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Gardner SWs and DLs in multi-ion dusty degenerate dense plasma

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The theoretical investigation has been made of the roles of the degeneracy and the dynamics of electrons and multi-ions (both positive and negative ions) on the DIA (dust ion-acoustic) solitary waves (SWs) and double layers (DLs) that are found to exit in a dusty degenerate dense plasma containing non-relativistic degenerate positive and negative ions and both non-relativistic and ultra relativistic electrons fluids, and the negatively charged dust grains. We have studied the nonlinear propagation of dust ion-acoustic waves in a dusty multi-ion dense plasma (with the constituents being degenerate, either non-relativistic or ultra-relativistic) and the propagation of such waves have been investigated by the reductive perturbation method. The K-dV, modified K-dV and standard Gardner (sG) equations have been derived, and numerically examined. The basic features SWs and DLs have been analyzed from the solution of sG equation. It has been shown that for the dependency of these properties on plasma parametric values, the degenerate plasma under consideration supports compressive or rarefactive SWs , but not DLs. It has been also found that the effects of degenerate pressures of electrons and multi-ion significantly modifies the basic features of the nonlinear waves that are found to exist in such a degenerate plasma with the presence of negatively charged dust grains. The relevance of our results to astrophysical objects is briefly discussed.

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I. INTRODUCTION

Recently, the physics of dusty plasma is receiving a great deal of attention[1–3]. Dusty plasmas are characterized as a low temperature multispecies ionized gas comprising electrons, and negatively (or positively) charged grains of micrometer or submicrometer size. The study of the collective effects in dusty plasmas is of significant interest. Charged dust grains are found to modify or even dominate wave propagation [4–7, 14], wave scattering [15–18], wave instability [19], self-similar plasma expansion [20], velocity modulation [21], charged particle transport [22], and ion trapping [23]. However, most of the studies on wave motions [4–7]in dusty plasma assume constant charge on the dust grains.

On the other hand, it has been found both theoretically and experimentally [8–10] that the presence of negative ions, which occur in many space and laboratory dusty plasma situations [11], significantly modify the charging of dust particles. Therefore, it is expected that the presence of such negative ions can modify or introduce new features in dust associated waves, particularly, in the DIA waves of Shukla and Silin [13] . Recently, motivated by laboratory experiments [12] have investigated the properties of DIA waves in a multi-ion dusty plasma by using reductive perturbation method which is valid only for small amplitude. But in many space and laboratory dusty plasmas the amplitude of DIA waves is not necessary to be small.

Now-a-days a number of authors have become interested to study the properties of matter under extreme conditions [24–29], which occur due to the combine effect of Pauli's exclusion principle and Heisenberg's uncertainty principle, depends only on the number density of constituent particles, but independent on it's own temperature [30–33]. This degenerate pressure has an important role to study the electrostatic perturbation in matters which exist in extreme conditions [24–26, 34, 35]. Electron degenerate pressure will halt the gravitational collapse of a star if its mass is below the Chandrasekhar limit (i.e. 1.44 solar masses) [36]. This is the pressure that prevents a white dwarf star from collapsing. Astrophysical aspects of high density like in many cosmic environments, compact astrophysical objects [37–40] and planetary systems have been recently discussed by Forton [41]. Examples of the latter are white and brown dwarf stars [42, 43], as well as massive Jupiter [44] which serves as the super-Earth terrestrial planets around other stars [45], and the benchmark for giant planets.

In case of such a compact object the degenerate electron number density is so high (in white dwarfs it can be of the order of 10^{30} cm^{-3}, even more [30–33]) that the electron Fermi energy is comparable to the electron mass energy and as a result the electron speed becomes comparable to the speed of light in vacuum. For such interstellar compact objects the equation of state for degenerate ions and electrons are mathematically explained by Chandrasekhar [26] for two limits, named as non-relativistic and ultra-relativistic limits. Chandrasekhar [24, 26] presented a general expression for the relativistic ion and electron pressures in his classical papers. The pressure for ion fluid can be given by the following equation

$$P_i = K_i n_i^{\alpha},$$

where

$$\alpha = \frac{5}{3}; \quad K_i = \frac{3}{5}\left(\frac{\pi}{3}\right)^{\frac{1}{3}}\frac{\pi\hbar^2}{m} \simeq \frac{3}{5}\Lambda_c\hbar c, \tag{2}$$

for the non-relativistic limit (where $\Lambda_c = \pi\hbar/mc = 1.2 \times 10^{-10}$ cm, and $\hbar$ is the Planck constant divided by 2π). While for the electron fluid,

$$P_e = K_e n_e^{\gamma}, \tag{3}$$

where

$$\gamma = \alpha; K_e = K_i \text{ for nonrelativistic limit, and} \tag{4}$$

$$\gamma = \frac{4}{3}; \quad K_e = \frac{3}{4}\left(\frac{\pi^2}{9}\right)^{\frac{1}{3}}\hbar c \simeq \frac{3}{4}\hbar c, \tag{5}$$

in the ultra-relativistic limit [24–26, 30, 31].

Recently, a large number of authors [30, 31, 33, 46–56], etc. have used the pressure laws (3) to (5) investigate the linear and nonlinear properties of electrostatic and electromagnetic waves, by using the non-relativistic quantum hydrodynamic (QHD) [46] and quantum-magnetohydrodynamic(Q-MHD) [49] models and by assuming either immobile ions or non-degenerate uncorrelated mobile ions. It turns out that the presence of the latter and degenerate ultra relativistic electrons with the pressure law (3-5) admits one-dimensional localized ion models (IMs) supported by linear and non linear ion inertial forces and the pressure of degenerate electron fluids in a dense quantum plasma that is unmagnetized. Furthermore, modified Volkov solutions of the Dirac equation for electrostatic and electromagnetic waves in relativistic quantum plasmas have been discussed by Mendonca and Serbeto [57]. Again in this present days, some authors [58–60] has made a number of theoretical investigations on the nonlinear propagation of electrostatic waves in degenerate quantum plasma. These investigations are mainly based on the electron equation of state, which are only valid for the non-relativistic limit. Some investigations have been also made of the nonlinear propagation of electrostatic waves in a degenerate dense plasma which are mainly based on the degenerate electron equation of state valid for ultra-relativistic limit [30–33].

