

Fuzzy based Two-Warehouse Inventory Model with Exponential Demand for Perishable Items

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ABSTRACT - In this study, inventory management for perishable items in e-commerce and retail settings poses a significant challenge due to the high deterioration rates, uncertainty in inventory costs, and increasing exponentially demands, often leads to stock-outs, excess wastes, and inflated expenses. The existing models often overlooks the combined effects of learning in operations, fuzzy uncertainties in costs, and dual-warehouse systems including both owned and rented warehouses, creates a gap in realistic optimization for dynamic environments. For addressing these issues, we propose a learning-based two-warehouse inventory model for perishable items including fuzzy environment, incorporating with learning effects in ordering costs for reducing repetition inefficiencies, while treating holding, shortages, and deterioration costs as triangular fuzzy numbers for handling imprecision. The model assumes an owned warehouse with limited capacity and another rented for overflows, exponential demand growths, and use of preservation technology for mitigating deterioration rates and extending items quality.

The total crisp and fuzzy inventory costs are also minimized with respect to cycle time, with considering stock depletes to zero, and fuzzy costs. The information is really

helpful for supplies and goods for making decisions about when to order more of these goods. The model helps people understand how things, like shipments and learning rates affect the time it takes to do things and the total cost of goods.

Key Elements: Learning Effect, Fuzzy Environment, Preservation Technology, Deterioration, Owned Warehouse, Rented Warehouse, Exponential demand.

1.1 INTRODUCTION

In real-world scenarios, the necessity for quick replenishment becomes extremely important, especially during times of strong demand like festival seasons or significant internet sales events. Employees that handle inventory and place orders gain experience over time, which boosts productivity and lowers order placement costs. This phenomenon is frequently referred to as the learning effect. Products that spoil rapidly, such food items, flowers, and various other perishable goods, are often handled by large retailers, particularly e-commerce platforms. These products need to be handled carefully since they might deteriorate quickly and cause large financial losses. Preservation technologies—such as refrigeration systems, regulated storage conditions, and sophisticated packaging techniques—are crucial in addressing this problem because they prolong the shelf life of items and slow down the general deterioration process. A fuzzy environment is incorporated into the suggested inventory model. In this case, triangular fuzzy integers that are marginally different from fixed values are used to represent uncertain cost factors. This method enables the model to capture the intrinsic imprecision found in actual supply chains. The main goal of the suggested model is to find the ideal cycle time at which the inventory level drops to zero in order to reduce the overall inventory cost under both crisp and fuzzy conditions. The centroid approach is then used to defuzzified the fuzzy costs, yielding a representative average value for the efficient decision-making system. This explains why inventory models are crucial in the modern world, particularly for online retailers who deal with perishable commodities. In this regarding, early works like [1] Pakkala and Acharya (1992) developed the deterministic inventory model for two warehouses rates in deteriorating items with finite replenishment rate. They focused on how to manage stock when items spoil over time and need extra storage. [2] Singh and Malik (2010) looked at inventory system with decaying items and variable holding cost. [3] Lee and Hsu (2009) proposed the two-warehouse production model with time-

dependent demands for deteriorating inventory items is close to our exponential demand but they didn't include rented warehouse costs or preservation. [4] Mandal and Giri (2017) made a two-warehouse integrated inventory model with stock-dependent demand and quantity discount offer. They included deterioration management system which helps in understanding how flaws in production affect storage needs. [5] Sheikh and Patel (2017) worked on two-warehouse inventory model under time-dependent demand and shortages with different deterioration rates. [6] De and Rawat (2011) developed a fuzzy inventory model using triangular fuzzy number without shortages. More recent, [7] Malik and Garg (2021) made an improved fuzzy inventory model includes fuzzy in costs but no learning effect under two warehouses. [8] Kumar (2021) also used fuzzy in EOQ models under fuzzy reasoning which helps in soft computing for inventory. On deterioration with shortages, [9] Singha et al (2019) assumed a scenario for deterioration system with shortages under two-warehouse system. They had selling price dependent demand and partial backlogged for perishable items. [10] Yadav et al. (2017) developed a model with trade credit policy for two-warehouse system by using deterioration system and conditionally permissible delay in payment. This adds finance angle but again, no fuzzy or exponential demand fully. [11] Jayaswal et al. (2019) looked at effects of learning with trade credit financing on retailer ordering policy. They showed how learning reduces costs over time. [12] Jayaswal et al. (2021) worked on the inventory system by using preservation technology and effect of learning in the two ware-houses. They included preservation investment which is key for perishables like in our model. [13] Mandal (2023) optimized fuzzy inventory model under stock-dependent linear trended demand with deteriorating items and variable holding cost. This includes fuzzy and deterioration but demand is linear not exponential. [14] Dhivya Lakshmi and Pandian (2021) made a production inventory model with exponentially declining deterioration and exponential demand rate matches our demand but no fuzzy or two-warehouse. [15] Mashud et al. (2021) had a sustainable production-inventory model for cleaner production in two-warehouse with advertisement and partial backlogging. [16] Tiwari et al. (2022) on sustainable inventory management for perishable products with fuzzy demand which emphasizes cleaner production. The carbon emissions are major issues for environments in this direction; [17] Alsaedi et al. (2023) designed a model with fuzzy environment and trade credit policies for perishable items under sustainability. [18] Alsaedi et al. (2023)

