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Generation of Associative Memories Using Cellular Automata

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Abstract

This paper presents the proposal of an associative memory implemented model with cellular automata. The model was applied to the iris plant database of the repertoire bases available by the UCI Machine Learning Repository. The model was compared with others by the reported performance making use of the k-fold cross validation.

Keywords: cellular automaton, associative memories, patterns classification.

1. Introduction

The concept of a cellular automaton (CA) was introduced in 1951 by John Von Newmann [1]. Von Newmann defines a cellular automaton as a space able to reproduce itself [2]. The cellular Automaton are mathematical models where the behavior of each one of the elements in the system depends of the local interaction with each other. A CA d-dimentional consist in a lattice or lattice d-dimentional extended infinitely that represents the "space", where each site of the lattice is called cell and have asociated a state variable, called the cell state that fluctuates on an infinite set, called state set. The time advances in discrete stages and the dynamic is given by an explicit rule called local funtion; the local function is used in each time stage for each cell to determine its new state from the current state of certain cells in its neighborhood. The cells alter their states synchronously in discrete time stages according to the local function. The Lattice is homogeneous so that all cells operate under the same local funtion. The state assigment to all the cells in the lattice is called a configuration, which is considered as the state of the total lattice. The cellular automatons have had a variety of applications in various science disciplines [3,4,5,6,7].

Oblivious to the field of cellular automata. there is the development and study of pattern recognition, and a problem of this arearefers to the patterns clasifications. The objective in the clasification consiste in partition the caracteristic space to generate regions, which will be assigned to a category or a class. Different patterns must be assigned in some of the created regions in the caracteristics space. In general, the full description of the classes is unknown. instead of this. there is a finite and reduced set of patterns that provides partial information about a specific problem.

Moreover, there is the development of associative memories, wich have been in force since the early 60's. The fundamental proposal of an associative memory is recover correctly full patterns from input patterns, wich can be alterated with additive noise, substractive or combined. The patterns clasification is one of the applications that are given to the asoctivas memories.

Several researchers have addressed the problem of developing models of associative memories [8,9,10,11,12,13] and have achieved important results for the field of research.

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2. Associative Memories.

An associative memory can be formulated as a system input and output which is divided into two phases:

Learning phase: $x \rightarrow [M] \leftarrow y$ (associative memory generation).

Recovering phase: $x \rightarrow [M] \rightarrow y$ (associative memory operation).

The input pattern is represented by a column vector denoted by **x** and the output pattern for a column vector denoted by **y**. Each one of the input patterns generate an association with the corresponding output pattern. The notation for an association is similar to an ordered pair (**x**, **y**).

The associative memory M is represented by a matrix whose component ij-th is m_{ij} [14]; the matrix M is generated from a finite set of associations previously known, called *fundamental set*. We denote by p the cardinality of the fundamental set.

The fundamental set is represented as follows:

$$\{(x_\mu, y_\mu) \mid \mu = 1,2,\ldots,p\}$$

The patterns that form the fundamental set associations are called *fundamental patterns*.

3. Cellular Automata

Be A_α a countable family of closed intervals in R such that meet the following conditions:

1. $\bigcup_{X \in A_\alpha} X = [a,b]$ for some $a,b \in \mathrm{R}$ or $\bigcup_{X \in A_\alpha} X = R.$
2. $[a_i, b_i] \in A_\alpha \Rightarrow b_i - a_i > 0.$
3. $[a_i, b_i.], [c_j, d_j.] \in A_\alpha \Rightarrow$ $[a_i, b_i] \cap [c_j, d_j] = \varnothing \vee$ $[a_i, b_i] \cap [c_j, d_j] = b_i = c_j$

Definition 3.1 Be $[a,b]$ an interval of R with $a \neq b$ and A_α a closed intervals family that satisfy 1,2 and 3. A *lattice* of dimentions 1 or 1-dimentional is the set $\mathrm{L} = \{x_i \times [a,b] \mid x_i \in A_\alpha\}$. If $A_{\alpha 1}, A_{\alpha 2}, \ldots, A_{\alpha n}$ are intervals families thet satisfy 1, 2 and 3 , so a lattice of dimention $n > 1$ is the set $\mathcal{L} = \{x_{\alpha 1} \times x_{\alpha 2} \times \cdots \times x_{\alpha n} \mid x_{\alpha 1} \in A_{\alpha i}\}$.