Still now, there is no theoretical investigation has been made to study the extreme condition of matter for both non-relativistic and ultra-relativistic limits on the propagation of electrostatic solitary waves (SWs) and double layers (DLs) in a dusty degenerate dense plasma system. Therefore, in our paper we have studied the properties of the SWs and DLs considering a dusty degenerate dense plasma containing degenerate electron-ion fluid (both non-relativistic and ultra-relativistic limits) with the negatively charged dust grains to study the basic features of the electrostatic soliton and double layer structures with the solutions of standard Gardner equation. Our considered model is relevant to compact interstellar objects (i.e. white dwarf, neutron star, black hole, etc.).

II. GOVERNING EQUATIONS

We consider the propagation of an electrostatic perturbation mode in a degenerate dense multi-ion dusty plasma containing ultra-relativistic degenerate electrons, non-relativistic degenerate inertial ions having both positive and negative ions and negative dust grains. The dynamics of the electrostatic waves propagating in such a plasma system is governed by

$$\frac{\partial n_s}{\partial t} + \frac{\partial}{\partial x}(n_s u_s) = 0, \tag{6}$$

$$\frac{\partial u_p}{\partial t} + u_p\frac{\partial u_p}{\partial x} + \frac{\partial \phi}{\partial x} + \frac{K_1}{n_p}\frac{\partial {n_p}^{\alpha}}{\partial x} = 0, \tag{7}$$

$$\frac{\partial u_n}{\partial t} + u_n\frac{\partial u_n}{\partial x} - \beta\frac{\partial \phi}{\partial x} + \frac{K_1}{n_n}\frac{\partial {n_n}^{\alpha}}{\partial x} = 0, \tag{8}$$

$$n_e\frac{\partial \phi}{\partial x} - K_2\frac{\partial n_e^{\gamma}}{\partial x} = 0, \tag{9}$$

$$\frac{\partial^2 \phi}{\partial x^2} = -\rho, \tag{10}$$

$$\rho = n_p - (1-\mu-\alpha_n)n_e - (1-\mu-\alpha_e)n_n, \tag{11}$$

where n_s $(s = p, n, e)$ is the plasma species number density normalized by its equilibrium value n_{s0} (n_{e0}), u_s is the plasma species ion fluid speed normalized by $C_{pm} = (m_e c^2/m_p)^{1/2}$ with m_e (m_p) being the electron (plasma ion species) rest mass mass and c being the speed of light in vacuum, ϕ is the electrostatic wave potential normalized by $m_e c^2/e$ with e being the magnitude of the charge of an electron, the time variable (t) is normalized by $\omega_{pm} = (4\pi e^2 n_0/m_p)^{1/2}$, and the space variable (x) is normalized by $\lambda_m = (m_e c^2/4\pi e^2 n_0)^{1/2}$, β is the ratio of negative and positive ion masses multiplied by their charge per ion, Z_j (where $j = p, n$), α_e is the ratio of the number density of electron and positive ion (n_{e0}/n_{p0}), α_n is the ratio of the number density of negative and positive ions (n_{n0}/n_{p0}) and μ is the ratio of the negative charged dust grains and positive ion $(Z_d n_{d0}/n_{p0})$. The constants $K_1 = n_{j0}^{\alpha-1} K_i/m_j c^2$ and $K_2 = n_{e0}^{\gamma-1} K_e/m_e c^2$. The equations of state used here are given by

$$P_i = K_i n_i^{\alpha}, \tag{12}$$

where

$$\alpha = \frac{5}{3}; \quad K_i = \frac{3}{5}\left(\frac{\pi}{3}\right)^{\frac{1}{3}} \frac{\pi\hbar^2}{m} \simeq \frac{3}{5}\Lambda_c \hbar c, \tag{13}$$

for the non-relativistic limit (where $\Lambda_c = \pi\hbar/mc = 1.2 \times 10^{-10}$ cm, and $\hbar$ is the Planck constant divided by 2π). While for the electron fluid,

$$P_e = K_e n_e^{\gamma}, \tag{14}$$

where

$$\gamma = \alpha; K_e = K_i \text{ for non} - \text{relativistic limit, and} \tag{15}$$

$$\gamma = \frac{4}{3}; \quad K_e = \frac{3}{4}\left(\frac{\pi^2}{9}\right)^{\frac{1}{3}} \hbar c \simeq \frac{3}{4}\hbar c, \tag{16}$$

in the ultra-relativistic limit [24–26, 30, 31].

III. DERIVATION OF K-DV EQUATIONS

To observe the electrostatic perturbations propagating in the ultra-relativistic degenerate dense plasma due to the effect of dispersion by analyzing the outgoing solutions of (6)−(9), we first introduce the stretched coordinates [61]

$$\xi = -\epsilon^{1/2}(x + V_p t), \tag{17}$$

$$\tau = \epsilon^{3/2} t, \tag{18}$$

where V_p is the wave phase speed (ω/k with ω being angular frequency and k being the wave number of the perturbation mode), and ϵ is a smallness parameter measuring the weakness of the dispersion $(0 < \epsilon < 1)$. We then expand n_s, n_e, u_s, and ϕ, in power series of ϵ:

$$n_s = 1 + \epsilon n_s^{(1)} + \epsilon^2 n_s^{(2)} + \cdots, \tag{19}$$

$$n_e = 1 + \epsilon n_e^{(1)} + \epsilon^2 n_e^{(2)} + \cdots, \tag{20}$$

$$u_s = \epsilon u_s^{(1)} + \epsilon^2 u_s^{(2)} + \cdots, \tag{21}$$

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots, \tag{22}$$

$$\rho = \epsilon \rho^{(1)} + \epsilon^2 \rho^{(2)} + \cdots, \tag{23}$$

and develop equations in various powers of ϵ. To the lowest order in ϵ, (6)−(22) give $u_s^{(1)} = -V_p n_s^{(}1)$, $n_p^{(1)} = \phi^{(1)}/(V_p^2 - K_1')$, $n_n^{(1)} = -\beta\phi^{(1)}/(V_p^2 - K_1')$, $n_e^{(1)} = \phi^{(1)}/K_2'$, and $V_p = \sqrt{K_2' \frac{(1+\beta)-\mu-\alpha_e}{1-\mu-\alpha_n} + K_1'}$ where $K_1' = \alpha K_1/(\alpha - 1)$ and $K_2' = \gamma K_2/(\gamma - 1)$. The relation $V_p = \sqrt{K_2' \frac{(1+\beta)-\mu-\alpha_e}{1-\mu-\alpha_n} + K_1'}$ represents the dispersion relation for the dust ion-acoustic type electrostatic waves in the degenerate plasma under consideration.