focused on reducing carbon emissions for defective items with credit financing under learning in fuzzy environment. [19] Alamri et al. (2023) had a supply chain model with credit financing and learning effect for imperfect quality items with fuzzy which adds green aspect. These bring in sustainability but our focus is more on cost with preservation. [20] Khan et al. (2023) studied learning effect in green supply chain inventory model under trade credit and deterioration. [21] Sebatjane and Adetunji (2020) proposed a three-tier supply chain model for perishable products with fuzzy parameters. [22] Paul et al. (2024) invested in a fuzzy two-warehouse inventory model with preservation technology.

RESEARCH GAP

Many studies have been conducted on inventory models for the deteriorating items; however, most of them consider the deterministic environments and do not incorporate uncertainty through fuzzy approaches. In real-world situations, several cost parameters such as holding cost, deterioration cost, and shortage cost are regularly uncertain due to market fluctuations, inflation, and changing the operational conditions. Therefore, representing these costs with precise numerical values may not always reflect the practical conditions. Fuzzy logic provides an effective context to deal with such imprecision by allowing the parameters to be represented using ranges or linguistic values such as low, medium, and high. The concept of fuzzy sets was first introduced by Zadeh (1965) and has since been extensively applied in the decision-making and inventory management problems. Later De and Rawat (2011) developed a fuzzy inventory model for the deteriorating items under without shortages.

Later studies further expanded the fuzzy-based inventory model by incorporating the different types demand patterns and operational policies. In addition, the concept of the learning effect has been explored in inventory management and supply chain management systems. Jayaswal et al. (2019) examined the influence of learning on ordering processes, demonstrating that repeated operations improve the efficiency and reducing the ordering costs over the time. However, this concept has not been effectively integrated within two-warehouse inventory systems that also include preservation technology. Yadav et al. (2017) incorporated the trade-related policies in a two-warehouse environment but did not integrate fuzzy learning effects. Similarly,

Mandal (2023) analyzed fuzzy inventory management systems for the deteriorating items under specific demand patterns such as linear demand but did not consider exponential demand growth. Other researchers, including Alamri et al. (2023), have introduced the sustainability aspects into inventory models; however, the simultaneous integration of the multiple factors remains limited. The objective of the proposed model is to minimize the total fuzzy inventory cost by obtaining the optimal cycle time at which the inventory level reaches zero. We use a method to make the fuzzy numbers clear and validate with numerical examples and sensitivity analysis. The changeable effects of parameters like shipments learning rate, deterioration rates are shown to have impacts on cost reduction. Thus, while literature has pieces of the puzzle of works this model puts them together for a picture useful for today’s digital markets. Our proposed model calculated the fuzzy cost, for the system and the changeable effect of inventory parameters are presented in the sensitivity part.

4.5 Notations and Assumptions

Notation	Description
$D(t)$	Rate of demand at any time t
$I_r(t)$	The buyer’s stock level in rented warehouse (RW)
$I_o(t)$	The buyer’s stock level in owned warehouse (OW)
Q	The buyer’s storage capacity in owned warehouse (OW)
θ_r	Deterioration rate in RW
θ_o	Deterioration rate in OW
t_1	Time where inventory reaches zero (decision variable)
t_2	Fixed cycle time for each production cycle
$S = S_o + \frac{S_1}{n^s}$	Set up cost per order, S_o , S_1 are fixed and variable ordering cost
f	Learning factor
ξ	Preservation cost per units for RW and OW
D_c	The cost per unit item for the deterioration in RW/OW
H_r	The cost per unit item for the storage of items in RW

H_o	The cost per unit item for the storage of items in OW
\tilde{D}_c	The cost per unit item for the fuzzy deterioration in RW/OW
$(\tilde{D}_{c1}, \tilde{D}_{c2}, \tilde{D}_{c3})$	Triangular fuzzy number for the fuzzy deterioration cost in RW/OW
\tilde{H}_r	The cost per unit item for the fuzzy storage of items in RW
$(\tilde{H}_{r1}, \tilde{H}_{r2}, \tilde{H}_{r3})$	Triangular fuzzy number for the fuzzy storage cost in RW
\tilde{H}_o	The cost per unit item for the storage of items in OW
$(\tilde{H}_{o1}, \tilde{H}_{o2}, \tilde{H}_{o3})$	Triangular fuzzy number for the fuzzy storage cost in OW
TIC(t_1)	Total inventory cost for the system in crisp model
$\tilde{TIC}(t_1)$	Total fuzzy inventory cost for the system in fuzzy model

