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Fuzzy Mathematical Fatigue Life Model for the Effect of North West India Rainfall

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ABSTRACT:
In this paper, we use the mathematical Fatigue Life distribution model to calculate the rainfall in the northwestern states of India, namely Uttar Pradesh, Uttarakhand, Delhi, Punjab, Himachal Pradesh, Jammu and Kashmir, and Rajasthan, and see which year received the most rainfall.

1. INDRODUCTION:

The North West Indian subdivisions by the Meteorological Department include 7 states in India.

- Uttar Pradesh
- Uttarakhand
- Delhi
- Punjab
- Himachal Pradesh
- Jammu & Kashmir
- Rajasthan

1.1. History of North West India:

Rainfall and temperature are the most important environmental factors influencing crop growth, development, and yield. The northwestern (NW) part of India is one of the main regions of food grain production of the country. North-East India displays, to a large extent, the character of tropical climate, especially in the valleys. The region has a monsoon climate with heavy to very heavy rains, confined within four summer months from June to September. There are three distinct seasons in the region: winter, summer, and the rainy season.

1.2. Uttar Pradesh:

Uttar Pradesh is a state in India, stretches back several millennia. The region shows the presence of human habitation dating back to between 85,000 and 73,000 years ago. Additionally, the region seems to have been domesticated as early as 6,000 BC. After independence in 1947, the United Provinces were renamed Uttar Pradesh in 1950.

Uttar Pradesh is a state in northern India. With over 241 million inhabitants, it is the most populated state in India as well as the most populous country subdivision in the world – more populous than all but four other countries outside of India – and accounting for 16.5 percent of the total population of India. The state is divided into 18 divisions and 75 districts. On 9 November 2000, a new state, Uttaranchal (now Uttarakhand), was created from Uttar Pradesh's western Himalayan hill region. The two major rivers of the state, the Ganges and its tributary Yamuna, meet at the Triveni Sangam in Prayagraj (formerly Allahabad), a Hindu pilgrimage site. Other notable rivers are Gomti and Saryu. The forest cover in the state is 6.1 per cent of the state's geographical area.

The climate of Uttar Pradesh (U.P) is primarily defined as humid subtropical with dry winter (*Cwa*) type with parts of Western U.P. as hot semi-arid (*BSh*) type. Alternatively, some authors refer to it as tropical monsoon. Variations do exist in different parts of the large state. The average annual rainfall is approximately 650 millimetres in the southwestern part of UP. The precipitation is around 1000 millimetres in the southeast and east of UP.

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1.3. Uttarakhand:

Uttarakhand is a Himalayan state in North India, nestled between the Tibetan Plateau and the Indo-Gangetic Plains. The name, which means "northern land" or "section" or "northern part" in Sanskrit was made popular in the 80s as part of the wider statehood struggle within the region. Initially, it was believed that due to harsh climate and mountainous terrain, this was a barren and uninhabited land. But after various excavations and the study of ancient literature, it is now established that the history of Uttarakhand goes back to Stone Age. Uttarakhand is popularly referred to as "the land of the gods" (Devbhumi) owing to its huge number of Hindu pilgrimage sites and proximity to Mount Kailash.

Uttarakhand state is located between 28°43' - 31°27' N latitudes and 77°34' - 81°02' E longitudes in the northern part of India. The state is mostly hilly and has international boundary with China (Tibet) in the north and Nepal in the east and state boundary with Himachal Pradesh in the northwest. The state has foothills areas in the south and southwest which are bounded by Uttar Pradesh.

Uttarakhand state has total geographical area of 53,485 sq. km, of which 86% is mountainous and 65% is covered by forest.The state is rich in natural resources especially water and forests with many glaciers, perennial rivers, dense forests and snow-capped mountain peaks. Annual rainfall ranges from 92 to 237 cm.

1.4. Delhi:

Delhi has been an important political centre of India as the capital of several empires. The recorded history of Delhi begins with the 8th century Tomar Rajputs kingdom. **Delhi**, city and national capital, and union territory, north-central India. The city of Delhi actually consists of two components: Old Delhi, in the north, the historic city; and New Delhi, in the south, since 1947 the capital of India, built in the first part of the 20th century as the capital of British India.

One of the country's largest urban agglomerations, Delhi sits astride (but primarily on the west bank of) the Yamuna River, a tributary of the Ganges (Ganga) River, about 100 miles (160 km) south of the Himalayas. The national capital territory embraces Old and New Delhi and the surrounding metropolitan region, as well as adjacent rural areas. Delhi receives an average annual precipitation of 774.4 mm.