We are interested in studying the nonlinear propagation of these dispersive dust ion-acoustic type electrostatic waves in a degenerate plasma. To the next higher order in ϵ, we obtain a set of equations

$$\frac{\partial n_s^{(1)}}{\partial \tau} - V_p \frac{\partial n_s^{(2)}}{\partial \zeta} - \frac{\partial}{\partial \zeta}[u_s^{(2)} + n_s^{(1)} u_s^{(1)}] = 0, \tag{24}$$

$$\frac{\partial u_p^{(1)}}{\partial \tau} - V_p \frac{\partial u_p^{(2)}}{\partial \zeta} - u_p^{(1)} \frac{\partial u_p^{(1)}}{\partial \zeta} - \frac{\partial \phi^{(2)}}{\partial \zeta} - K_1' \frac{\partial}{\partial \zeta}\left[n_p^{(2)} + \frac{(\alpha-2)}{2}(n_p^{(1)})^2\right] = 0, \tag{25}$$

$$\frac{\partial u_n^{(1)}}{\partial \tau} - V_p \frac{\partial u_n^{(2)}}{\partial \zeta} - u_n^{(1)} \frac{\partial u_n^{(1)}}{\partial \zeta} + \beta \frac{\partial \phi^{(2)}}{\partial \zeta} - K_1' \frac{\partial}{\partial \zeta}\left[n_n^{(2)} + \frac{(\alpha-2)}{2}(n_n^{(1)})^2\right] = 0, \tag{26}$$

$$\frac{\partial \phi^{(2)}}{\partial \zeta} - K_2' \frac{\partial}{\partial \zeta}\left[n_e^{(2)} + \frac{(\gamma-2)}{2}(n_e^{(1)})^2\right] = 0, \tag{27}$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} = (1-\mu-\alpha_n) n_e^{(2)} - n_p^{(2)} + (1-\mu-\alpha_e) n_n^{(2)}. \tag{28}$$

Now, combining (24)-(28) we deduce a Korteweg-de Vries equation

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \zeta} + B \frac{\partial^3 \phi^{(1)}}{\partial \zeta^3} = 0, \tag{29}$$

where

$$A = \frac{(V_p^2 - K_1')^2}{2V_p[1+(1-\mu-\alpha_e)]\beta}\left[\frac{[3V_p^2 + K_1'(\alpha-2)][1+(1-\mu-\alpha_e)]\beta^2}{(V_p^2-K_1')^3} + \frac{(\gamma-2)(1-\mu-\alpha_n)\beta^2}{{K_2'}^2}\right], \tag{30}$$

$$B = \frac{(V_p^2 - K_1')^2}{2V_p[1+(1-\mu-\alpha_e)]\beta}. \tag{31}$$

The stationary solitary wave solution of (29) is

$$\phi^{(1)} = \phi_m^{(1)} \operatorname{sech}^2\left(\frac{\xi}{\Delta}\right), \tag{32}$$

where special coordinates, $\xi = \zeta - u_0\tau$, amplitude,$\phi_m = 3u_0/A$, and width,$\Delta = (4B/u_0)^{1/2}$.

IV. DERIVATION OF MODIFIED K-DV EQUATION

To examine electrostatic perturbations propagating in the relativistic degenerate dense plasma due to the effect of dispersion by analyzing the outgoing solutions of equations (6-10), we now introduce the new set of stretched coordinates for the modified K-dV equation is [61]

$$\zeta = \epsilon(x - V_p t), \tag{33}$$

$$\tau = \epsilon^3 t, \tag{34}$$

To the lowest order in ϵ, using equations (33,34, and 19-22), into the equations (6-10), we find the same results as we have had for the solitons for K-dV equation.

To the next higher order in ϵ, we obtain a set of equations, which, after using the values of $u_p^{(1)}$, $u_n^{(1)}$, $n_p^{(1)}$, $n_n^{(1)}$, and $n_e^{(1)}$ can be simplified as

$$u_p^{(2)} = \frac{V_p\phi^{(2)}}{V_p^2 - K_1'} + [K_1' V_p(\alpha-2) + 3V_p^3]\frac{(\phi^{(1)})^2}{2(V_p^2-K_1')^3} - \frac{V_p(\phi^{(1)})^2}{(V_p^2-K_1')^2}, \tag{35}$$

$$u_n^{(2)} = -\frac{\beta V_p\phi^{(2)}}{V_p^2 - K_1'} + [K_1' V_p\beta^2(\alpha-2) + 3\beta^2 V_p^3]\frac{(\phi^{(1)})^2}{2(V_p^2-K_1')^3} - \frac{\beta^2 V_p(\phi^{(1)})^2}{(V_p^2-K_1')^2}, \tag{36}$$

$$n_p^{(2)} = \frac{\phi^{(2)}}{V_p^2 - K_1'} + [K_1'(\alpha-2) + 3V_p^2]\frac{(\phi^{(1)})^2}{2(V_p^2-K_1')^3} \tag{37}$$

$$n_n^{(2)} = -\frac{\beta\phi^{(2)}}{V_p^2 - K_1'} + \beta^2\,[K_1'(\alpha-2) + 3V_p^2]\frac{(\phi^{(1)})^2}{2(V_p^2 - K_1')^3} \tag{38}$$

$$n_e^{(2)} = \frac{\phi^{(2)}}{K_2'} - \frac{(\gamma-2)(\phi^{(1)})^2}{2(K_2')^2}, \tag{39}$$

$$\rho^{(2)} = \frac{1}{2}\mathring{A}(\phi^{(1)})^2, \tag{40}$$

where

$$\mathring{A} = -\frac{(\gamma-2)(1-\mu-\alpha_n)}{(K_2')^2} + [(1-\mu-\alpha_e)\beta^2 - 1]\,[\frac{3V_p^2 + K_1'(\alpha-2)}{(V_p^2 - K_1')^3}], \tag{41}$$

