially useful for law enforcement applications such as detecting explosives and illegal drugs [2]. Nonlinear optical phenomena occur typically at high optical intensities [3], and most of the wave equations involved are governed by the Nonlinear Schrödinger Equations (NLSE). Optical solitons have been regarded as the next generation technology for high capacity optical communications, chiefly because of their promise to transmit signals over long distances. Agrawal [4] describes the soliton as the particle like properties of pulse envelopes in dispersive nonlinear medium that under certain conditions, the envelope not only propagates undistorted but survives collisions just as particles do. There are basically two types of solitary waves. Xie [5] distinguished the temporal and the spatial solitary waves based on the nonlinear Schrödinger equations associated with them. For propagation along the z – axis, the transverse dimension for the temporal solitary wave

is in terms of time *t*, and the dimensionless pulse envelope *U* (*z*,*t*) evolves according to the NLSE as [5]

$$i\frac{\partial U}{\partial z}-\frac{\beta_2}{2}\frac{\partial^2 U}{\partial t^2}+\nu_{NL}|U|^2U=0 \qquad (1)$$

where $\beta_2=\dfrac{d^2k}{d\omega^2}$ is the second derivative of the linear wave number *k* with respect to angular frequency ω, and $v_{NL}$ is the nonlinear coefficient. $\beta_2$ is called the group velocity dispersion (GVD) that changes sign as one moves from normal to anomalous dispersion. It also requires a material medium and/or a waveguide in order to exist. For $\beta_2$ = 0, the pulse will experience the nonlinear change as a self phase modulation (SPM). So, when nonlinearity counteracts or balances dispersion in nonlinear media, one obtains temporal solitary wave. Maxwell's laws govern the interaction of bodies which are electrically charged. The bodies and their charges may either be stationary or in motion [6]. Materials in which two of the components of the dielectric constant are equal (such as $\varepsilon_x=\varepsilon_y\neq\varepsilon_z$) are termed *uniaxial* crystals. This class includes trigonal, tetragonal and hexagonal crystals. Syms and Cozens [7] explained that for uniaxial crystals, two of the refractive indices are identical ($n_x = n_y = n_o$), and $n_z = n_e$, and, that this particular ellipsoid can be characterized by two values of refractive indices; $n_o$ (the ordinary index), which corresponds to $n_x$ or $n_y$, and $n_e$ (extraordinary index) corresponding to $n_z$. In this work, the Nonlinear Schrödinger Equation (NLSE) for Fourth Order is numerically simulated and stable dynamic pulse profiles for THz radiation are obtained.

## 2. NONLINEAR SCHRÖDINGER EQUATION FOR FOURTH ORDER HARMONICS

The Nonlinear Schrödinger Equation (NLSE) is an example of a universal nonlinear model that describes many physical nonlinear scenarios [8]. The mathematical description of solitons requires the solution of nonlinear Schrödinger equation (NLSE) in a dispersive medium. The Maxwell's equations provide the most fundamental description of electric and magnetic fields. According to Wang [9], THz pulses generated from an optoelectronic crystal can be calculated using the Maxwell's equations. For a neutral dielectric medium (one with no free charges, i.e $\rho = 0$), the four Maxwell's equations are;

$$\nabla . \mathbf{D} = 0 \tag{2}$$

$$\nabla . \mathbf{B} = 0 \tag{3}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{4}$$

$$\nabla \times \mathbf{H} = -\frac{\partial \mathbf{D}}{\partial t} \tag{5}$$

where **E** is the electric field , **B** is the magnetic field ,$\mathbf{D} \equiv \varepsilon E$ is the electric flux and **H** is the magnetic intensity. For a non magnetic media,

$$\mathbf{B} \equiv \mu_o \mathbf{H} \tag{6}$$

where $\mu_o$ is the permeability of free space (~ $4\pi$ x$10^{-7}$ N/A$^2$). Duffin [10] defines the electric flux as

$$\mathbf{D} = \varepsilon_o \mathbf{E} + \mathbf{P} \tag{7}$$

where $\varepsilon_o$ is the permittivity of free space (~ 8.85x$10^{-12}$ C$^2$/N.m$^2$), **P** is the polarization, and is the dielectric dipole moment per unit volume of the medium. The Maxwell's equations are used to obtain the transverse field's wave equation of the form