Assumptions

1. The value of lead time is not considered. The replenishment rate of the perishable items is unlimited but size of the lot is finite.
2. The repair policy is not considered for the deteriorating items in the whole cycle length of the inventory system.
3. The deteriorating cost, storage cost in OW and storage cost in RW are taken as triangular fuzzy number.
4. The rate of demand of the perishable items depends on the time, $D(t) = e^{kt}$, $k > 0$.
5. The buyer's storage cost in RW is more than OW (assumed). Firstly, the perishable items have preferred in OW and after that in RW (assumed).

4.5 Model formulation

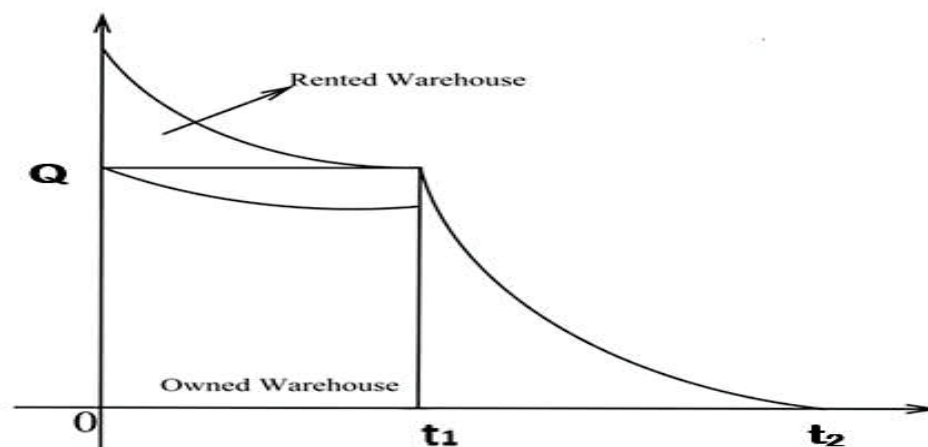


Figure 1: Graphical representation of two-warehouses inventory

Figure 1. shows how inventory depletes in both the warehouses. First, we discussed on the beginning of the cycle, the inventory level in RW is high, and in OW is Q. During the time 0 to t_1 , customer demand is fulfilled only from RW. During this interval $[0, t_1]$, the inventory in the RW decreases due to customer demand $D(t) = e^{kt}$ and deterioration at rate θ_r (reduced by preservation technology). Some items also deteriorate in OW during the same period, but at a lower rate θ_o . When the RW stock reaches zero at time t_1 , then from t_1 to t_2 all demand is fulfilled from OW. In OW, inventory decreases from the beginning due to demand and deterioration, and finally reaches zero at time t_2 . This completes the cycle, after which the next order arrives. The initial stock in OW (Q_o) is calculated using the boundary condition that inventory reaches zero at time t . Now we present the mathematical model. The inventory level in RW during $[0, t_1]$ reduces due to demand and deterioration, following the differential equation

$$\frac{dI_r(t)}{dt} + \theta_r I_r(t) = -e^{kt}, \quad 0 \leq t \leq t_1 \quad \dots(1)$$

with boundary condition $I_r(t_1) = 0$. Solving this gives the expression for $I_r(t)$. Similarly, for OW, during $[0, t_1]$ the differential equation is

$$\frac{dI_{o1}(t)}{dt} + \theta_o I_{o1}(t) = 0, \quad 0 \leq t \leq t_1 \quad \dots(2)$$

$$\frac{dI_{o2}(t)}{dt} + \theta_o I_{o2}(t) = -e^{kt}, \quad t_1 \leq t \leq t_2 \quad \dots(3)$$

4.6.1 Formulation of model under crisp environment

Therefore, the rate of change of inventory in RW and OW follows these differential equations for time 0 to t_1 :

$$\frac{dI_r(t)}{dt} + \theta_r I_r(t) = -e^{kt}, \quad 0 \leq t \leq t_1 \quad \dots(1)$$

$$\frac{dI_{o1}(t)}{dt} + \theta_o I_{o1}(t) = 0, \quad 0 \leq t \leq t_1 \quad \dots(2)$$

$$\frac{dI_{o2}(t)}{dt} + \theta_o I_{o2}(t) = -e^{kt}, \quad t_1 \leq t \leq t_2 \quad \dots(3)$$

With the boundary conditions $I_r(t_1) = 0, I_{o1}(0) = Q, I_{o2}(t_2) = 0$ respectively.