To further higher order of ϵ, we obtain a set of equations

$$\frac{\partial n_s^{(1)}}{\partial\tau} - V_p\frac{\partial n_s^{(3)}}{\partial\zeta} + \frac{\partial u_s^{(3)}}{\partial\zeta} + \frac{\partial}{\partial\zeta}[u_s^{(2)}n_s^{(1)} + n_s^{(2)}u_s^{(1)}] = 0, \tag{42}$$

$$\frac{\partial n_p^{(1)}}{\partial\tau} - V_p\frac{\partial n_p^{(3)}}{\partial\zeta} + \frac{\partial u_p^{(3)}}{\partial\zeta} + \frac{\partial}{\partial\zeta}[u_p^{(2)}n_p^{(1)} + n_p^{(2)}u_p^{(1)}] = 0, \tag{43}$$

$$\frac{\partial n_n^{(1)}}{\partial\tau} - V_p\frac{\partial n_n^{(3)}}{\partial\zeta} + \frac{\partial u_n^{(3)}}{\partial\zeta} + \frac{\partial}{\partial\zeta}[u_n^{(2)}n_n^{(1)} + n_n^{(2)}u_n^{(1)}] = 0, \tag{44}$$

$$\frac{\partial u_p^{(1)}}{\partial\tau} - V_p\frac{\partial u_p^{(3)}}{\partial\zeta} + \frac{\partial}{\partial\zeta}[u_p^{(1)}u_p^{(2)}] + \frac{\partial\phi^{(3)}}{\partial\zeta} + K_1'\frac{\partial}{\partial\zeta}[n_p^{(3)} + (\alpha-2)(n_p^{(1)}n_p^{(2)}) + \frac{(\alpha-2)(\alpha-3)}{6}(n_p^{(1)})^3] = 0, \tag{45}$$

$$\frac{\partial u_n^{(1)}}{\partial\tau} - V_p\frac{\partial u_n^{(3)}}{\partial\zeta} + \frac{\partial}{\partial\zeta}[u_n^{(1)}u_n^{(2)}] - \beta\frac{\partial\phi^{(3)}}{\partial\zeta} + K_1'\frac{\partial}{\partial\zeta}[n_n^{(3)} + (\alpha-2)(n_n^{(1)}n_n^{(2)}) + \frac{(\alpha-2)(\alpha-3)}{6}(n_n^{(1)})^3] = 0, \tag{46}$$

$$\frac{\partial\phi^{(3)}}{\partial\zeta} - K_2'\frac{\partial}{\partial\zeta}[n_e^{(3)} + (\alpha-2)(n_e^{(1)}n_e^{(2)}) + \frac{(\alpha-2)(\alpha-3)}{6}(n_e^{(1)})^3] = 0, \tag{47}$$

$$\frac{\partial^2\phi^{(1)}}{\partial\zeta^2} = -\rho^{(3)}, \tag{48}$$

$$\rho^{(3)} = n_p^{(3)} - (1-\mu-\alpha_e)n_n^{(3)} - (1-\mu-\alpha_n)n_e^{(3)}. \tag{49}$$

Now combining equations (44) - (49) and using the values of $n_i^{(1)}$, $n_i^{(2)}$, $u_i^{(1)}$, $u_i^{(2)}$, and $\rho^{(2)}$, we obtain an equation of the form

$$\frac{\partial\phi^{(1)}}{\partial\tau} + \beta\{\phi^{(1)}\}^2\frac{\partial\phi^{(1)}}{\partial\zeta} + B\frac{\partial^3\phi^{(1)}}{\partial\zeta^3} = 0, \tag{50}$$

where the value of B is as before and β is given by

$$\beta = \alpha B \tag{51}$$

$$\alpha = \frac{15V_p^4 + 12K_1'V_p^2 + 18K_1'V_p^2(\alpha-2) + 3(K_1'(\alpha-2))^2}{2(V_p^2 - K_1')^5} + \frac{K_1'(\alpha-2)(\alpha-3)}{2(V_p^2 - K_1')^4} - \frac{(1-\mu)(6-7\gamma+2\gamma^2)}{2(K_2')^3} \tag{52}$$

$$B = \frac{(V_p^2 - K_1')^2}{2V_p[1 + (1-\mu-\alpha_e)\beta]} \tag{53}$$

We call equation (50) as modified K-dV equation for planner geometry. The stationary solitary solution of equation (50) is given by

$$\phi^{(1)} = \phi_m sech(\frac{\xi}{\Delta}), \tag{54}$$

where the special coordinate, $\xi = \zeta - u_0\tau$, the amplitude is $\phi_m = \sqrt{\frac{6u_0}{\beta}}$, the width is $\Delta = \sqrt{\frac{1}{\gamma\phi_m}}$ and u_0 is the plasma species speed at equilibrium.

