Generalized γ-Axiom for Advanced Engineering Applications

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ABSTRACT

In this paper we discuss the properties, characterization of γ- open sets , relations between regular open set , γ- open set and semi open set, γ- adherent, γ-closure of a subset A of a topological space, γ- irresolute function between topological spaces and equivalence relation γ- correspondence on the set of a topologies of a set X .The inter relationship between γ-axiom and various Separation axioms.

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1. INTRODUCTION

A subset A of a topological space (X,τ) is said to be γ-open iff there exists a regularly open set R such that R⊆A⊆cl(R)The Paper is organized as follows.In section (2) Properties and characterization of γ-open sets and relations between regular open sets, γ-opensets and semi open sets.

It is interesting to note that

(1)The complement of a γ-open set is again a γ-open set.

(2)Neither the union nor the intersection of two γ-open sets is γ-open. In Section (3) ,γ-adherant, μ-closure of a subset A of a topological space,γ-irresolute function between topological space and equivalence relation γ-correspondance on the set of a topologies of a set X.Section (4) is devoted to the study of γ-axiom due to Sharma[27]. Separation axioms γT_0, γT_1 and γT_2 which are generalizations of separation axioms T_0,T_1 and T_2 respectively are studied. It is interesting to note in a topological space all the three axioms γT_0, γT_1 and γT_2 coincide. Therefore these axioms are referred as μ-axiom.The interrelationship between μ-axiom and various separation axioms semi T_2, semi T_1, semi T_0 and γT_0 are discussed.

2.γ-open sets

Definition : 2.1

A subset A of a topological space (X, τ) is said to be γ-**open** iff there exist a regularly open set R such that R⊆A⊆cl(R).

Theorem :2.2

A subset A of a topological space (X, τ) is said to be γ-**open** iff there exist a regularly closed set F such that int(F)⊆A⊆F.

Proof :

Assume A is a γ-open set.
Since A is γ-open, there exist a regularly open set R such that R⊆A⊆cl(R).
R=int(cl(R) ⟹ cl(R) = cl(int(cl(R)))
⟹ F= cl(int(F)).
we getA⊆F
Hence int(F)⊆A⊆F.
Hence there exist a regularly closed set F such that int(F)⊆A⊆F
Let R=int(F)
Consider int(cl(R))=int(cl(int(F)))=int(F)=R,
Hence R⊆A⊆cl(R)
Hence A is γ-open.

Theorem :2.3

A necessary and sufficient condition for a set A in a topological space (X,τ) is said to be γ-open is, int(cl(A))⊆A⊆cl(int(A)).

Proof :

Assume A is a γ-open set in (X, τ).
Since A is γ-open,there exists a regularly open set R such that R⊆A⊆cl(R).
int(cl(R))⊆ int(cl(A))
R ⊆ int(cl(A))
A⊆cl(R) ⟹ int(cl(A)) ⊆ int(cl(R))=R
Hence int(cl(A))⊆R
Hence int(cl(A))⊆A
R⊆A, R⊆int(A) ⟹ cl(R)⊆cl(int(A))
A⊆cl(R), A⊆cl(int(A))
Hence int(cl(A))⊆A⊆cl(int(A))
Conversely assume int(cl(A))⊆A⊆cl(int(A))
Let R= int(cl(A))
Then R= int(cl(A))⊆A⊆cl(int(A)) =cl(R).
Therefore R⊆A⊆cl(R).

Theorem :2.4

A is γ-open in a topological space (X, τ) iff A is semiopen as well as semi closed in (X, τ).

Proof :

Assume A is γ-open, we get int(cl(A))⊆A⊆cl(int(A)) (by theorem 2..3) ⟹ int(cl(A))⊆A and A⊆cl(int(A)).
A⊆cl(int(A)), for R=cl(A), we have R⊆A⊆cl(R)
int(cl(A))⊆A, for R=cl(A), we have int(R)⊆A⊆cl(A)=R
Conversely assume A is semi open as well as semi closed.
R⊆A⟹ cl(R)⊆cl(int(A))
Hence A⊆cl(R)⊆cl(int(A))
int(R)⊆A⊆R for some closed set R
A⊆R ⟹ cl(A)⊆R
Then int(cl(A))⊆int(R)⊆A
Hence int(cl(A))⊆A⊆cl(int(A))
Hence by theorem 2.3, we have A is γ-open.

