Functionally Graded Material Plate Subject to Thermomechanical Loading: A Review

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Abstract— **The material property of the functionally graded material (FGM) can be tailored to accomplish the specific demands in various engineering utilizations to achieve the advantage of the properties of individual material. This is possible due to the material composition of the FGM changes sequentially in a preferred direction. The thermo-mechanical deformation of FGM structures have attracted the attention of many researchers in the past few years in different engineering applications which include design of aerospace structures, heat engine components and nuclear power plants etc. The varying nature of FGMs makes design and analysis more challenging compared to traditional materials. In this analysis technique properties are employed for each layer, and thus the actual distribution of the phases does not explicitly affect the structural solution for the plate. The concept of FGMs hinges on materials science and mechanics due to the integration of the material and structural considerations into the final design of structural components. Because of the many variables that control the design of functionally graded microstructures, full utilization of the FGMs potential requires the development of appropriate modeling strategies for their response to combined thermo-mechanical loads.**

This paper reviews the major work in functionally graded materials (FGMs) with an emphasis on the plate. Various aspects of plate theory, finite element formulation methods, software used, material gradient functions, boundary conditions and various combinations of functionally graded materials are reflected in this paper.

Keywords— FGM; plate; functions.

I. Introduction

The material property of the FGM can be tailored to accomplish the specific demands in various engineering utilizations to achieve the advantage of the properties of individual material. This is possible due to the material composition of the FGM changes sequentially in a preferred direction. The thermo-mechanical deformation of FGM structures have attracted the attention of many researchers in the past few years in different engineering applications which include design of aerospace structures, heat engine components and nuclear power plants etc. A huge amount of published literature has been observed for evaluation of thermo mechanical behavior of functionally gradient material plate using finite element techniques. It includes both linearity and non-linearity in various areas. A few of published literature highlights the importance of the topic. -leveled equations, graphics, and tables are not prescribed, although the various table text styles are provided. The formatter will need to create these components, incorporating the applicable criteria that follow.

II. DeformAtion theories of plate

To ascertain the distribution of stress and displacement for a plate subjected to a given set of forces requires a consideration of number of basic conditions. It consist physical laws, material properties, and geometry and surfaces forces. A number of approaches have been employed to study the static bending problems of FGM plates. The assessment of thermo-mechanical deformation behavior of functionally graded plate structures considerably depends on the plate model kinematics. **G. N. Praveen and J. N. Reddy (1997) [1]** developed the equations of motion based on the combination of the first order plate theory and the von Karman strains. The von Karman plate theory accounts for moderately large deflections and small strains. The First-order plate theory is based on the displacement field which can be expressed in the form of Equation (2.1):

$$u(x,y,z,t)=u_0\ (x,\ y,\ t)+z\Phi_x\ (x,y,t)$$
$$v(x,\ y,z,t)=v_0\ (x,y,t)+z\Phi y\ (x,y,t)$$
$$w(x,y,z,t)=w_0\ (x,y,t) \qquad (2.1)$$

Where (u_0,v_0, w_0, Φx, Φy) are unknown functions to be determined.

J. N. Reddy (2000) [2] showed the effect of the material distribution on the deflections and stresses using the linear third-order theory and non-linear first-order theory. **J.N. Reddy and Zhen-Qiang Cheng (2001) [7]** used higher order plate theory to analyze the FGM plate deformations. **Bhavani V. Sankar (2002) [9]** developed a simple Euler–Bernoulli-type beam theory based on the assumption that plane sections remain plane and normal to the beam axis. **Senthil S. Vel and R.C. Batra (2003) [10]** adopted the uncoupled quasi-static linear thermoelasticity theory neglecting the change in temperature due to deformations. **L. F. Qian and R. C. Batra (2004) [12]** solved the problem by using a higher order shear and normal deformable plate theory (HONSDPT) since In the HOSNDPT the transverse normal and shear stresses are computed from equation of the plate theory rather than from the balance of linear momentum. The displacement components are assumed to be as in Equation (2.2) in HONSDPT:

$$u(x,y,z,t)=u_0\ (x,y,t)+z\Phi_x\ (x,y,t)-c_1z^3(\phi_x + \partial w_o/\partial x)$$

$$v(x,y,z,t)=v_0\ (x,y,t)+z\Phi y\ (x,y,t)\ c_1z^3(\phi_y + \partial w_o/\partial y)$$