V. DERIVATION OF STANDARD GARDNER EQUATION

It is obvious from (42) that $A = 0$ since $\phi^{(1)} \neq 0$. One can find that $A = 0$ at its critical value $\mu = (\mu)_c = 2/3$ (which is a solution of $A = 0$). So, for μ around its critical value μ_c, $A = A_0$ can be expressed as

$$A_0 = s(\frac{\partial A}{\partial \mu})_{\mu=\mu_c} \; |\mu - \mu_c| = s\epsilon, \tag{55}$$

where where $|\mu - \mu_c|$ is a small and dimensionless parameter, and can be taken as the expansion parameter ϵ, i.e. $|\mu - \mu_c| \simeq \epsilon$, and $s = 1$ for $\mu < \mu_c$ and $s = -1$ for $\mu > \mu_c$. So, $\rho^{(2)}$ can be expressed as

$$\epsilon^{(2)}\rho^{(2)} \simeq \epsilon^{(3)}\frac{1}{2}s(\phi^{(1)})^{(2)}, \tag{56}$$

which, therefore, must be included in the third order Poisson's equation. To the next higher order of the ϵ, we obtain a set of equations:

$$\frac{\partial n_s^{(1)}}{\partial \tau} - V_p\frac{\partial n_s^{(3)}}{\partial \zeta} + \frac{\partial u_s^{(3)}}{\partial \zeta} + \frac{\partial}{\partial \zeta}[u_s^{(2)}n_s^{(1)} + n_s^{(2)}u_s^{(1)}] = 0, \tag{57}$$

$$\frac{\partial n_p^{(1)}}{\partial \tau} - V_p\frac{\partial n_p^{(3)}}{\partial \zeta} + \frac{\partial u_p^{(3)}}{\partial \zeta} + \frac{\partial}{\partial \zeta}[u_p^{(2)}n_p^{(1)} + n_p^{(2)}u_p^{(1)}] = 0, \tag{58}$$

$$\frac{\partial n_n^{(1)}}{\partial \tau} - V_p\frac{\partial n_n^{(3)}}{\partial \zeta} + \frac{\partial u_n^{(3)}}{\partial \zeta} + \frac{\partial}{\partial \zeta}[u_n^{(2)}n_n^{(1)} + n_n^{(2)}u_n^{(1)}] = 0, \tag{59}$$

$$\frac{\partial u_p^{(1)}}{\partial \tau} - V_p\frac{\partial u_p^{(3)}}{\partial \zeta} + \frac{\partial}{\partial \zeta}[u_p^{(1)}u_p^{(2)}] + \frac{\partial \phi^{(3)}}{\partial \zeta} + K_1'\frac{\partial}{\partial \zeta}[n_p^{(3)} + (\alpha-2)(n_p^{(1)}n_p^{(2)}) + \frac{(\alpha-2)(\alpha-3)}{6}(n_p^{(1)})^3] = 0, \tag{60}$$

$$\frac{\partial u_n^{(1)}}{\partial \tau} - V_p\frac{\partial u_n^{(3)}}{\partial \zeta} + \frac{\partial}{\partial \zeta}[u_n^{(1)}u_n^{(2)}] - \beta\frac{\partial \phi^{(3)}}{\partial \zeta} + K_1'\frac{\partial}{\partial \zeta}[n_n^{(3)} + (\alpha-2)(n_n^{(1)}n_n^{(2)}) + \frac{(\alpha-2)(\alpha-3)}{6}(n_n^{(1)})^3] = 0, \tag{61}$$

$$\frac{\partial \phi^{(3)}}{\partial \zeta} - K_2'\frac{\partial}{\partial \zeta}[n_e^{(3)} + (\alpha-2)(n_e^{(1)}n_e^{(2)}) + \frac{(\alpha-2)(\alpha-3)}{6}(n_e^{(1)})^3] = 0, \tag{62}$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} = -\rho^{(3)} - \frac{s}{2}(\phi^{(1)})^{(2)}, \tag{63}$$

$$\rho^{(3)} = n_p^{(3)} - (1-\mu-\alpha_e)n_n^{(3)} - (1-\mu-\alpha_n)n_e^{(3)}. \tag{64}$$

Now combining (57-64), we obtain an equation of the form:

$$\frac{\partial \phi^{(1)}}{\partial \tau} + sB\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \zeta} + \beta{\phi^{(1)}}^2\frac{\partial \phi^{(1)}}{\partial \zeta} + B\frac{\partial^3 \phi^{(1)}}{\partial \zeta^3} = 0. \tag{65}$$

where the values of β and B are same as before in the modified KdV equation. This equation (65) is called standard Gardner (sG) eqaution.

The exact analytical solution of (65) is not possible. Therefore, we have numerically solved (65), and have studied the effects of planar geometry DIA GSs and DLs. The stationary SW and DL solution of the sG equation [i.e. (65)] is obtained by considering a moving frame (moving with speed u_0) $\xi = \zeta - u_0\tau$, and imposing all the appropriate boundary conditions for the SW and DL solution, including $\phi^{(1)} \to 0$, $d\phi^{(1)}/d\zeta \to 0$, $d^2\phi^{(1)}/d\zeta^2 \to 0$ at $\zeta \to -\infty$. These boundary conditions allow us to have two solutions to express the sG equation [i.e. (65)], as one is the stationary SW solution and another is DL solution. The stationary SW solution of sG equation [i.e. (65)] can be written as

$$\phi^{(1)} = \left[\frac{1}{\phi_{m2}} - \left(\frac{1}{\phi_{m2}} - \frac{1}{\phi_{m1}}\right)\cosh^2\left(\frac{\xi}{\delta}\right)\right]^{-1}, \tag{66}$$

where $\phi_{m1,2}$ is given by

$$\phi_{m1,2} = \psi_m\left[1 \mp \sqrt{1 + \frac{U_0}{V_0}}\right], \tag{67}$$

$$V_0 = \frac{s^2 B}{6\alpha}, \tag{68}$$

$$U_0 = \frac{sB}{3}\phi_{m1,2} + \frac{\beta}{6}{\phi_{m1,2}}^2 \tag{69}$$

$$\delta = \frac{2}{\sqrt{-\gamma\phi_{m1}\phi_{m2}}} \tag{70}$$

$$\gamma = \frac{\alpha}{6} \tag{71}$$

$$\phi_m = \frac{s}{\alpha} \tag{72}$$

and δ is the width of the SWs, and is given by

$$\delta = \frac{2}{\sqrt{-\gamma\phi_{m1}\phi_{m2}}}. \tag{73}$$

Now, the stationary DL solution of sG equation can be written as

$$\phi^{(1)} = \frac{\phi_m}{2}\left[1 + \tanh\left(\frac{\xi}{\Delta}\right)\right], \tag{74}$$