$$\text{or} \quad I_r(t) = \frac{1}{(k + \theta_r)} \left(e^{kt_1} \cdot e^{\theta_r(t_1-t)} - e^{kt} \right) \quad \dots(4)$$

$$I_{o1}(t) = Q e^{-\theta_0 t} \quad \dots(5)$$

$$I_{o2}(t) = \frac{1}{(k + \theta_0)} \left(e^{kt_2} \cdot e^{\theta_0(t_2-t)} - e^{kt} \right) \quad \dots(6)$$

Due to continuity of $I_o(t)$ at $t=t_1$, it follows from the above system of equations (5) and (6), we get

$$Q = \frac{1}{(k + \theta_0)} \left(e^{(k+\theta_0)t_2} - e^{(k+\theta_0)t_1} \right) \quad \dots(7)$$

Ordering cost (O_c) is

$$S = S_o + \frac{S_1}{n_f} \quad \dots(8)$$

The inventory storage cost in RW is

$$\begin{aligned} IHC_r &= H_r \left(\int_0^{t_1} I_r(t) dt \right) \\ &= H_r \left(\int_0^{t_1} \left\{ \frac{1}{(k + \theta_r)} \left(e^{kt_1} \cdot e^{\theta_r(t_1-t)} - e^{kt} \right) \right\} dt \right) \\ &= \frac{H_r}{(k + \theta_r)} \left(\frac{e^{kt_1}}{\theta_r} (e^{\theta_r t_1} - 1) - \frac{1}{k} (e^{kt_1} - 1) \right) \quad \dots(9) \end{aligned}$$

The inventory holding cost in OW is

$$\begin{aligned} IHC_o &= H_o \left(\int_0^{t_1} I_{o1}(t) dt + \int_{t_1}^{t_2} I_{o2}(t) dt \right) \\ &= H_o \left(\int_0^{t_1} \{ Q e^{-\theta_0 t} \} dt + \int_{t_1}^{t_2} \left\{ \frac{1}{(k + \theta_0)} \left(e^{kt_2} \cdot e^{\theta_0(t_2-t)} - e^{kt} \right) \right\} dt \right) \end{aligned}$$

$$= \frac{H_o}{(k + \theta_0)} \left(\frac{1}{\theta_0} \left(e^{(k+\theta_0)t_2} - e^{kt_2} - e^{(k+\theta_0)t_1} + e^{kt_1} \right) + \frac{1}{k} \left(e^{kt_1} - e^{kt_2} \right) \right) \quad \dots(10)$$

The inventory cost due to deterioration in RW is

$$\begin{aligned} IDC_r &= D_c \left(\int_0^{t_1} \theta_r \cdot I_r(t) dt \right) \\ &= D_c \cdot \theta_0 \left(\int_0^{t_1} \left\{ \frac{1}{(k + \theta_r)} \left(e^{kt_1} \cdot e^{\theta_r(t_1-t)} - e^{kt} \right) \right\} dt \right) \\ &= \frac{D_c \cdot \theta_0}{(k + \theta_r)} \left(\frac{e^{kt_1}}{\theta_r} \left(e^{\theta_r t_1} - 1 \right) - \frac{1}{k} \left(e^{kt_1} - 1 \right) \right) \quad \dots(11) \end{aligned}$$

The inventory cost due to deterioration in OW is

$$\begin{aligned} IDC_o &= D_c \left(\int_0^{t_1} \theta_o \cdot I_{o1}(t) dt + \int_{t_1}^{t_2} \theta_o \cdot I_{o2}(t) dt \right) \\ &= D_c \cdot \theta_0 \left(\int_0^{t_1} \{ Q e^{-\theta_o t} \} dt + \int_{t_1}^{t_2} \left\{ \frac{1}{(k + \theta_o)} \left(e^{kt_2} \cdot e^{\theta_o(t_2-t)} - e^{kt} \right) \right\} dt \right) \\ &= \frac{D_c \cdot \theta_0}{(k + \theta_o)} \left(\frac{1}{\theta_o} \left(e^{(k+\theta_o)t_2} - e^{kt_2} - e^{(k+\theta_o)t_1} + e^{kt_1} \right) + \frac{1}{k} \left(e^{kt_1} - e^{kt_2} \right) \right) \dots(12) \end{aligned}$$

The preservation inventory cost for the time period (t_1) is $PIC_c = \xi \cdot t_1$ Now, putting the all values from the equations (8) to (12) in the below equation (13) of total inventory cost (TIC), we get