Corollary :2.5 A set A in a topological space (X, τ) is
γ-open iff A=s-cl-s-int(A) and A=s-int-s-cl(A).

Remark : 2.6 By definition of a γ-open set it follows that every regularly open set is a γ-open set and every γ-open set is semi open set,
i.e., R O (X, τ) ⊆ V O(X, τ) ⊆ S O ⊆ (X, τ)
However the converse of the above statements are not true in general as shown by the following examples.

Example : 2.7
Let X={a,b,c} and τ ={∅,X,{a},{b},{a,b}} Then {a,c} is a γ-open set but not regularly open.

Example : 2.8
Let X={a,b,c} & τ ={∅,X,{a}} then the set {a,b} is a semi open which is not γ-open.

Theorem :2.9
A γ-open set A is regularly open if A⊆int(cl(A))

Proof :
Assume A is γ--open, there exists a regularly open set O such that O⊆A⊆cl(O) and A⊆int(cl(A))
int(cl(A))⊆O⊆A⟹int(cl(A))⊆A
Hence A is regularly open**.**

Theorem :2.10 A semi open set A is γ-open if int(cl(A)) ⊆ A

Proof :
Since A is semi open, there exists an open set O such that O⊆A⊆cl(O)
Hence A⊆cl(O)⊆cl(int(A))
Hence A⊆cl(int(A))
By our assumption, int(cl(A))⊆A
Hence int(cl(A))⊆A⊆cl(int(A))
Hence by theorem 2.3, we get A is γ-open.

Theorem :2.11 A semi closed set A is γ-open if A⊆cl(int(A))

Remark : 2.12
Neither the union nor the intersection of two γ-open sets is γ-open.

Example : 2.13
Let X={a,b,c} and τ ={∅,X,{a},{b},{a,b}} then {a,c} and {b,c} are γ-open sets. But {a,c}∩{b,c}={c} is not a γ-open sets.

Example : 2.14
Let X={a,b,c} and τ ={∅,X,{a},{b},{a,b}} then {a} and {b} are γ-open sets but {a}∪{b}={a,b} is not a γ-open set.

Theorem :2.15
The complement of γ-open set is again a γ-open set.

Proof :
Assume A is γ-open, there exists a regularly open set R such that R⊆A⊆cl(R)
Therefore we have X-R ⊇ X-A ⊇ X-cl(R)
Hence int(X-R)⊆X-A⊆X-R
R is regularly open, X-R is regularly closed.
Hence by Theorem 2.2, and from (1), we have A is γ-open.

Theorem :2.16 If A is a γ-open set then
(a) int(cl(A))=int(A)
(b) cl(R)=cl(A), where R is a regularly open set such that R⊆A⊆cl(R).

Theorem :2.17
If A and R are regularly open sets and S is γ-open such that R⊆S⊆cl(R), then A∩S=∅, implies A∩S=∅.

Proof : Assume A and R are regularly open sets and S is γ--open such that
R⊆S⊆cl(R)
Since A∩R=∅, therefore R⊆X-A.
Hence X-A is closed.
Therefore R⊆X-A implies cl(R) ⊆ X-A
S⊆cl(R), we get S⊆X-A.
Hence A∩S=∅.

Theorem :2.18
Intersection of a γ-open set S and regularly open set U is a γ-open set.

Proof :
Since S is γ-open, there exists a regularly open set R
Case(i): S∩U = ∅
Let S∩U =∅
It is obvious that then S∩U is a γ-open set.
Case(ii): S∩U≠∅
we get R∩U ≠∅ (by Theorem 2.17)
Since R⊆S, we get R∩U⊆ S∩U
If x∈S∩U, then either x belongs to U and R (or) x belongs to U and S-R.
Case (a): x belongs to U and R.
Then x∈U∩R⊆cl(U∩R)
Hence U∩R⊆S∩U⊆cl(U∩R)
Therefore, S∩U is a γ-open set.
Case (b): x belongs to U and S-R.
Then x belongs to U and x is a limit point of R, since S⊆cl(R).
Let N be a γ-neighbourhood of x.
Then N∩U is a γ-neighbourhood of x and (N∩U)∩R ≠ ∅ which implies N∩(U∩R) ≠ ∅
Hence x belongs to cl(U∩R)
Hence R∩U⊆S∩U⊆cl(R∩U)
Hence S∩U is a γ-open set.