where Δ is the width of the DLs, and is given by

$$\Delta = \sqrt{-\frac{24}{\phi_m^2\alpha}}. \tag{75}$$

VI. NUMERICAL ANALYSIS

It is clear from (74) and (75) that DLs exist if and only if $\mu_0 < 0$. It is obvious from figures 9 and 10 that $\mu > \mu_c$ which confirm us that the DLs are associated with positive potential only. The parametric regimes for the existence of the positive DLs are not bounded by the lower and upper surface plot of μ, and the DLs exist for parameters corresponding to any point in between two ($\mu_0 = 0$) surface plots. It may be noted here that if we would neglect the higher order nonlinear term [viz. the third term of (65) or the term containing $\phi^{(3)}$], but would keep the lower order nonlinear term [viz. the second term of (65) or the term containing $\phi^{(2)}$], we would obtain the solitary structures that are due to the balance between nonlinearity (associated with $\phi^{(2)}$ only) and dispersion [5, 62]. On the other hand, in our present work, we have kept both the terms containing $\phi^{(2)}$ and $\phi^{(3)}$, and have obtained the DL structures which are formed due to the balance between the nonlinearity (associated with $\phi^{(2)}$ and $\phi^{(3)}$) and dispersion.

It may be added here that the dissipation (which is usually responsible for the formation of the shock-like structures [63, 64]) is not essential for the formation of the solitary and DL structures [65, 66]. The stationary DL solution of the sG equation, and the conditions for the existence of DLs clearly imply that the DL structures predicted in our present investigation is not due to the dissipation (which has been neglected in our present investigation), but is due to the coexistence of positively and negatively charged dust where we have taken the values of α_e between 0.1 to 0.6 and α_p between 0.1 to 0.4.

In the figures 1-8 we have tried to show the solitary profiles obtained from the SWs solution of sG equation (66) due to the effect of μ on the potential, $\phi^{(1)}$ for the case of electron-ion being non-relativistic degenerate and ion being non-relativistic degenerate and electron being ultra-relativistic degenerate. From the figures 9-10 obtained from DLs solution of sG (74) we have observed the effect of μ on the potential, $\phi^{(1)}$ for the double layers obtained from the solution of sG equation (65) when we have considered the value of μ for the both case of relativistic limits.

By the careful observation on the figures 1-10 it has become clear that the terms μ has an great effect on the potential, $\phi^{(1)}$ of the DIA SWs and DLs. Because of the critical value of μ we get both compressive and rarefactive SWs with the positive and negative potential, but only positive potential with DLs. Again potential, $\phi^{(1)}$ increases more rapidly for ion being non-relativistic degenerate and electron being ultra-relativistic degenerate than for both electron-ion being non-relativistic degenerate. But we get only positive potential, $\phi^{(1)}$, for the DL (9-10) obtained from the solution of sG equation, whatever the value of μ, i.e it does not depend on the value of μ.

VII. DISCUSSION

We have represented a theory for a DIA SWs and DLs in a electron and multi-ion degenerate dense dusty plasma system. The latter is a source for dissipation, and is responsible for the formation of the DIA SWs and DLs. From

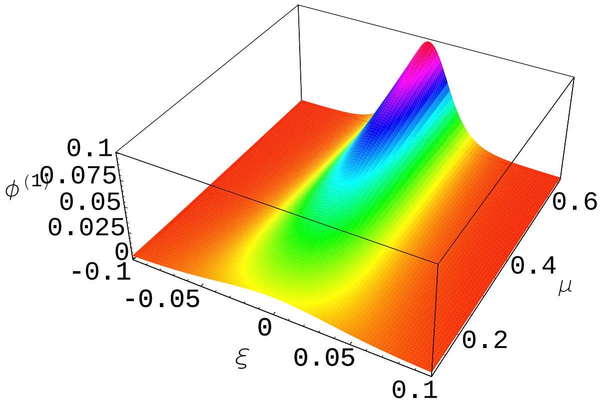

FIG. 1: Showing the effect of μ on solitary waves (potential structure) for both electron-ion being non-relativistic degenerate when $\mu < 0.66$, $\alpha_e \to 0.4$ and $\alpha_p \to 0.3$

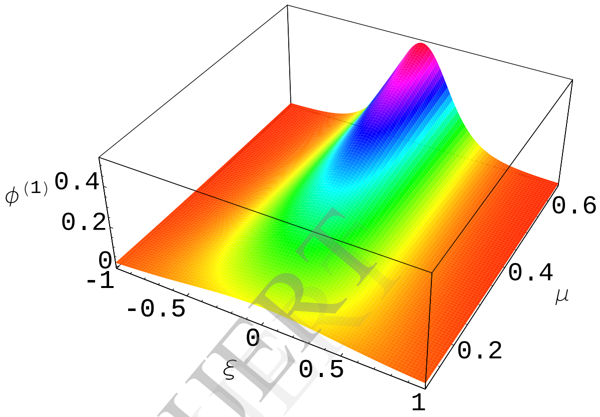

FIG. 2: Showing the effect of μ on solitary waves (potential structure) for ion being non-relativistic degenerate and electron being ultra-relativistic degenerate when $\mu < 0.66$, $\alpha_e \to 0.4$ and $\alpha_p \to 0.3$.

this investigation it has been found that the presence of the negative ions significantly reduces the magnitude of the charge of the negatively charged dust, and under a certain condition [9] it cab cause the charge neutralization of the dust particles; the presence of negative ions can change the dust gpolarity. This prediction agrees with the recent experimental observations [9, 10]. The resulting negative ion velocity is directed outwards; it turns out that, although the main evaluation is by diffusion, the negative ion front propagation speed is nearly constant for constant electrons and ions temperatures because of degenerate. The negative ion front is new type of nonlinear structure, different from hydrodynamic nonlinear waves, and beyond the classification of dissipative structures done in [67]. The evolution of ion fronts in the afterglow of electronegative plasma is very important, since it determines the time needed for negative ions to reach the wall and thus influence surface reactions in plasma processing.