The climate of Delhi is an overlap between monsoon-influenced and semi-arid, with high variation between summer and winter temperatures and precipitation. Summer starts in early April and peaks in late May or early June, with average temperatures near 38 °C (100 °F) although occasional heat waves can result in highs close to 45 °C (113 °F) on some days and therefore higher apparent temperature. The monsoon starts in late June and lasts until mid-September, with about 797.3 mm (31.39 inches) of rain. The average temperatures are around 29 °C (84 °F), although they can vary from around 25 °C (77 °F) on rainy days to 35–40 °C (95–104 °F) during dry spells.

1.5. Punjab:

The History of Punjab refers to the past history of Punjab region which is a geopolitical, cultural, and historical region in the northwest of South Asia, comprising western Punjab province in Pakistan and eastern Punjab state in India. It is believed that the earliest evidence of human habitation in Punjab traces to the Soan valley of the Pothohar,between the Indus and the Jhelum rivers, where Soanian culture developed between 774,000 BC and 11,700 BC.

Some two-fifths of Punjab's population is engaged in the agricultural sector, which accounts for a significant segment of the state's gross product. Punjab produces an important portion of India's food grain and contributes a major share of the wheat and rice stock held by the Central Pool . Much of the state's agricultural progress and productivity is attributable to the so-called Green Revolution, an international movement launched in the 1960s that introduced not only new agricultural technologies but also high-yielding varieties of wheat and rice.

Punjab has an inland subtropical location, and its climate is continental, being semiarid to subhumid. In June, the warmest month, daily temperatures in Ludhiana usually reach about 100 °F (upper 30s C) from a low in the upper 70s F (mid-20s C). In January, the coolest month, daily temperatures normally rise from the mid-40s (about 7 °C) into the mid-60s F (upper 10s C). The average annual rainfall is 273 to 676 mm .

1.6. Himachala Pradesh:

Himachal Pradesh was established in 1948 as a Chief Commissioner's Province within the Union of India. The province comprised the hill districts around Shimla and southern hill areas of the former Punjab region. Thus Himachal emerged as the eighteenth state of the Indian Union. After independence the Chief Commissioner's province of H.P. came into being on 15 April 1948. Himachal became a part C state on 26 January 1950 with the implementation of the Constitution of India. Himachal Pradesh became Union Territory on 1 November 1956. On 18 December 1970 the State of Himachal Pradesh Act was passed by Parliament and the new state came into being on 25 January 1971.

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There is a huge variation in the climatic conditions of Himachal Pradesh due to variation in altitude (360–6500 metres). The climate varies from hot and sub-humid tropical (450–900 metres) in the southern low tracts, warm and temperate (900–1800 metres), cool and temperate (1800–2400 metres) and cold glacial and alpine (2400–4800 metres) in the northern and eastern high elevated mountain. Pollution is affecting the climate of almost all the states of India. The average annual rainfall is 1,251(mm). The rainy season start at the end of the month of June.

The spring season starts from mid February to mid April. The weather is pleasant and comfortable in the season. The rainy season start at the end of the month of June. The landscape lushes green and fresh. During the season streams and natural springs are replenished. The average annual rainfall is 1,251(mm).

1.7. Jammu & Kashmir:

Jammu and Kashmir, also known as Kashmir and Jammu, was a princely state in a subsidiary alliance under British East India Company rule from 1846 to 1858 and under the paramountcy of the British Crown, from 1858 until the Partition of India in 1947, when it became a disputed territory, now administered by three countries: China, India, and Pakistan. Jammu and Kashmir is an State of India with population of Approximate 1.25 Crores. The population of Jammu and Kashmir state is 12,541,302. The density of Jammu and Kashmir state is 56 per sq km. Jammu and Kashmir State is spread over 222,236 Sq Km.

The inter mountainous valley of Kashmir has unique geographical setting and it is located 130 between the Greater Himalayas in the north and Pir Panjal ranges in the south, roughly within the latitude and longitude ranges of 330 55´ to 34o 50´ and 74o 30´ to 75o 131 35´ respectively. The heights of these 132 mountains range from about 3,000 to 5,000 m and the mountains strongly influence the weather and 133 climate of the region.

State of Jammu and Kashmir is home to several Himalayan glaciers. Jhelum, Indus, Tawi, Ravi and Chenab are major rivers flowing through the state. the state of Jammu comprises of three distinct climatic regions: cold arid desert areas of Ladakh, temperate Kashmir Valley, and the humid sub-tropical region of Jammu. The mean annual rainfall in cold arid zone varies from 100 mm to 300 mm received by western disturbance. The average rainfall annually is usually between 700 and 750 mm.