$$TIC(t_1) = \frac{1}{t_2} (S_c + IHC_r + IHC_o + IDC_r + IDC_o + PIC_c)$$

$$= \frac{1}{t_2} \left[\begin{aligned} & \left[S_o + \frac{S_1}{n^f} \right] + \frac{H_r}{[k + \theta_r]} \left[\frac{e^{k t_1}}{\theta_r} [e^{\theta_r t_1} - 1] - \frac{1}{k} [e^{k t_1} - 1] \right] \\ & + \frac{H_o}{[k + \theta_o]} \left[\frac{1}{\theta_o} [e^{(k+\theta_o)t_2} - e^{k t_2} - e^{(k+\theta_o)t_1} + e^{k t_1}] + \frac{1}{k} [e^{k t_1} - e^{k t_2}] \right] \\ & + \frac{D_c \cdot \theta_o}{[k + \theta_r]} \left[\frac{e^{k t_1}}{\theta_r} [e^{\theta_r t_1} - 1] - \frac{1}{k} [e^{k t_1} - 1] \right] \\ & + \frac{D_c \cdot \theta_o}{k + \theta_o} \left[\frac{1}{\theta_o} [e^{(k+\theta_o)t_2} - e^{k t_2} - e^{(k+\theta_o)t_1} + e^{k t_1}] + \frac{1}{k} [e^{k t_1} - e^{k t_2}] \right] \end{aligned} \right] \quad \dots(13)$$

4.6.2 Crisp model with fuzzy system

In this section, the deterioration cost and storage cost are modeled within a fuzzy environment using the triangular fuzzy numbers. The consequential fuzzy total cost is subsequently defuzzified using the centroid method to obtain a crisp value for the analysis work.

$$\tilde{TIC} = \frac{1}{t_2} \left[\begin{aligned} & \left[S_o + \frac{S_1}{n^f} \right] + \left[\frac{\tilde{H}_{o1} + \tilde{H}_{o2} + \tilde{H}_{o3}}{3} \right] \frac{1}{(k + \theta_r)} \left[\frac{e^{k t_1}}{\theta_r} [e^{\theta_r t_1} - 1] - \frac{1}{k} [e^{k t_1} - 1] \right] \\ & + \left[\frac{\tilde{H}_{o1} + \tilde{H}_{o2} + \tilde{H}_{o3}}{3} \right] \frac{1}{k + \theta_o} \left[\frac{1}{\theta_o} [e^{(k+\theta_o)t_2} - e^{k t_2} - e^{(k+\theta_o)t_1} + e^{k t_1}] + \frac{1}{k} [e^{k t_1} - e^{k t_2}] \right] \\ & + \left[\frac{\tilde{D}_{c1} + \tilde{D}_{c2} + \tilde{D}_{c3}}{3} \right] \frac{\theta_o}{k + \theta_r} \left[\frac{e^{k t_1}}{\theta_r} [e^{\theta_r t_1} - 1] - \frac{1}{k} [e^{k t_1} - 1] \right] \\ & + \left[\frac{\tilde{D}_{c1} + \tilde{D}_{c2} + \tilde{D}_{c3}}{3} \right] \frac{\theta_o}{k + \theta_o} \left[\frac{1}{\theta_o} [e^{(k+\theta_o)t_2} - e^{k t_2} - e^{(k+\theta_o)t_1} + e^{k t_1}] + \frac{1}{k} [e^{k t_1} - e^{k t_2}] \right] + \xi \cdot t_1 \end{aligned} \right] \quad \dots(14)$$

Accordingly, the total fuzzy cost obtained from Equation (13) is defuzzified using the centroid method in order to find an equivalent crisp value for further analysis work. The total fuzzy cost per unit time is.

4.7 Solution Method.

To find the best cycle time T that gives the lowest total cost, we use simple mathematics. We take the total crisp cost TIC(t) or the defuzzified fuzzy cost (after centroid method) and find its derivative with respect to t. We set $\frac{dTIC(t_1)}{dt} = 0$ for the fuzzy case). This equation gives the condition for minimum cost. Because the cost

function has exponential terms from demand and deterioration, it is not easy to solve by hand, so we solve it numerically using Mathematica software. For making sure it is really minimum, we check the second derivative. If $\frac{d^2T\tilde{C}(t_1)}{dt^2} > 0$ at that point, then the cost curve is convex, so the solution is minimum.

4.8 Numerical Example

In this section, we are discussing inventory parameters for the model and for calculation of total cost and cycle time, we used the Mathematica software. The inventory parameters and decision variable have been shown in the Table 4.2.

Table 4.2: Model's inventory parameters

Inventory parameters inputs	Numerical values of the inventory parameters
S_o	300\$ per order
S_1	90\$ per order
n	6
f	0.26
H_r	0.65 \$ per unit item
H_o	0.50 per unit\$,
θ_r	0.065
θ_o	0.074
R	0.5
t_2	1.2 years
$(\tilde{D}_{c1}, \tilde{D}_{c2}, \tilde{D}_{c3})$	(0.35, 0.40, 0.45)
$(\tilde{H}_{r1}, \tilde{H}_{r2}, \tilde{H}_{r3})$	(0.72, 0.74, 0.76)
$\tilde{H}_{o1}, \tilde{H}_{o2}, \tilde{H}_{o3}$	(0.25, 0.30, 0.35)