Electrostatic perturbation for degenerate dense relativistic multi-ion plasma with an additional particle, negatively charged dust, has been studied. The formation of nonlinear electrostatic propagation modes, particularly the solitary waves and double layers profiles obtain from the solution of standard Gardner equation has been theoretically investigated. We have found that the degenerate electron and multi-ion pressures and the number density of the negatively charged dust grains have significant effects on the potential of the electrostatic profiles (shown in 1-10).

Our present investigation is different from the related investigations [30–33] in the way that we have considered the pressure of all the constituent particles (electrons and multi-ion), as the whole system is degenerate and all the particles should follow the equation of state (12)−(16) whatever the limit is (non relativistic or ultra-relativistic),

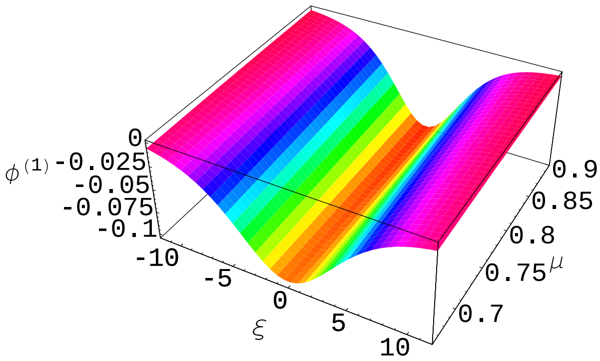

FIG. 3: Showing the effect of μ on solitary waves (potential structure) for both electron-ion being non-relativistic degenerate when $\mu \geq 0.66$, $\alpha_e \to 0.4$ and $\alpha_p \to 0.3$.

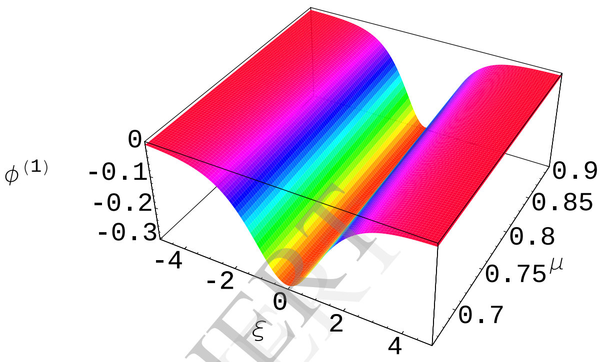

FIG. 4: Showing the effect of μ on solitary waves (potential structure) for ion being non-relativistic degenerate and electron being ultra-relativistic degenerate when $\mu \geq 0.66$, $\alpha_e \to 0.4$ and $\alpha_p \to 0.3$.

again the effect of static charged dust grains. From this point of view our present investigation is more acceptable and the system constituents have made the validity of our investigations greater than the previous works [30–33].

In our numerical analysis we have used a wide range of the degenerate plasma parameters, which are relevant for many cosmic environments and compact astrophysical objects. The results of the present investigation is, therefore, expected to be useful in understanding the dispersion properties of the electrostatic waves in such cosmic environments and compact astrophysical objects [37, 38]. We have investigated the DIA GSs and corresponding the DLs in a dusty plasma system (composed of negative dust grains, degenerate electrons and ions), by deriving the sG equation. The K-dV solitons and finite amplitude DLs investigated earlier, are not valid for $\mu = \mu_c$, which vanishes the nonlinear coefficients of the K-dV equation. However, the DIA GSs and DLs investigated in our present work are valid for around $\mu = \mu_c$. The results, which have been obtained from this investigation can be pinpointed as follows:

1. The dusty plasma system under consideration supports the finite amplitude GSs and DLs, whose basic features (polarity, amplitude, width, etc.) depend on the degenerate ions and dust number densities.

2. GSs are shown to exist around $\mu = \mu_c$, and are found to be different from the K-dV solitons, which do not exist for around $\mu = \mu_c$ and mK-dV solitons which exist for around $\mu = \mu_c$, but have only one types of polarity and have no corresponding DL solution.

3. At $\mu < \mu_c$, negative GSs exist, whereas at $\mu > \mu_c$, positive GSs exist.

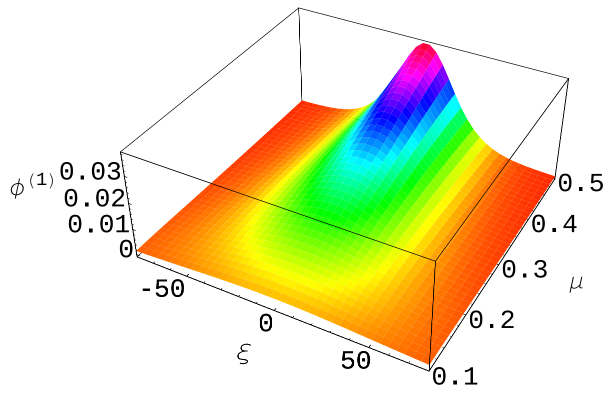

FIG. 5: Showing the effect of μ on solitons (potential structure) for both electron-ion being non-relativistic degenerate when $\mu < 0.66$, $\alpha_e \rightarrow 0.4$ and $\alpha_p \rightarrow 0.3$.

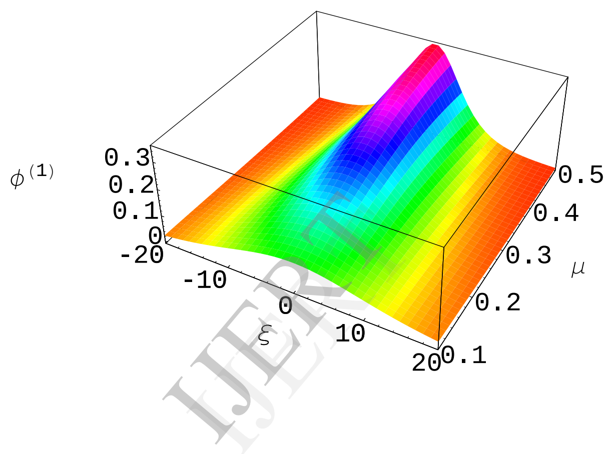

FIG. 6: Showing the effect of μ on solitons (potential structure) for ion being non-relativistic degenerate and electron being ultra-relativistic degenerate when $\mu < 0.66$, $\alpha_e \rightarrow 0.4$ and $\alpha_p \rightarrow 0.3$.