1.8. Rajasthan:

The history of human settlement in the western Indian state of Rajasthan dates back to about 100,000 years ago. Around 5000 to 2000 BCE many regions of Rajasthan belonged as the site of the Indus Valley Civilization. Kalibangan is the main Indus site of Rajasthan, here fire altars have been discovered, similar to those found at Lothal. Around 2000 BCE, Sarasvati River flowed through the Aravalli mountain range in the state.

Rajasthan state of northwestern India, located in the northwestern part of the Indian subcontinent. It is bounded to the north and northeast by the states of Punjab and Haryana, to the east and southeast by the states of Uttar Pradesh and Madhya Pradesh, to the southwest by the state of Gujarat, and to the west and northwest by the provinces of Sindh and Punjab in Pakistan.

The capital city is Jaipur, in the east-central part of the state. The Aravalli (Aravali) Range forms a line across the state running roughly from Guru Peak on Mount Abu (5,650 feet [1,722 metres]), near the town of Abu in the southwest, to the town of Khetri in the northeast. About three-fifths of the state lies northwest of that line, leaving the remaining two-fifths in the southeast. Those are the two natural divisions of Rajasthan. The northwestern tract is generally arid and unproductive, although its character shifts gradually from desert in the far west and northwest to comparatively fertile and habitable land toward the east. The region includes the Thar (Great Indian) Desert. The annual rainfall in the East and West Rajasthan is about 64.9 cm and 32.7 cm respectively.

2. FATIGUE LIFE DISTRIBUTION

The Birnbaum–Saunders distribution, also known as the fatigue life distribution, is a probability distribution used extensively in reliability applications to model failure times. There are several alternative formulations of this distribution in the literature. It is named after Z. W. Birnbaum and S. C. Saunders. The Fatigue life distribution is closed under scalar multiplication and under reciprocation; its median coincides with the BS scale parameter; it has different shapes for its probability density function (PDF), which cover high, medium, and low asymmetry levels .

This distribution was developed to model failures due to cracks. A material is placed under repeated cycles of stress. The j^{th} cycle leads to an increase in the crack by X_i amount. The sum of the X_i is assumed to be normally distributed with mean $n\mu$ and variance $n\sigma^2$. The probability that the crack does not exceed a critical length ω is

$$P(X \leq \omega) = \Phi\left(\frac{\omega - n\mu}{\sigma\sqrt{n}}\right)$$

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where Φ is the cdf of normal distribution.
The more usual form of this distribution is:

$$F(x;\alpha,\beta) = \Phi\left(\frac{1}{\alpha}\left[\left(\frac{x}{\beta}\right)^{0.5} - \left(\frac{\beta}{x}\right)^{0.5}\right]\right)$$

Here α is the shape parameter and β is the scale parameter.

2.1 Proprerties

The Birnbaum–Saunders distribution is unimodal with a median of β.

$$\text{Mean} = \mu\left(1+\frac{\gamma^2}{2}\right)$$

$$\text{Variance} = (\gamma\,\mu)^2\left(1+\frac{5\gamma^2}{4}\right)$$

Given a data set that is thought to be Birnbaum–Saunders distributed the parameters' values are best estimated by maximum likelihood.
If T is Birnbaum–Saunders distributed with parameters α and β then T^{-1} is also Birnbaum-Saunders distributed with parameters α and β^{-1}.

2.2 Probability density function

The general formula for the probability density function (pdf) is

$$f(x) = \frac{\sqrt{\frac{x-\mu}{\beta}}+\sqrt{\frac{\beta}{x-\mu}}}{2\gamma(x-\mu)}\phi\left(\frac{\sqrt{\frac{x-\mu}{\beta}}-\sqrt{\frac{\beta}{x-\mu}}}{\gamma}\right) \quad x>\mu;\gamma,\beta>0$$

where γ is the shape parameter, μ is the location parameter, β is the scale parameter, and φ s the probability density function of the standard normal distribution.

2.3 Standard Fatigue Life Distrinution

The case where μ = 0 and β = 1 is called the standard fatigue life distribution. The pdf for the standard fatigue life distribution reduces to

$$f(x) = \frac{\sqrt{x}+\sqrt{\frac{1}{x}}}{2\gamma x}\phi\left(\frac{\sqrt{x}-\sqrt{\frac{1}{x}}}{\gamma}\right) \quad x>0;\gamma>0$$

Since the general form of probability functions can be expressed in terms of the standard distribution, all of the subsequent formulas are given for the standard form of the function.