\tilde{T}_1^*	0.854 year
$\tilde{Z}_3(\tilde{T}_1^*)$	6765 \$

4.8 Sensitivity Analysis

Table 4.3 : Effect of shipments on the Fuzzy Cycle Length and Total Inventory Fuzzy cost

Number of shipments	Fuzzy Cycle Length (t_1^*)	Total Inventory Fuzzy Cost $TIC(t_1^*)$
1	0.854	6915
2	0.854	6896
3	0.854	6858
4	0.854	6807
5	0.854	6787
6	0.854	6765

Table 4.4 : Effect of learning rate on the Fuzzy Cycle Length and Total Inventory Fuzzy cost

Learning factor (f)	Fuzzy Cycle Length (t_1^*)	Total Inventory Fuzzy Cost $TIC(t_1^*)$
0.22	0.854	6987
0.23	0.854	6920
0.24	0.854	6895
0.25	0.854	6790
0.26	0.854	6765

Table 4.5 : Effect of deteriorating rate in RW on the Fuzzy Cycle Length and Total Inventory Fuzzy cost

Deteriorating rate in RW	Fuzzy Cycle Length (t_1^*)	Total Inventory Fuzzy Cost $\tilde{TIC}(t_1^*)$
0.040	0.654	6843
0.045	0.685	6815
0.050	0.754	6801
0.055	0.797	6784
0.065	0.854	6765

Table 4.6 : Effect of deteriorating rate in OW on the Fuzzy Cycle Length and Total Inventory Fuzzy cost

Deteriorating rate in OW	Fuzzy Cycle Length (t_1^*)	Total Inventory Fuzzy Cost $\tilde{TIC}(t_1^*)$
0.074	0.854	6765
0.075	1.86	6785
0.076	2.10	6796
0.077	2.43	6820
0.078	2.57	6838

Table 4.7 : Effect of Fuzzy storage cost in RW on the Fuzzy Cycle Length and Total Inventory Fuzzy cost

Fuzzy storage cost in RW	Fuzzy Cycle Length (t_1^*)	Total Inventory Fuzzy Cost $\tilde{TIC}(t_1^*)$
(0.69, 0.71, 0.73)	0.894	6753
(0.70, 0.72, 0.74)	0.879	6757

(0.71, 0.73, 0.75)	0.867	6762
(0.72, 0.74, 0.76)	0.854	6765

Table 4.8 : Effect of Fuzzy storage cost in OW on the Fuzzy Cycle Length and Total Inventory Fuzzy cost

Fuzzy storage cost in OW	Fuzzy Cycle Length (t_1^*)	Total Inventory Fuzzy Cost $TIC(t_1^*)$
(0.25, 0.30, 0.35)	0.854	6765
(0.30, 0.35, 0.40)	0.892	6743
(0.35, 0.40, 0.45)	0.961	6728
(0.40, 0.45, 0.50)	1.01	6714

Table 4.9: Impact of Fuzzy deterioration cost on the Fuzzy Cycle Length and Total Inventory Fuzzy cost

Fuzzy deterioration cost	Fuzzy Cycle Length (t_1^*)	Total Inventory Fuzzy Cost $TIC(t_1^*)$
(0.35, 0.40, 0.45)	0.854	6765
(0.40, 0.45, 0.50)	0.765	6741
(0.45, 0.50, 0.55)	0.565	6721
(0.50, 0.55, 0.60)	0.342	6695