$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{\varepsilon_o c^2}\frac{\partial^2 \mathbf{P}}{\partial t^2} \tag{8}$$

Equation (8) is the fundamental electromagnetic wave equation of which the THz radiation is an example, and it shows how the electric field depends upon the dipole moment, P of the medium. According to Agrawal [11], the NLSE can be solved exactly using inverse scattering method (ISM) that is similar to the Fourier Transform Method. However, the NLSE belongs to a class of non-integrable partial differential equations that are not amenable to ISM, it is the variational method (VM) that proves to be a more suitable substitute for the ISM [12]. The stationary solitary wave solution for NLSE was used to generate a stationary pulse profile for the THz radiation. The propagation of THz wave in a nonlinear medium is governed by the wave equation derived from Maxwell's equation given as in equation (8). The solution of equation (8.) is of the form [13]

$$E(x,y,z,t) = \Gamma F(x,y) A(z,t)\, e^{-i(\omega t - \beta z)} \tag{9}$$

where *A(z,t)* is the amplitude of the pulse envelope, Γ is a normalization constant, and *F(x,y)* is the field distribution in the (*x,y*) plane and corresponds to the mode structure. Timtere [14] obtained the dimensionless NLSE for the fourth order harmonics as

$$i\frac{\partial U}{\partial \xi} - \frac{\delta}{2}\frac{\partial^2 U}{\partial \tau^2} - \alpha_o U + \delta_o |U| U + \delta_1 \nu |U|^3 U = 0$$
(10)

where $U$ is the dimensionless pulse amplitude (height), $\xi$ is the dimensionless pulse distance, $\nu$ is the nonlinear coefficient, and $\tau$ is the dimensionless time. $\delta = \pm 1$ defines normal or anomalous dispersion propagation. For $\delta_o = \pm 1$ implies $n_1 > 0$ or $n_1 < 0$, and $\delta_1 = \pm 1$ implies $n_3 > 0$ or $n_3 < 0$. For most nonlinear applications, it is the anomalous dispersion ($\beta_2 < 0$) that is applicable [12]. This is because in the study of nonlinear effects, it is in the anomalous dispersion regime that optical fibers support solitons through a balance between the dispersive and nonlinear effects. This means that in this work $\delta = -1$. However, in equation (10), $\delta_o$ and $\delta_1$ take positive values of +1. This is because the fourth and the fifth terms that correspond to the second ($\chi^{(2)}$) and the fourth ($\chi^{(4)}$) order nonlinearities are positive for the three uniaxial crystals studied in a work by [15].



## 3. GENERATION OF DYNAMIC THZ PULSE PROFILES

One requires a numerical simulation of the dimensionless 4$^{th}$ order NLSE to test the authenticity of the NLSE developed. The other major reason for the simulation is to establish the balance between the nonlinear coefficient, $\nu$ in the fifth term of equation (10) and the group velocity dispersion (GVP) in the second term of equation (10). The explicit finite – difference scheme is a numerical approach that is often necessary for an understanding of the nonlinear effects in fibre – like waveguides [11]. This scheme was used to solve the NLSE generated from equation (10). The values of the nonlinear coefficient $\nu$ were selected from a work by [13], where the allowed values for the nonlinear coefficient, $\nu$ saturates at 1. The values of $\tau_o$ were chosen based on the fact that good dynamic profiles, and very small dispersion and nonlinear effects are mostly observed during short pulse duration [4]. The selection of the order of the solitary wave is also based on the fact that small numerical instabilities are pronounced when N$\leq 2$. In this work, the input pulse for all the dynamic pulse profiles is the Gaussian shape, which is a component of the input THz pulse generated by [16].

## 4. RESULTS AND DISCUSSION

One should note that pulses emitted from many lasers (where THz radiation is generated) are approximated by Gaussian shapes, but other pulse shapes such as the hyperbolic – secant pulse shape that occur naturally in relation to optical solitons could be considered [4]. Figs. 1 – 4 represent the dynamic pulse evolution of THz radiation at different values of nonlinear coefficient $v$, dimensionless pulsewidth $\tau_o$, and the order of the solitary wave N

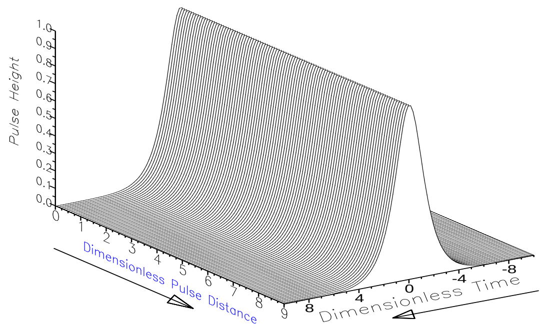


Fig. 1: The dynamic THz pulse profile for nonlinear coefficient $\nu = 0.004$, the dimensionless pulsewidth is of magnitude $\tau_o = 1$ unit, and the solitary wave
order N = 1.