4. The magnitude of the amplitude of positive and negative GSs increases with μ, but also increases with u_0.
5. The DLs having large width we have found only positive potential, no negative DLs are formed.
6. The magnitude of the amplitude of the DLs increases with the increase of μ, but also increases with the increase of u_0.

We have carried out SWs and DLs by deriving the sG equations for an unmagnetized plasma system containing degenerate electrons (non-relativistic or ultra relativistic limits) and degenerate ions being non-relativistic limit and the negatively charged charged dust grains. We have shown the existence of compressive (hump shape) and rarefactive (dip shape) DIA SWs and only positive potential for DLs for both limits. Generally the DIA SWs and DLs are completely different from the K-dV and modified K-dV solitary waves. It may be stressed here that the results of this investigation should be useful for understanding the nonlinear features of electrostatic disturbances in laboratory plasma conditions. Our investigation would also be useful to study the effects of degenerate pressure in interstellar and space plasmas [68], particularly in stellar polytropes [69], hadronic matter and quark-gluon plasma [70], protoneutron stars [71], dark-matter halos [72] etc. Further it can be said that the analysis of shock structures, vortices, double-layers etc. in a nonplanar geometry where the degenerate pressure can play the significant role, are also the problems of great importance but beyond the scope of the present work. To conclude, we propose to perform a laboratory

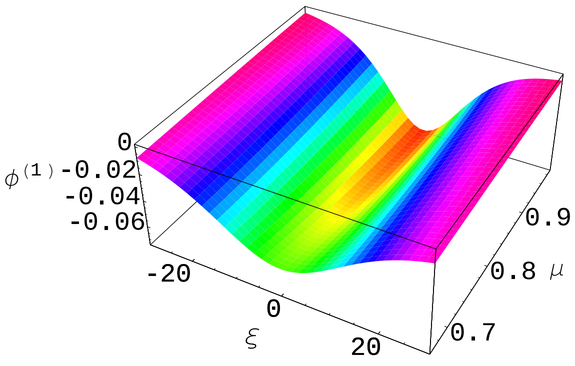

FIG. 7: Showing the effect of μ on solitons (potential structure) for both electron-ion being non-relativistic degenerate when $\mu \geq 0.66$, $\alpha_e \to 0.4$ and $\alpha_p \to 0.3$.

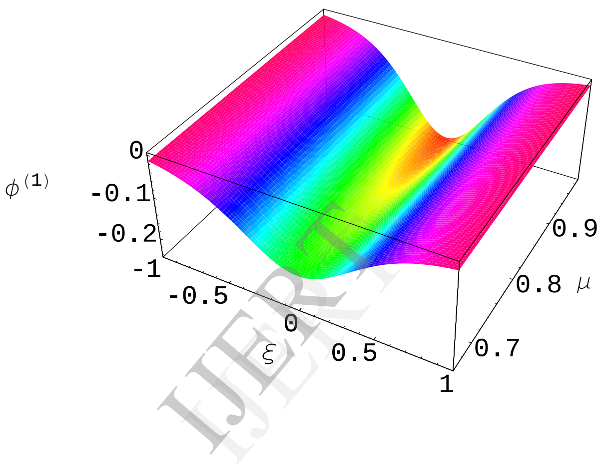

FIG. 8: Showing the effect of μ on solitons (potential structure) for ion being non-relativistic degenerate and electron being ultra-relativistic degenerate when $\mu \geq 0.66$, $\alpha_e \to 0.4$ and $\alpha_p \to 0.3$.

experiment which can study such special new features of the DIA solitons propagating in dusty plasma in presence of degenerate electrons and ions

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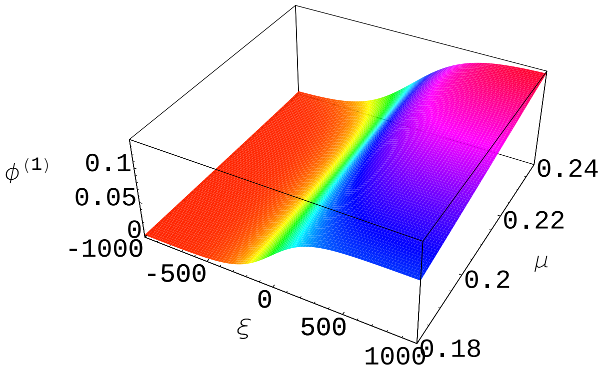

FIG. 9: Showing the effect of μ on double layers (potential structure) for both electron-ion being non-relativistic degenerate, $0.01 < \mu < 1$, $\alpha_e \to 0.4$ and $\alpha_p \to 0.3$.

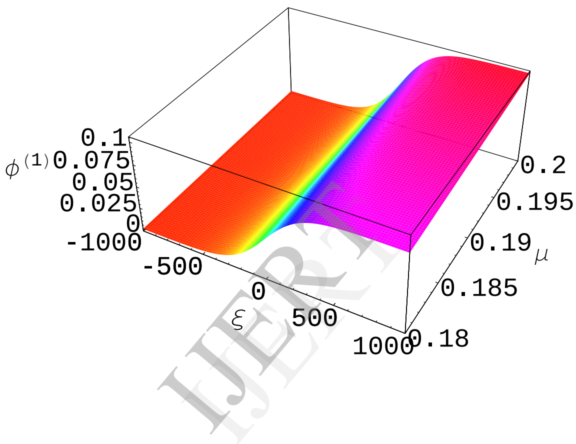

FIG. 10: Showing the effect of μ on double layers (potential structure) for ion being non-relativistic degenerate and electron being ultra-relativistic degenerate $0.01 < \mu < 1$, $\alpha_e \to 0.4$ and $\alpha_p \to 0.3$..

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