Theorem :2.19 If B is a subset of a topological space (X, τ) such that A⊆B⊆cl(A). Then B is γ-open if A is γ-open.
Hence the claim.

Definition : 2.20

Let (X, τ) be a topological space. Then the set of all regular open set forms a base for a topology τ^* on X called the **semi regularization topology** of X such that $\tau^* \subseteq \tau$. The space (X, τ^*) is called the **semi regularization space** of (X, τ).

Definition : 2.21

A topological space (X, τ) is said to be **semi regular** iff $\tau = \tau^*$.

Lemma : 2.22

R O (X,)=int V O (X, τ), where int V O (X, τ) denotes the collection of the interior of γ-open sets in a topological space (X, τ).

Proof :

Let (X, τ) be a topological space.

Claim: R O (X,)=int V O (X, τ)

Let A ∈ int V O (X, τ) .

Then A=int(B) for B ∈ int V O (X, τ).

A=int(B)=int(cl(B)) (by theorem 2.1.16(a))

Therefore A ∈ R O (X, τ)

HenceintVO(X,)⊆RO(X,τ) ....(1)

Then R ∈ V O (X,), (by Remark 2.1.6)

Therefore R ∈ int V O (X, τ)

Hence R O (X, τ) ⊆ int V O (X, τ) ....(2)

From (1) and (2), we have R O (X, τ)=int V O (X, τ)

Remark : 2.23

By Lemma 2.22, we get that the collection of γ-open sets generates a topological space on (X, τ).

Theorem :2.24

In a semi regular space (X, τ), int V O (X, τ) generates topology τ on X.

Proof :

Assume (X, τ) is a semi-regular space.

To prove int V O (X, τ) generates topology τ on X.

By Lemma 2.22 collection of int V O (X, τ) forms a base for a topology τ^* on X such that $\tau^* \subseteq \tau$.

Since X is semi-regular , $\tau = \tau^*$

Hence int V O (X, τ) generates the topology τ .

Theorem :2.25

Let A⊆Y⊆X, where Y is a regularly open subspace of a topological space (X, τ). Then A ∈ V O (X, τ) implies A ∈ V O (Y, τ_y).

Proof :

Since A is γ-open in X, there is a regularly open set R in X such that $R \subseteq A \subseteq cl_x R$.

Hence $R \cap Y \subseteq A \cap Y \subseteq (cl_x R) \cap Y$

Hence A ∈ V O (Y, τ_y).

Theorem :2.26

If R is a regularly-open subspace of a topological space (X, τ) and V a γ-open set in X, then R∩V is γ-open in R.

Proof :

Assume R is a regularly open set in a topological space (X, τ) and V is γ-open set in X.

By Theorem 2.18, we get R∩V is a γ-open set in X.

Since R is a regularly open subspace of X, we get R∩V is a γ-open set in R

(by theorem 2.18).

Theorem :2.27

Let Y be a subspace of a topological space (X, τ) and A ∈V O (Y, τ_y). Then A∈ V O (X, τ) iff Y is γ-open in X.

Proof :

Since A ∈ V O (Y, τ_y), we get A ∈ V O (X, τ)

Hence Y is regularly open.

Hence Y is a γ-open in Y. i.e., Y ∈ V O (Y, τ_y).

Y ∈ V O (X, τ).

Hence Y is γ-open in X.

Conversely assume Y is γ-open in X.

Since A ∈V O (Y, τ_y), there exists a regularly open set R in Y such that $R \subseteq A \subseteq cl_y R$.

Let R_1 be a regularly open set in X such that $R_1 \cap Y = R$.

Then by Theorem 2.18, R is γ-open in X.

Hence

$R_1 \cap Y \subseteq A \subseteq cl_y(R_1 \cap Y) = \{cl_x(R_1 \cap Y)\} \cap Y \subseteq cl_x(R_1 \cap)$ implies

$R \subseteq A \subseteq cl_x R$ (since $R = R_1 \cap Y$)

By applying Theorem 2.19, we have A ∈V O (X, τ).

Hence A ∈ V O (X, τ).

Definition : 2.28

A subset M of a topological space (X, τ) is called a γ-**neighbourhood** of x⊆X if there exists a γ-open set V in X such that X∈V⊆M

Theorem : 2.29

Let Y be a subspace of a X and A be a γ-neighbourhood of x in Y. Then A is a γ - neighbourhood of x in X iff Y is γ-open in X.