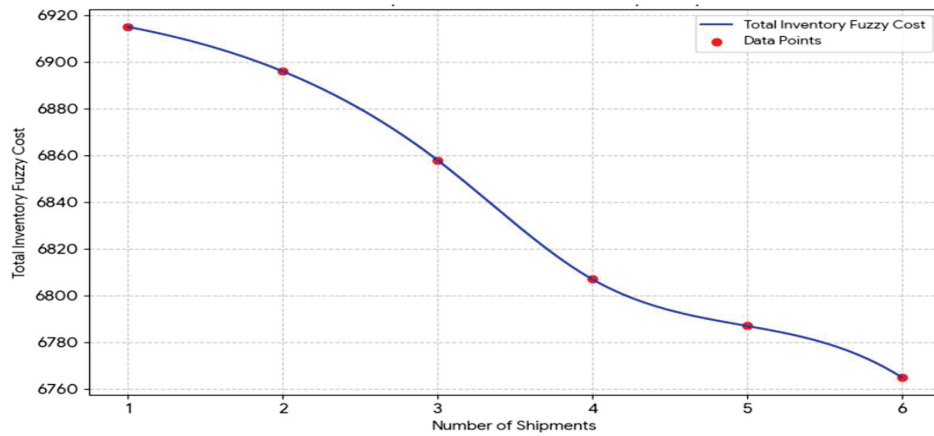


Figure 4.5: Effect of shipments on Total Inventory Fuzzy cost

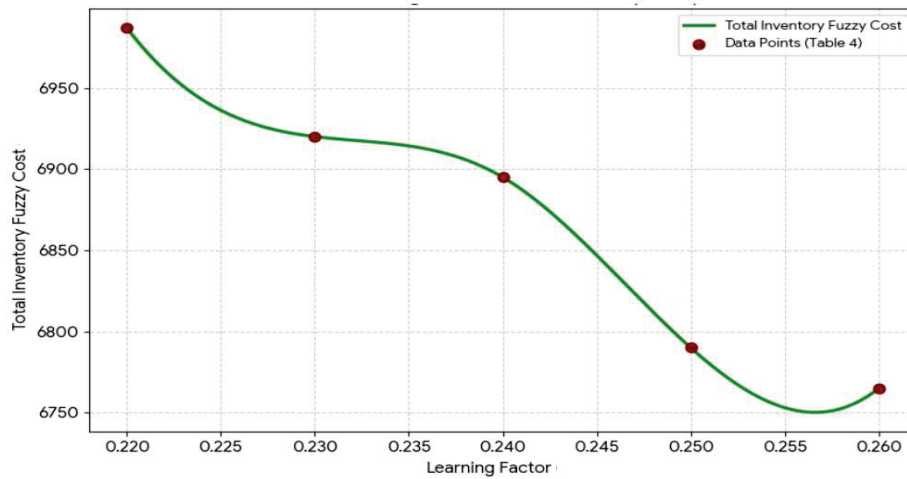


Figure 4.6: Effect of learning rate on the Total Inventory Fuzzy cost

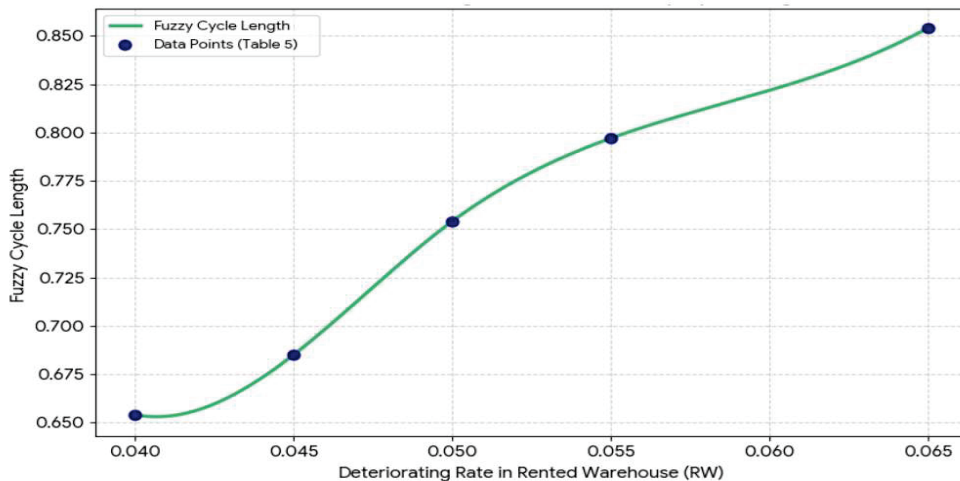


Figure 4.7 : Effect of deterioration rate in RW on the Fuzzy Cycle Length

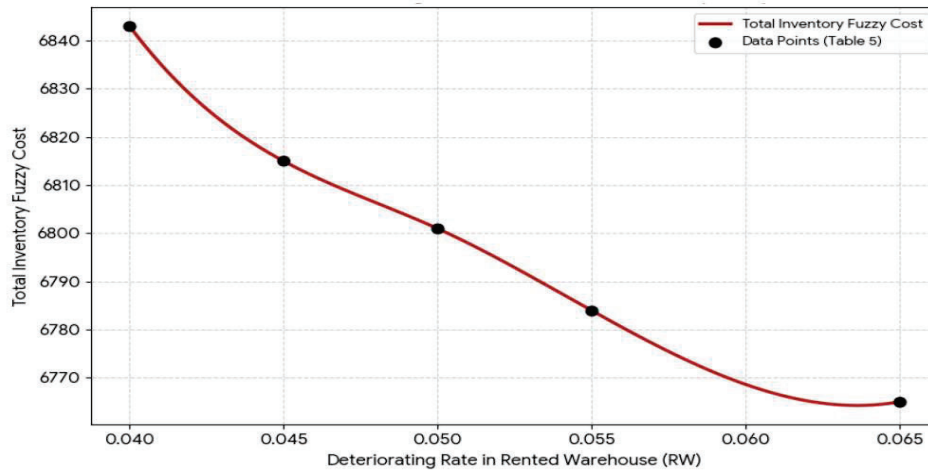


Figure 4.8 : Effect of deterioration rate in RW on the Total Inventory Fuzzy cost