Definition 3.2 Be $r \in R$ a *lattice* 1-dimentional is regular if $[a_i, b_i] = r$ for each $[a_i\ , b_i\ ] \in A_\alpha$. A lattice n-dimentional is regular if $[a_{a_{ik}}, b_{a_{ik}}] = r$ for each $[a_{a_{ik}}, a_{a_{ik}}] \in A_{\alpha_i}$ for *i* = 1, ..., *n*.

Definition 3.3 Be $\mathcal{L}$ a lattice. A cellule, cell or site is an elemnt of $\mathcal{L}$. This is, a cell is an elemnt of the form $[a_{\alpha_{1_k}}, b_{\alpha_{1_k}}] \times \cdots \times [a_{\alpha_{n_k}}, b_{\alpha_{n_k}}]$ with $[a_{\alpha_{i_k}}, b_{\alpha_{i_k}}] \in A_{\alpha_i}$ for $i = 1, \ldots, n.$.

Definition 3.4 Be ***L*** a lattice, and is r a cell of ***L***. A neighbourhood of size $n \in \mathcal{N}$ to r , is the set $v(r) := \{\{k_{1,}\ k_{2,\ \ldots,} k_n\} \mid k_j$ is a cell of $\mathcal{L}$ for each $j\}$.

Definition3.5 Be $n \in N$. A cullular automata, is a tuple $(\mathcal{L}, S, \mathcal{N}, f)$ such that:

1. $\mathcal{L}$ is a regular lattice.
2. S is a finite set of states
3. $\mathcal{N}$ is a defined neighborhoods set as follows.

$\mathcal{N} = \{\mathcal{N}(r) \mid r$ is a cell and $\mathcal{N}(r)$ is a neighborhood of r of size $n\}$

4. $f : \mathcal{N} \rightarrow S$ is a function called transition function.

Definition 3.6 Is $Q = (\mathcal{L}, S, \mathcal{N}, f)$ and $W = (\mathcal{L}, S, \mathcal{N}', g)$ two CA. Is defined the CA composition of the $CA\ Q$ y W in the time $t = t_0$ denotated as $W * Q$ by the CA $W * Q = (\mathcal{L}, S, \mathcal{N}, f)$ where h, f y g are relationated as follows:

$$C_{t_0+1}(r) = f(\{C_{t_0}(i) : i \in N(r)\}$$

$$C_{t_0+2}(r) = g(\{C_{t_0}(i) : i \in N'(r)\}$$

$$C_{t_0+2}(r) = h(\{C_{t_0}(i) : i \in N(r)\}$$

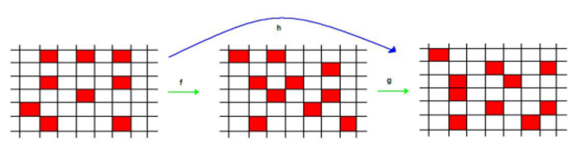

Fig. 2. Example of a CA composition.

4. Proposed model

This section will build the associative memory by Cellular Automata.

In what follows, consider the set $A = \{0,1\}$ an the fundamental set $CF = \{(x^\mu, y^\mu) \mid \mu = 1,2,\ldots, p\}$ with $x^\mu \in A^n$ y $y^\mu \in A^m$.

The lattice $\mathcal{L}$ for the CA will be composed by the matrix of size $2m \times 2n$ with the first index the couple $(0,0)$.

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The set $S=\{0,1\}$ is the finite states set

Is $I=\{i\in Z \mid i=2k$ for some $k=0,1,2,...,n-1]\}=\{0,2,4,...,2(n)-2\}$ and $J=\{j\in Z \mid j=2k+1$ for some $k=0,1,2,...,m-1]\}=\{1,3,5,...,2m-1\}$.

Considerate the partition of $\mathcal{L}$ formed by the subsets family $IJ=\{v_{(i,j)} \mid (i,j)\in I\times J\}$ with $v_{(i,j)}=(i,j),(i,j-1),(i+1,j),(i+1,j-1)\}$. Inasmuch as IJ is a partition of $\boldsymbol{L}$, given v^l exist an unique IJ such that $v^l=v_{(i,j)}$. For example, if $l=(3,0)$, so $l\in v^{(3,0)}=v_{(2,1)}=\{(2,1),(2,0),(3,1),(3,0)\}$.