3.γ-irresolute functions

Definition : 3.1

A function f:X⊆Y is said to be γ-**irresolute** iff for any γ-open set V of Y, $f^{-1}(V)$ is γ-open in X.

Theorem : 3.2

An almost-continuous and almost-open mapping f:X⊆Y is γ-**irresolute.**

Theorem : 3.3

An identity mapping on a topological space (X, τ) is γ-irresolute.

Definition : 3.4

A point x in a topological space (X, τ) is said to be γ-**adherent** of a subset G of X if every γ-open set containing x has a non empty intersection with G.

Definition : 3.5

The set of all γ-adherent points of a set G is called γ-**closure** of G or the intersection of all γ-open sets containing G is known as γ-closure of G and denoted as γ-cl(G).

Theorem : 3.6

If A is a subset of a topological space (X, τ) , then

$A \subseteq$ s-cl(A) $\subseteq \gamma$ -cl(A) $\subseteq \delta$ -cl(A).

Proof :

Since every regularly open set is γ-open and every γ-open set is semi open, the Theorem follows.

Definition : 3.7

A subset A in (X, τ) is said to be δ_v **- closed** if A= γ -cl(A).

Remark : 3.8

Two different topologies τ_1 and τ_2 on a set X may have same class of γ-open sets. This leads to define the notation of γ-correspondence.

Definition : 3.9

Two topologies τ_1 and τ_2 on a set X are said to be γ **–correspondent** if V O (X, τ_1) = V O (X, τ_2).

Example : 3.10

Let X={a,b,c} and τ_1 = {∅,{a},{b,c},X} and τ_2 = {∅,{a},{c},{a,c},{b,c},X}, then V O (X, τ_1) = V O (X, τ_2).

Theorem : 3.11

The relation of γ-correspondence on the collection of all topologies on a set X is an equivalence relation.

4.γ-**axiom:**

Definition : 4.1

A topological space (X, τ) is said to be γ**T**$_0$ if for each pair of distinct points x,y of X there exists a γ-open set G containing x but not y (or) a γ-open set T containing y but not x.

Definition : 4.2

A topological space (X, τ) is said to be γ**T**$_1$ if for each pair of distinct points x,y of X there exists a γ-open set G containing x but not y (or) a γ-open set H containing y but not x.

Definition : 4.3

A topological space (X, τ) is said to be γ**T**$_2$ if for each pair of distinct points x , y of X there exists a γ-open set G and H such that $x \in G$, $y \in H$ and $G \cap H = \emptyset$

Theorem : 4.4

For a topological space (X, τ) the following are equivalent.

a) X is γT_0
b) X is γT_1
c) X is γT_2

Proof:

Let (X, τ) be topological space.
Let x , y $\in$ X such that $x \neq y$.
Since X is γ-T_0 , i.e., $x \in G$ and $y \notin G \Longrightarrow x \notin X$-G and $y \in X$-G
Let X-G be H.
Then $y \in H$ also $G \cap H = \emptyset$
Hence for $x \neq y \in X$ there exists γ-open set G containing x but not y and a γ-open set H containing y but not x.
Hence X is γT_1.
Assume X is γT_1
Let x , y $\in$ X such that $x \neq y$.
Since X is γT_1 , there exists a γ-open set G containing x but not y and a γ-open set H containing y but not x.
Since $x \in G$ and $y \notin G$,we get $x \notin X$-G and $y \in X$-G.
Let H=X-G.
Then H is a γ--open set since G is a open set.
Also $G \cap H = \emptyset$
Hence there exists γ-open sets G and H such that $x \in G$, $y \in H$ and $G \cap H = \emptyset$
Therefore X is γT_2.
As γT_2 axiom is stronger than γT_0 axiom, the result follows.

Remark : 4.5

The three axioms γT_0, γT_1, γT_2 coincide. Hence we shall refer the three axioms as γ-axiom.

Definition : 4.6

A topological space (X, τ) is said to be γ-**space** if for each pair of distinct points x , y of X there exists a γ-open set G containing x but not y.

Proposition : 4.7

A γ-space X is semi T_2.

Proof:

Since every γ-open set is semi open, we have X is semi T_2.

Example : 4.8

Let X={a,b,c,d}, and τ ={∅, {a}, {b}, {c} ,{a,b}, {a,c}, {b,c}, {b,d}, {a,b,c}, {a,b,d}, {b,c,d}, X}. Then X is a semi T_2 space but not a γ-space.