2.4 Cumulative Distribution Function

The formula for the cumulative distribution function is

$$F(x) = \Phi\left(\frac{\sqrt{x}-\sqrt{\frac{1}{x}}}{\gamma}\right) \quad x>0;\gamma>0$$

where Φ is the cumulative distribution function of the standard normal distribution.

2.5 Quantile Function

The formula for the quantile function is

$$G(p) = \frac{1}{4}\left[\gamma\Phi^{-1}(p)+\sqrt{4+\left(\gamma\Phi^{-1}(p)\right)^2}\right]^2$$

where Φ^{-1} is the quantile function of the standard normal distribution.

3. APPLICATION

3.1 Application - I
In this work, We observe the normal monthly rainfall for the state Uttar Pradesh which is the state of North West India from the month January to December during the year 2012. We present a model of Rainfall for the state Uttar Pradesh in India using the Fatigue Life distribution with two parameters.
We have constructed a model of Rainfall that produces result that qualitatively and quantitatively agree with experimental observation.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Rainfall (mm)	26	5.75	3.95	5.55	0.45	16.35	273.45	233.9	163.7	1.0	0.2	1.9

Table 3.1.1 Rainfall of Uttar Pradesh in 2012

The scale and shape parameters of Fatigue life distribution in Table 3.1.1, γ = 3.6399 μ = 8.46115. The triangular fuzzy number is γ = [3.1021, 3.6399, 4.1777] and μ = [7.92335, 8.46115, 8.99895] are corresponding α cuts are γ = [3.1021+0.5378α , 4.1777-0.5378 α] μ = [7.92335+0.5378 α , 8.99895-0.5378 α]

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α	low γ	up γ	low μ	up μ	low E(X)	up E(X)
0	3.1021	4.1777	7.92335	8.99895	46.04665	87.52908
0.1	3.15588	4.12392	7.97713	8.94517	47.70156	85.00915
0.2	3.20966	4.07014	8.03091	8.89139	49.3978	82.53895
0.3	3.26344	4.01636	8.08469	8.83761	51.13583	80.11801
0.4	3.31722	3.96258	8.13847	8.78383	52.91612	77.74586
0.5	3.371	3.9088	8.19225	8.73005	54.73914	75.42203
0.6	3.42478	3.85502	8.24603	8.67627	56.60536	73.14607
0.7	3.47856	3.80124	8.29981	8.62249	58.51524	70.9175
0.8	3.53234	3.74746	8.35359	8.56871	60.46924	68.73586
0.9	3.58612	3.69368	8.40737	8.51493	62.46784	66.60068
1	3.6399	3.6399	8.46115	8.46115	64.5115	64.5115

Table 3.1.2 Lower and Upper alpha cut for mean

3.2 Application - II

In this work, We observe the normal monthly rainfall for the state Uttarakhand which is the state of North West India from the month January to December during the year 2012.

We present a model of Rainfall for the state Uttarakhand in India using the Fatigue Life distribution with two parameters.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Rainfall (mm)	53.4	13.4	30.8	46.6	10.7	48.0	389.8	461.2	223.3	8.8	4.6	19.2

Table 3.2.1 Rainfall of Uttarakhand in 2012

The scale and shape parameter of Fatigue life distribution in Table 3.2.1, $\gamma = 1.6981$ $\mu = 47.012$. The triangular fuzzy number is $\gamma = [\ 1.1603, 1.6981, 2.2359\ ]$ & $\mu = [\ 46.4742, 47.012, 47,5498\ ]$ are corresponding α cuts are $\gamma = [1.1603+0.5378\alpha\ ,\ 2.2359-0.5378\ \alpha\ ]$ & $\mu = [\ 46.4742+0.5378\ \alpha\ ,\ 47.5498-0.5378\ \alpha\ ]$