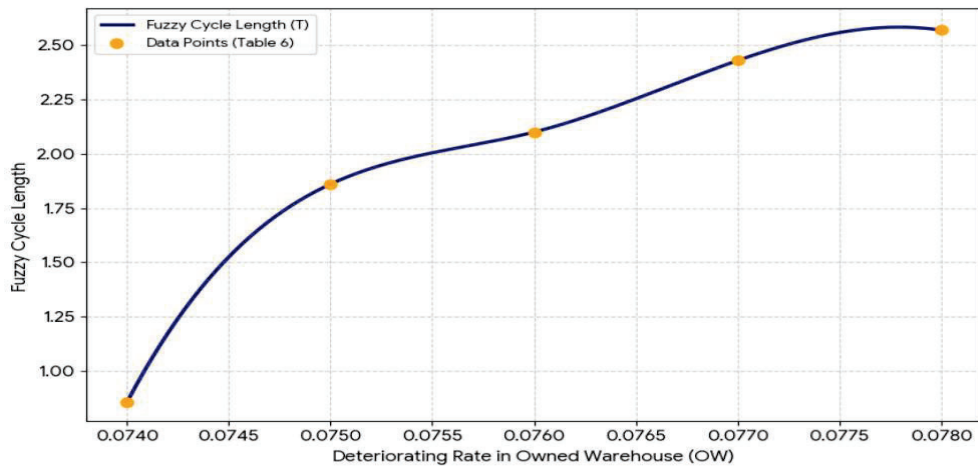


Figure 4.9 : Effect of deterioration rate in OW on the Fuzzy Cycle Length

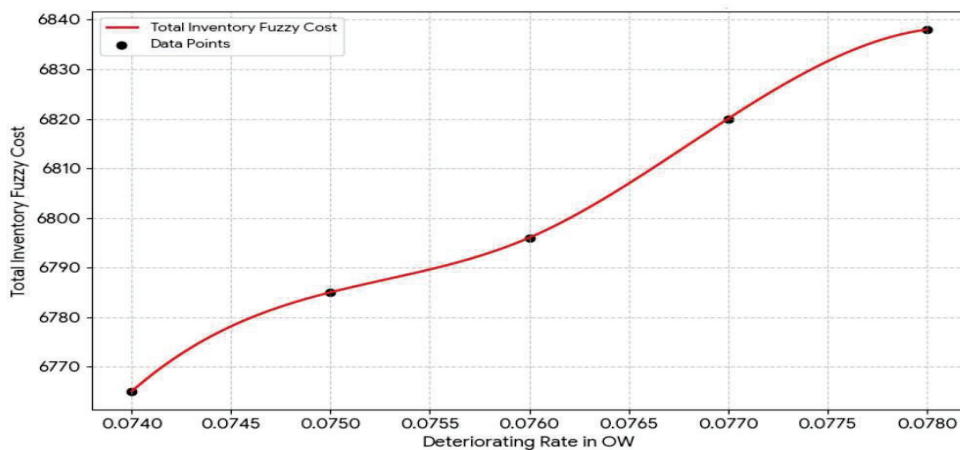


Figure 4.10: Effect of deterioration rate in OW on the Total Inventory Fuzzy cost

4.11 CONCLUSION

We created an inventory model that uses a fuzzy environment and a learning effect especially for items that spoil quickly like fruits and vegetables. In today's world online shopping is growing fast with websites like Flipkart, Amazon and Myntra so sellers have to handle items like dairy products and medicines carefully because they deteriorate quickly and can cause big financial losses if not managed properly. Our model includes learning in ordering cost so with every repeated order the process improves mistakes reduce and setup cost comes down over time. We used triangular numbers for costs like holding, deterioration and shortages because these values are never exact in the actual market. They change due to price changes, weather effects or customer reactions. This fuzzy way makes the model much closer to supply chain situations than old models.

The analysis shows that parameters like the number of shipments, learning factor, deterioration rates in the owned and rented warehouses and fuzzy storage costs all have effects. More shipments lower the fuzzy cost a lot without changing the cycle time. A higher learning factor reduces cost by improving efficiency. A higher deterioration rate extends the cycle time slightly. The fuzzy costs keep the results stable with uncertainty. These findings help retailers make decisions on ordering, preservation investment and handling uncertain costs to reduce waste and keep customers happy. This proposed model brings together key features that were not combined well before. Like exponential growing demand, preservation technology learning effect in ordering and fuzzy handling of uncertain parameters. By finding the cycle time that minimizes the total fuzzy cost the model gives a strong and practical tool for inventory management in dynamic and uncertain environments.

4.12 Future Scope

The model can be extended for carbon emission, inflation, shortages and stochastic demand to make the model more realistic. It would also be an idea to make preservation technology a separate decision. This means the model can decide how much to spend on things like cooling and packaging to reduce deterioration. The model can be used to make supply chains more sustainable and to reduce waste.

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