From the previous fact is defined the neighbourhood set

$$N=\{v^l \mid l\in L)$$

Definition 4.1 Consider the set A^k. Is defined the projected funtion of the $i-th$ component $(1\le i\le k)\,\Pr_i : A^k\to A$ as

$$\Pr_i(z)=z_i, con \quad z=(z_{1,}z_{2,...,}z_k)$$

Proposition 4.2 If $(y_{i,}x_j)\in\Pr_{yx}=\{(y_{i,}x_j)\mid y_i=\Pr_i(y)$ and $x_j=\Pr_j(x)\}$, so $(2_j-2+y_i,2_i-2+x_j)\in v_{(2_j-2,2i-1)}$.

Demonstration Must be $v_{(2_j-2,2i-1)}=\{(2j-2,2i-1),(2j-2,2i-2),.$

$(2j-1,2i-1),(2j-1,2i-2)\}$.

Inasmuch as $(y_i,x_j)\in\Pr_{yx}$, so $y_i=\Pr_i(y)$ y $x_i=\Pr j(x)$ and in as much as $x\in A^n$ and $\in A^m$,

so $y_i, x_j\in\{0,1\}$, , then

1. if $y_i=x_j=0$, so $(2j-2+y_i,2i-2+x_j)=(2j-2,2i-2)\in v_{(2j-2,2i-1)}$.

2. if $y_i=0\,y$ $x_j=1$, so $(2j-2+y_i,2i-2+x_j)=(2j-2,2i-1)\in v_{(2j-2,2i-1)}$.

3. if $y_i=1$ y $x_j=0$, so $(2j-2+y_i,2i-2+x_j)=(2j-1,2i-2)\in v_{(2j-2,2i-1)}$.

4. if $y_i=x_j=1$, so $(2j-2+y_i,2i-2+x_j)=(2j-1,2i-1)\in v_{(2j-2,2i-1)}$.

Is defined the set $\mathcal{L}_{CF}=\{(2j-2+y_i^u,2i-2+y_j^u \mid 1\le\mu\le p,1\le i\le myl\le j\le n\}\subseteq l.$

.

Consider the cellular automata $Q=(\mathcal{L},S,\mathcal{N},f_Q)$ and $\mathcal{W}=(\mathcal{L},S,\mathcal{N}',f_w)$ with $\mathcal{N}'=IJ$, y $f_Q:\mathcal{N}\to S, fw:\mathcal{N}'\to S$ defined as follows:

$$f_Q(v^{(i,j)})=\begin{cases}1\,if\,(i,j)\in\mathcal{L}_{CF}\\ 1\,if\,(i,j)\notin\mathcal{L}_{CF}\end{cases}$$

$$f_w(v_{(i,j)})=\begin{cases}1\;in\;the\;position\,(i+1,j)\;if\;(i,j-1)=1\\ 1\;in\;the\;position\,(i,j-1)\;if\;(i+1,j)=1\end{cases}$$

It defines the CA associative (CAA) in its rearning phase as $\mathcal{W}*Q=(\mathcal{L},S,\mathcal{N},f_{\mathcal{A}})$

For the learning phase, is represented two different algorithms, aim is recover an associative pattern associated to an input pattern. First is represented the *max* algorithm to recover the patterns and follows the *min* algorithm to recover the patterns.

Algorithm to recovering patterns *max*

INPUT : Pattern that recognized $\tilde{x} = \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \vdots \\ \tilde{x}_n \end{pmatrix}$

OUTPUT: Recovered pattern $\tilde{y} = \begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_n \end{pmatrix}$

PROCESS:

$for\ i = 1,2,\ldots,m$
  $\tilde{y}_i = 1$
    $for\ j = 1,2,\ldots,n$
      $if\ \tilde{x}_j == 0\ \&\&(2j-1, 2i-2) == 1$
        $continue$
      $if\ \tilde{x}_j == 1\ \&\&((2j-2, 2i-2)\ ||\ (2j-1, 2i-2) == 1)$
        $continue$
      $else$
        $\tilde{y}_i = 0$
        $break$
    $end\ second\ for$
$end\ first\ for$