α	low γ	up γ	low μ	up μ	low E(X)	up E(X)
0	1.1603	2.2359	46.4742	47.5498	77.75822	166.4064
0.1	1.21408	2.18212	46.52798	47.49602	80.81887	160.5757
0.2	1.26786	2.12834	46.58176	47.44224	84.02113	154.8949
0.3	1.32164	2.07456	46.63554	47.38846	87.36544	149.3637
0.4	1.37542	2.02078	46.68932	47.33468	90.85229	143.9815
0.5	1.4292	1.967	46.7431	47.2809	94.48212	138.7479
0.6	1.48298	1.91322	46.79688	47.22712	98.25542	133.6624
0.7	1.53676	1.85944	46.85066	47.17334	102.1727	128.7247
0.8	1.59054	1.80566	46.90444	47.11956	106.2343	123.9341
0.9	1.64432	1.75188	46.95822	47.06578	110.4408	119.2902
1	1.6981	1.6981	47.012	47.012	114.7926	114.7926

Table 3.2.2 Lower and Upper alpha cut for mean

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3.3 Application - III

In this work, We observe the normal monthly rainfall for the state Delhi which is the state of North West India from the month January to December during the year 2012. We present a model of Rainfall for the state Delhi in India using the Fatigue Life distribution with two parameters. We have constructed a model of Rainfall that produces result that qualitatively and quantitatively agree with experimental observation.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Rainfall (mm)	4.6	0.1	0.3	13.6	3.6	4.4	57.9	163.5	57.1	4.0	0.6	3.9

Table 3.3.1 Rainfall of Delhi in 2012

The scale and shape parameter of Fatigue life distribution in table 3.3.1 $\gamma = 3.1529$ $\mu=4.2156$, The triangular fuzzy number is $\gamma = [\ 2.6151, 3.1529, 3.6907\ ]$ & $\mu = [\ 3.6778, 4.2156, 4.7534\ ]$ are corresponding α cuts are $\gamma = [\ 2.6151+0.5378\alpha\ ,\ 3.6907-0.5378\ \alpha\ ]$ $\mu = [\ 3.6778+0.5378\ \alpha\ , 4.7534-0.5378\ \alpha\ ]$

α	**low γ**	**up γ**	**low μ**	**up μ**	**low E(X)**	**up E(X)**
0	2.6151	3.6907	3.6778	47.5498	16.25357	371.394
0.1	2.66888	3.63692	3.73158	47.49602	17.02145	361.6154
0.2	2.72266	3.58314	3.78536	47.44224	17.81556	351.9951
0.3	2.77644	3.52936	3.83914	47.38846	18.63637	342.5328
0.4	2.83022	3.47558	3.89292	47.33468	19.48435	333.228
0.5	2.884	3.4218	3.9467	47.2809	20.35995	324.0802
0.6	2.93778	3.36802	4.00048	47.22712	21.26365	315.0889
0.7	2.99156	3.31424	4.05426	47.17334	22.19592	306.2537
0.8	3.04534	3.26046	4.10804	47.11956	23.15722	297.5741
0.9	3.09912	3.20668	4.16182	47.06578	24.14801	289.0497
1	3.1529	3.1529	4.2156	47.012	25.16877	280.6799

Table 3.3.2 Lower and Upper alpha cut for mean

3.4 Application - IV

In this work, We observe the normal monthly rainfall for the state Punjab which is the state of North West India from the month January to December during the year 2012. We present a model of Rainfall for the state Punjab in India using the Fatigue Life distribution with two parameters. We have constructed a model of Rainfall that produces result that qualitatively and quantitatively agree with experimental observation.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Rainfall (mm)	35.8	2.9	1.9	20.2	0.8	9.6	67.7	105.1	83.5	2.8	0.4	8.2

Table 3.4.1 Rainfall of Punjab in 2012

The scale and shape parameter of Fatigue life distribution in Table 3.4.1 $\gamma =2.2535$ $\mu=7.7669$. The triangular fuzzy number is $\gamma = [\ 1.7157, 2.2535, 2.7913\ ]$ & $\mu = [\ 7.2291, 7.7669, 8.3047\ ]$ are corresponding α cuts are $\gamma = [1.7157+0.5378\alpha\ ,\ 2.7913-0.5378\ \alpha\ ]$ & $\mu = [\ 7.2291+0.5378\ \alpha\ , 8.3047-0.5378\ \alpha\ ]$

α	**low γ**	**up γ**	**low μ**	**up μ**	**low E(X)**	**up E(X)**
0	1.7157	2.7913	7.2291	8.3047	17.86899	40.65714
0.1	1.76948	2.73752	7.28288	8.25092	18.68445	39.16718
0.2	1.82326	2.68374	7.33666	8.19714	19.53121	37.71693
0.3	1.87704	2.62996	7.39044	8.14336	20.40973	36.30591
0.4	1.93082	2.57618	7.44422	8.08958	21.32049	34.93365
0.5	1.9846	2.5224	7.498	8.0358	22.26395	33.5997