Remark : 4.10

(i) Since every γ-space is semi T_2 and semi T_2 implies semi T_1 , we get every γ-space is semi T_1.
(ii) Every semi T_1 space need not be a γ-space.

Example : 4.11

Let X={a,b,c,d}, and τ ={∅, {a}, {c}, {a,c}, {c,d}, {a,c,d},{b,c,d},X}. Then the space X is a semi T_1 space which is not a γ-space.

Example : 4.12

Let X={a,b,c} and τ ={∅,{a},X}. Then X is a semi T_0 space but not a γ-space.

Proposition :4.13

Every rT_0 space is a γ-space.

Proof:

Let X be a rT_0 space.
Then for each pair of distinct points x , y of X there exists a regularly open set G containing x but not y.
Since every regularly open set is a γ-open set , G is a γ-open set.
Then for each pair of distinct points x , y of X there exists a γ-open set G containing x but not y.
Hence X is a γ-space.
Hence every rT_0 space is a γ-space.

Remark : 4.14

The converse of proposition 4.13 is not true.

Example : 4.15

Let X={a,b,c,d}, and τ ={∅,{a},{b},{a,b}, X}. Then X is a γ-space but not rT_0.

Remark : 4.16

The concepts of a space being a γ-space and T_0-space are mutually independent.

Example : 4.17

Let X={a,b,c}, and τ ={∅,{a},{a,b}, X}. Then X is a T_0-space but not a γ-space.

Example : 4.18

Let X={a,b,c,d}, τ ={∅, {a}, {b}, {a,b}, X}. Then X is a γ-space but it is not a T_0-space.

Remark : 4.19

The axiom of γ-space is independent of the T_1-axiom.

Example : 4.20

Let X be countable set, τ ={∅,X,A} where A ⊂ X such that X-A is finite.

Example : 4.21

Let X={a,b,c,d} and τ ={ ∅,{a}, {b}, {a,b},{d}, {a,d}, {b,d}, {a,b,c}, {a,b,d},X}. Then X is a γ-space but not T_1.

Theorem : 4.22

A space X is a γ-space iff each singleton set in X is δ_v-closed.

Proof:

Let X be a γ -space.
Let x,y ∈ X such that x ≠ y.
Since X is a γ-space, there exists a γ-open set containing y but not x.
Then y ∉ γ -cl{x}.
Hence γ -cl{x} ⊆ {x}
Therefore {x} = γ -cl{x}.
Hence {x} is δ_v-closed.
Conversely let x ≠ y ∈ X.
Then by the hypothesis, {x} and {y} are δ_v-closed.
i.e., {x} = γ -cl{x} and {y} = γ -cl{y}.
Therefore y ∉ γ -cl{x} and x ∉ γ -cl{y}.
Hence there exists a γ-open set containing x but not y.
Hence X is a γ-space.

Theorem : 4.23

The necessary and sufficient condition for a space X to be a γ-space is that for each x ∈ X there exist a γ-open set U of X containing x such that the subspace U is a γ-space.

Theorem : 4.24

Every regularly open subset Y of a γ-space X is a γ-space.

Proof:

Let Y be regularly open subset of a γ-space X.
Claim: Y is a γ-space.
Let x,y ∈ X such that x ≠ y.
Then x ≠ y ∈ X.
Since X is a γ-space, there exist a γ-open set G containing x but not y.
Hence G ∩ Y is a γ-open set in Y (by Theorem 2.26), containing x but not Y.
Hence Y is a γ-space.

Theorem : 4.25

If f is a γ-irresolute function from a space X to a γ-space Y.Then X is a γ-space.

Theorem :4.26

The product of two γ-spaces is a γ-space.

CONCLUSION

This paper is an attempt to generalize **μ**-open sets due to Sharma [25] to fuzzyTopological spaces.

μ-open sets and **μ**-axiom due to Sharma[25,26] are analyzed. **μ**-open sets, its properties and characterizations are analyzed .

μ-adherent, **μ**-closure of a subset A of a topological space, **μ**-irresolute function between topological spaces and the relation **μ**-correspondence on the set of the topologies on a set X are analyzed.

separation axioms μT_0, μT_1 and μT_2 and their equivalence in topological spaces are discussed. Properties and characterizations of **μ**-spaces are analyzed.

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