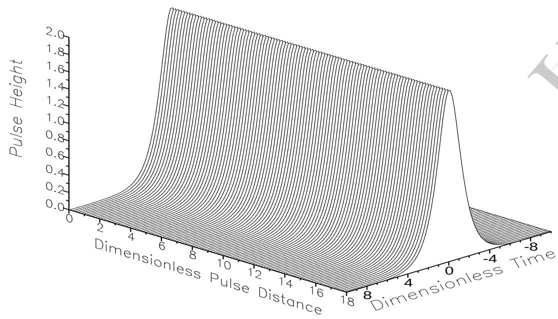


Fig. 2: The dynamic THz pulse profile for nonlinear coefficient $\nu = 0.004$, the dimensionless pulsewidth is of magnitude $\tau_o = 2$ units, and the solitary wave order N = 2.

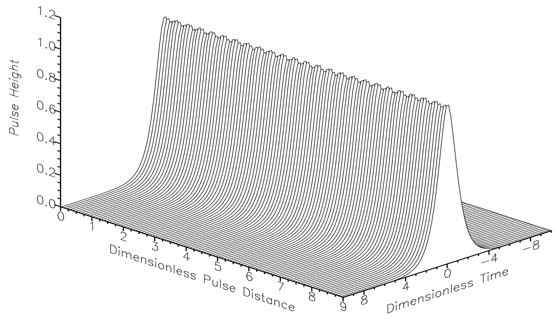


Fig. 3: The dynamic THz pulse profile for nonlinear coefficient $\nu = 2$, the dimensionless pulsewidth is of magnitude $\tau_o = 0.8$, and the solitary wave order N = 1.

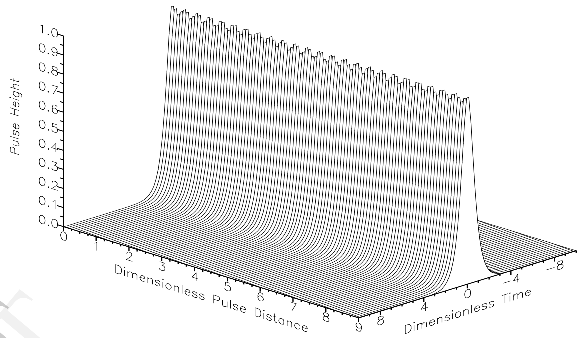


Fig. 4: The dynamic THz pulse profile for nonlinear coefficient $\nu = 1$, the dimensionless pulsewidth is of magnitude $\tau_o = 0.5$, and the solitary wave order N = 1.



In Figure 1, the dynamic pulse profile is numerically stable throughout the dimensionless distance. This could be as a result of the value of $v$ used, which is within the allowed range of values as obtained by [13]. In Figure 2, the dynamic pulse profile is also numerically stable throughout the dimensionless distances. The change in the value of the pulsewidth from 1 to 2, and the order of the solitary wave from 1 to 2 has not caused any significant change in the pulse profile. This could be as a result of the value of $v$ used, which is within the allowed range. In Figs. 3 and 4, the dynamic pulse profiles produce regular periods. These periods could be due to the increase in the value of the nonlinear coefficient. This periodicity is one of the characteristics of a solitary wave. The periodicity could be desirable if it is not much pronounced. In general, dispersion and nonlinearity act together along the length of the fibre – like THz waveguide. Within this length, there is a great deal of smooth temporal THz profile, which is very useful for optical communication systems [11].

## 5. CONCLUSION

In this work, we have generated temporal THz pulses that could be propagated in a nonlinear uniaxial crystal. The results show that stable dynamic THz pulse profiles could be obtained within the allowed range of values of the nonlinear coefficient. The work has also shown that THz radiation could be used in optical telecommunications in a uniaxial crystal fibre – like waveguides as a carrier wave, just as lasers in silica optical fibers.

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