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0.6	2.03838	2.46862	7.55178	7.98202	23.24058	32.30357
0.7	2.09216	2.41484	7.60556	7.92824	24.25084	31.04482
0.8	2.14594	2.36106	7.65934	7.87446	25.29519	29.82296
0.9	2.19972	2.30728	7.71312	7.82068	26.37412	28.63754
1	2.2535	2.2535	7.7669	7.7669	27.48808	27.48808

Table 3.4.2 Lower and Upper alpha cut for mean

3.5 Application - V

In this work, We observe the normal monthly rainfall for the state Himachal Pradesh which is the state of North West India from the month January to December during the year 2012. We present a model of Rainfall for the state Himachal Pradesh in India using the Fatigue Life distribution with two parameters. We have constructed a model of Rainfall that produces result that qualitatively and quantitatively agree with experimental observation.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Rainfall (mm)	112.1	66.4	43.2	64.1	11.5	25.5	207.1	311.6	152.1	2.9	6.7	31.6

Table 3.5.1 Rainfall of Himachal Pradesh in 2012

The scale and shape parameter of Fatigue life distribution in Table 3.5.1, γ=1.6073 μ=35.913. The triangular fuzzy number is γ = [1.0695, 1.6073, 2.1451] & μ = [35,3752, 35.913, 36.4508] are corresponding α cuts are γ = [1.0695+0.5378α , 2.1451-0.5378 α] μ = [35.3752+0.5378 α , 36.4508-0.5378 α]

α	**low γ**	**up γ**	**low μ**	**up μ**	**low E(X)**	**up E(X)**
0	1.0695	2.1451	35.3752	36.4508	55.60681	120.3141
0.1	1.12328	2.09132	35.42898	36.39702	57.78038	115.9904
0.2	1.17706	2.03754	35.48276	36.34324	60.06291	111.784
0.3	1.23084	1.98376	35.53654	36.28946	62.45488	107.6945
0.4	1.28462	1.92998	35.59032	36.23568	64.95676	103.7214
0.5	1.3384	1.8762	35.6441	36.1819	67.569	99.86433
0.6	1.39218	1.82242	35.69788	36.12812	70.29207	96.12274
0.7	1.44596	1.76864	35.75166	36.07434	73.12645	92.49619
0.8	1.49974	1.71486	35.80544	36.02056	76.0726	88.9842
0.9	1.55352	1.66108	35.85922	35.96678	79.13098	85.58631
1	1.6073	1.6073	35.913	35.913	82.30206	82.30206

Table 3.5.2 Lower and Upper alpha cut for mean

3.6 Application – VI

In this work, we observe the normal monthly rainfall for the state Jammu & Kashmir which is the state of North West India from the month January to December during the year 2012. We present a model of Rainfall for the state Jammu & Kashmir in India using the Fatigue Life distribution with two parameters.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Rainfall (mm)	120.2	116.7	53.3	112.5	60.1	42.9	121.0	256.4	138.5	14.9	12.7	67.5

Table 3.6.1 Rainfall of Jammu & Kashmir in 2012

The scale and shape parameter of Fatigue life distribution in Table 3.6.1, γ=0.92999 μ=64.065 The triangular fuzzy number is γ = [0.39219, 0.92999, 1.46779] & μ = [63.5272, 64.065, 64.6028] are corresponding α cuts are γ = [0.39219+0.5378α , 1.46779-0.5378 α] μ = [63.5272+0.5378 α , 64.6028-0.5378 α]

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α	low γ	up γ	low μ	up μ	low E(X)	up E(X)
0	0.39219	1.46779	63.5272	64.6028	68.41285	134.1932
0.1	0.44597	1.41401	63.58098	64.54902	69.90377	129.0795
0.2	0.49975	1.36023	63.63476	64.49524	71.58115	124.1606
0.3	0.55353	1.30645	63.68854	64.44146	73.44548	119.4362
0.4	0.60731	1.25267	63.74232	64.38768	75.49721	114.9057
0.5	0.66109	1.19889	63.7961	64.3339	77.73682	110.5687
0.6	0.71487	1.14511	63.84988	64.28012	80.16477	106.4246
0.7	0.76865	1.09133	63.90366	64.22634	82.78153	102.4732
0.8	0.82243	1.03755	63.95744	64.17256	85.58756	98.71376
0.9	0.87621	0.98377	64.01122	64.11878	88.58333	95.14597
1	0.92999	0.92999	64.065	64.065	91.76931	91.76931