Algorithm to recovering patterns *min*

INPUT : Pattern to recognized $\tilde{x} = \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \vdots \\ \tilde{x}_n \end{pmatrix}$

OUTPUT: Recovered Pattern $\tilde{y} = \begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_n \end{pmatrix}$

PROCESS:

$for\ i = 1,2,\ldots,m$
  $\tilde{y}_i = 0$
    $for\ j = 1,2,\ldots,n$
      $if\ \tilde{x}_j == 1\ \&\&(2j-2, 2i-1) == 1$
        $continue$
      $if\ \tilde{x}_j == 0\ \&\&((2j-2, 2i-1)\ ||\ (2j-1, 2i-1) == 1)$
        $continue$
      $else$
        $\tilde{y}_i = 1$
        $break$
    $end\ second\ for$
$end\ first\ for$

Example 4.3. Be $m = 4, n = 3$ and $p = 3$. The fundamental set $CF = \{(x^1, y^1), (x^2, y^2), (x^3, y^3)\}$ is given by:

$$x^1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\ y^1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$x^2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\ y^2 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$x^3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\ y^3 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

- The lattice $\mathcal{L}$ is composed by the matrix of size $2m \times 2n = 8 \times 6$.
- The states set $S = \{0,1\}$.
- The Neighbourhood set is given by $N = \{v^1 : 1 \in \mathcal{L}\}$
- The next set $\mathcal{L}_{CF}$, is one in which f_Q take the value of 1 and 0 in its complement. (Figure 3a)

$$\mathcal{L}_{CF} = \{(2j-2+y_i^u, 2i-2+x_j^u) \mid 1 \le \mu \le 3, 1 \le i \le 4, 1 \le j \le 3\} = \{(0,0), (1,0), (2,0), (4,0), (5,0), (0,1), (3,1), (4,1), (0,2), (1,2), (2,2), (3,2), (4,2), (0,3), (2,3), (5,3), (0,4), (2,4), (4,4), (0,5), (2,5), (4,5), (0,6), (2,6), (3,6), (4,6), (5,6), (1,7), (2,7), (4,7)\}$$

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- Applying f_W to the previous CA, is obteined the CAA that it shows in the figure 3b.
- Now apply the algorithm to recovering patterns *max* and *min* from the CAA

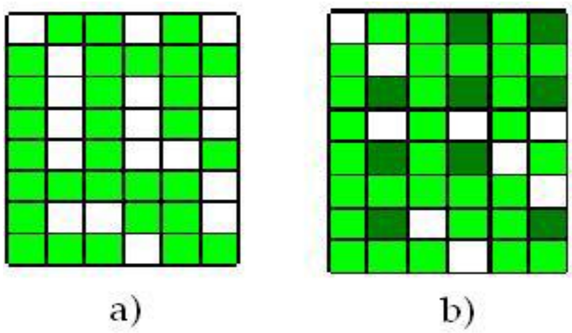

Figure3. Configuration of the CA of the example 5.3, in a) after to apply f_Q and in b) after to apply f_W.

Consider the input pattern

$$x^1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

The table 1 shows the initial an the final value for each component of the output vector y^1 when is applied the *max* algorithm for pattern recovery. The first column shows the value por the variable *i* considerated the first **for** of the algorithm. the second column is the value of y_i^1 for default, wich is 1. The third column are the differents values for the cycle of the variable j, the fourth column is the respective value tha has x_j^1, the fifth column is the condition that must comply in the algorithm depends if the value of x_j^1 is 0 o 1.

Finally the sixth column shows the final value of the y_i^1 component.