Table 3.6.2 Lower and Upper alpha cut for mean

3.7 Application - VII

In this work, We observe the normal monthly rainfall for the state Rajasthan which is the state of North West India from the month January to December during the year 2012. We present a model of Rainfall for the state Rajasthan in India using the Fatigue Life distribution with two parameters.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Rainfall (mm)	1.1	0.0	0.0	6.35	12.05	10.5	104.4	261.8	109.1	0.95	0.0	0.75

Table 3.7.1 Rainfall of Rajasthan in 2012

The scale and shape parameter of Fatigue life distribution γ=2.53515 μ=11.39415. The triangular fuzzy number is γ = [1.99735, 2.53515, 3.07295] & μ = [10.85635, 11.39415, 11.93195] are corresponding α cuts are γ = [1.99735+0.5378α , 3.07295-0.5378 α] & μ = [10.85635+0.5378 α , 11.93195-0.5378 α]

α	low γ	up γ	low μ	up μ	low E(X)	up E(X)
0	1.99735	3.07295	10.85635	11.93195	32.51155	68.26878
0.1	2.05113	3.01917	10.91013	11.87817	33.86032	66.01523
0.2	2.10491	2.96539	10.96391	11.82439	35.25251	63.8135
0.3	2.15869	2.91161	11.01769	11.77061	36.68859	61.66312
0.4	2.21247	2.85783	11.07147	11.71683	38.16902	59.56363
0.5	2.26625	2.80405	11.12525	11.66305	39.69427	57.51456
0.6	2.32003	2.75027	11.17903	11.60927	41.26481	55.51544
0.7	2.37381	2.69649	11.23281	11.55549	42.88111	53.56581
0.8	2.42759	2.64271	11.28659	11.50171	44.54362	51.6652
0.9	2.48137	2.58893	11.34037	11.44793	46.25282	49.81314
1	2.53515	2.53515	11.39415	11.39415	48.00917	48.00917

Table 3.7.2 Lower and Upper alpha cut for mean

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3.8 Discussion

α	UP	Uttarakhand	Delhi	Punjab	HP	J & K	Rajasthan
0	46.04665	77.75822	16.25357	17.86899	55.60681	68.41285	32.51155
0.1	47.70156	80.81887	17.02145	18.68445	57.78038	69.90377	33.86032
0.2	49.3978	84.02113	17.81556	19.53121	60.06291	71.58115	35.25251
0.3	51.13583	87.36544	18.63637	20.40973	62.45488	73.44548	36.68859
0.4	52.91612	90.85229	19.48435	21.32049	64.95676	75.49721	38.16902
0.5	54.73914	94.48212	20.35995	22.26395	67.569	77.73682	39.69427
0.6	56.60536	98.25542	21.26365	23.24058	70.29207	80.16477	41.26481
0.7	58.51524	102.1727	22.19592	24.25084	73.12645	82.78153	42.88111
0.8	60.46924	106.2343	23.15722	25.29519	76.0726	85.58756	44.54362
0.9	62.46784	110.4408	24.14801	26.37412	79.13098	88.58333	46.25282
1	64.5115	114.7926	25.16877	27.48808	82.30206	91.76931	48.00917

Table 3.8.1 Lower fuzzy mean value of North West India

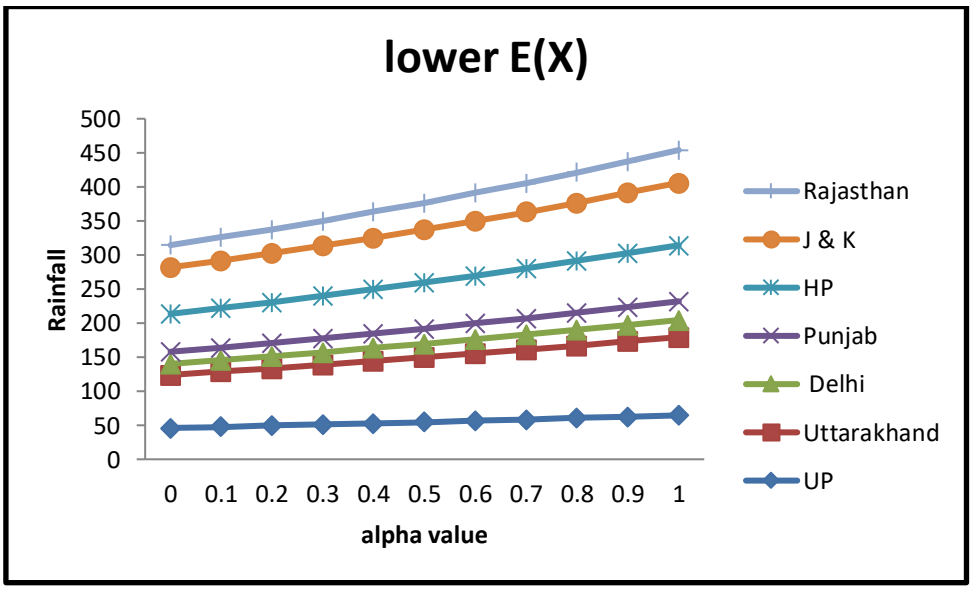

Fig: 3.8.1 Fuzzy lower mean value of North West India

α	UP	Uttarakhand	Delhi	Punjab	HP	J & K	Rajasthan
0	87.52908	166.4064	371.394	40.65714	120.3141	134.1932	68.26878
0.1	85.00915	160.5757	361.6154	39.16718	115.9904	129.0795	66.01523
0.2	82.53895	154.8949	351.9951	37.71693	111.784	124.1606	63.8135
0.3	80.11801	149.3637	342.5328	36.30591	107.6945	119.4362	61.66312
0.4	77.74586	143.9815	333.228	34.93365	103.7214	114.9057	59.56363
0.5	75.42203	138.7479	324.0802	33.5997	99.86433	110.5687	57.51456
0.6	73.14607	133.6624	315.0889	32.30357	96.12274	106.4246	55.51544
0.7	70.9175	128.7247	306.2537	31.04482	92.49619	102.4732	53.56581
0.8	68.73586	123.9341	297.5741	29.82296	88.9842	98.71376	51.6652
0.9	66.60068	119.2902	289.0497	28.63754	85.58631	95.14597	49.81314
1	64.5115	114.7926	280.6799	27.48808	82.30206	91.76931	48.00917

Table 3.8.2 Upper fuzzy mean value of North West India

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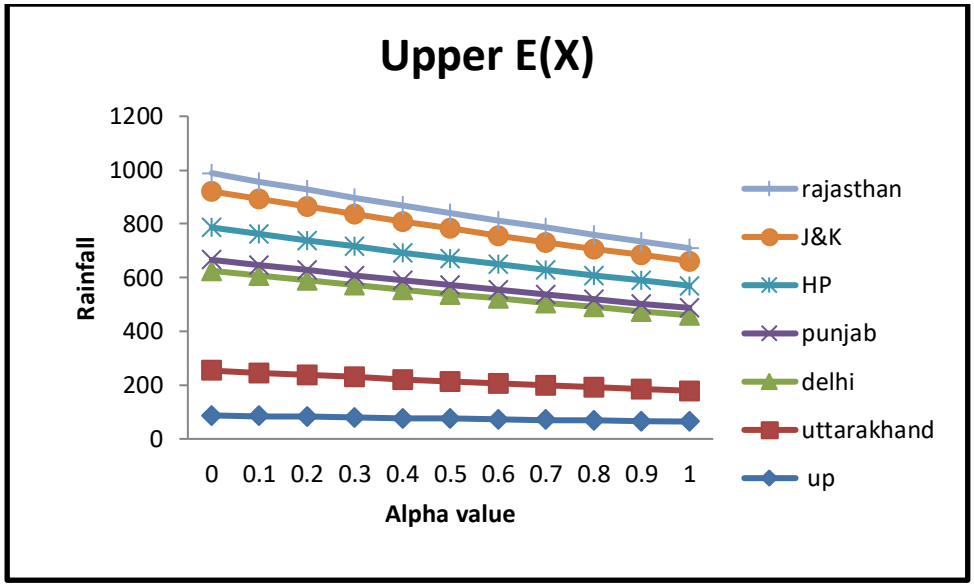

Fig: 3.8.2 Fuzzy Upper mean value of North West India

4. CONCLUSION

This chapter has provided a brief introduction to the fatigue Life distribution and their application in fuzzy Mathematical modeling. Finally in this paper, if the fuzzy mean is increasing in lower alpha cut values and it decreases in upper alpha cut values. In this work, we observe the normal monthly rainfall of the North West India from the month January to December during the year 2012. Using the Fatigue life distribution we present the lower and upper fuzzy mean value of North West India. In this observation the state Rajasthan has the highest lower and upper mean value in North east India . Finally we conclude that Rajasthan gets highest level of rainfall during the year 2012.

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