Table 1. Configuration of the CA of the example 5.3 in a) after to apply f_Q and in b) after to apply f_W.

i	y_i^1	j	x_j^1	*condition*	$\rightarrow y_i^1$
1	1	1 2	1 0	$(2j-2,2i-2) == 1$ $(2j-1,2i-2) != 1$	1 0
2	1	1 2 3	1 0 0	$(2j-2,2i-2) == 1$ $(2j-1,2i-2) == 1$ $(2j-1,2i-2) != 1$	1 1 0
3	1	1 2	1 0	$(2j-2,2i-2) == 1$ $(2j-1,2i-2) != 1$	1 0
4	1	1 2 3	1 0 0	$(2j-2,2i-2) == 1$ $(2j-1,2i-2) == 1$ $(2j-1,2i-2) == 1$	1 1 1

From the table 1 we have $y^1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$. So recover the pattern y^1 when is presentated the input pattern x^1 using the *max* algorithm for pattern recovery.

Similarly to the above table, Table 2 shows the initial and the final value for each component of the output vector y^1 when is applied the *min* algorithm for pattern recovery.

Table 2. Configuration of the CA of the example 5.3 in a) after to apply f_Q and in b) after to apply f_W.

i	y_i^1	j	x_j^1	*condition*	$\rightarrow y_i^1$
1	0	1 2 3	1 0 0	$(2j-2,2i-1) == 1$ $(2j-1,2i-1) == 1$ $(2j-1,2i-1) == 1$	0 0 0
2	0	1 2 3	1 0 0	$(2j-2,2i-1) == 1$ $(2j-2,2i-1) == 1$ $(2j-1,2i-1) == 1$	0 0 0
3	0	1 2 3	1 0 0	$(2j-2,2i-1) == 1$ $(2j-2,2i-1) == 1$ $(2j-2,2i-1) == 1$	0 0 0
4	0	1	1	$(2j-2,2i-1) != 1$	1

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From the table 2 we have $y^1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$. So, recover the pattern y^1 when is presentated the pattern x^1 using the *min* algorithm for pattern recovery.

5. Experiments and results

For the experimental part was used the Iris database provided by the University of California Irvine Machine Learning Repository available in *http://www.ics.uci.edu/~mlearn/mlrepository.html*. The Iris database count with 150 instances, each instance with 4 real attributes without information loss, divided in 3 classes: Iris Setosa, Iris Versicolour and Iris Virgínica. To validate the test, it was considered the *k-fold cross validation* method with k = 10. The CAA was applied in its learning phase. For the recovering phase it was applied the *max* algorithm and the *min* algorithm. The figure 4 shows the CAA configuration in its learning phase, and the table 3 shows the result of the model compared with another results applied at the Iris Plant database.

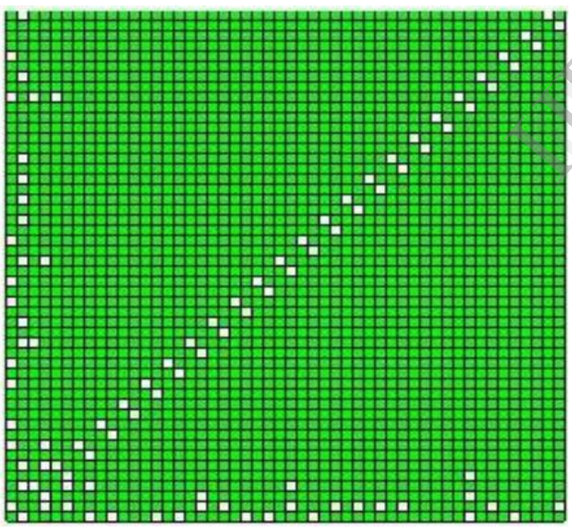

Figure 4. CAA configuration in its learning phase for the Iris Plant database.

Table 3. Comparison of proposed model with other models.

Model	Iris Plant (%)
Bayesian Network (K2) [15]	93.20
Adaboost NB [15]	94.80
Bagging NB [15]	95.53
NBTree [15]	93.53
LogitBostDS [15]	94.93
K-Means [16]	89
Neural Gas [16]	91.7
MLP [17]	95.99
NBT [17]	93.99
PART [17]	94.66
ACA with *max* recuperation	99.33
ACA with *min* recuperation	99.33

6. Conclusions

It has been presented a model of associative memory based on cellular automata that we call CAA. For the learning phase the CAA is builded from a fundamental set. For the recovering there is two algorithms: the *max* and the *min* recovering algorithms. The model was applied to the database of Iris Plant from the databases available in the repertory by the UCI Machine Learning Repository. The model was compared with other models by their showed yield.

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