$$w(x,y,z,t)=w_0\ (x,y,t) \qquad (2.2)$$

L.F. Qian, R.C. Batra and L.M. Chen (2004) [12] computed all components of the stress tensor using higher order shear and normal deformable plate theory (HOSNDPT) and a meshless local Petrov–Galerkin (MLPG) method. **K.Y. Dai, G.R. Liu, X. Han, K.M. Lim (2005) [15]** established the weak form formulation based on the first-order shear deformation theory (FSDT) and variational principle accounting for thermo-electro-mechanical coupling. **Shyang-Ho Chi, Yen-Ling Chung (2006) [17]** obtained series solutions based on the classical plate theory and Fourier series expansion. **M. Mahdavian (2009) [23]** the equilibrium and stability equations for FGMs are obtained on the basis of classical plate theory. **Ashraf M. Zenkour and Daoud S. Mashat (2010) [24]** presented sinusoidal shear deformation plate theory (SPT) to obtain the buckling of the plate under different types of thermal loads. **S.S. Alieldin, A.E. Alshorbagy and M. Shaat (2011) [28]** exploited the first-order shear deformation plate (FSDT) model to investigate the mechanical behavior of laminated composite and functional graded plates. **J. Suresh Kumar, B. Sidda Reddy, C. Eswara Reddy, K. Vijaya Kumar Reddy (2011) [27]** investigated the behavior of functionally graded material (FGM) plates with material variation parameter (n), boundary conditions, aspect ratios and side to thickness ratios using higher order displacement model. They derived equations of motion for higher order displacement model using principle of virtual work. They obtained the nonlinear simultaneous equations by Navier's method. **Mohammad Talha and B N Singh (2012) [33]** reported formulations based on higher order shear deformation theory with a considerable amendment in the transverse displacement using finite element method (FEM). **Mostapha Raki, Reza Alipour and Amirabbas Kamanbedast (2012) [34],** derived equilibrium and stability equations of a rectangular plate made of functionally graded material (FGM) under thermal loads based on the higher order shear deformation plate theory.

III. ANALYSIS TECHNIQUES INCLUDING FINITE ELEMENT METHOD

Finite Element Analysis (FEA) is a vital step for the design of structures or components formed by heterogeneous objects. The main objective of the FEA-based design of heterogeneous objects is to simultaneously optimize both geometry and material distribution over the design domain. However, the accuracy of the FEA-based design wholly depends on the quality of the finite element models. Therefore, there exists an increasing need for generating finite element models adaptive to both geometric complexity and material distribution. The FEA-based design of heterogeneous objects is a popular research topic extensively published. **G. N. Praveen and J. N. Reddy (1997) [1]** developed finite element model and employed four-noded rectangular isoparametric element which has five degrees of freedom. They chose a regular mesh of 8 x 8 linear elements for the convergence studies. **Z.-Q. Cheng and R.C. Batra (1999) [4]** obtained deformations due to the temperature varying only in the thickness direction analytically and those due to the mechanical load by the method of asymptotic expansion **J. N. Reddy (2000) [2]** developed linear and nonlinear finite element model and Navier solutions that account for the thermo mechanical coupling and geometric non-linearity. **J.N. Reddy and Zhen-Qiang Cheng (2001) [7]** formulated the asymptotic formulations in the form of the transfer matrix and validate the asymptotic expansion to any desired degrees of numerical accuracy. **Bhavani V. Sankar (2002) [9]** solved thermo elastic equilibrium equations for a functionally graded beam in closed-form to obtain the axial stress distribution. **Senthil S. Vel and R.C. Batra (2003) [10]** used the Laplace transformation technique to reduce equations governing the transient heat conduction to an ordinary differential equation (ODE) in the thickness coordinate.

They solved the elasticity problem for each instantaneous temperature distribution by using displacement functions. **L. F. Qian and R. C. Batra (2004) [12]** used the mesh less local Petrov–Galerkin method (MLPG) method mainly because it does not require any background mesh to evaluate different integrals appearing in the weak formulation of the problem. **K.Y. Dai, G.R. Liu, X. Han, K.M. Lim (2005) [15]** used the element-free Galerkin method is to derive the shape functions using the moving least squares (MLS) method. **A.J.M. Ferreira, R.C. Batra, C.M.C. Roque, L.F. Qian and P.A.L.S. Martins (2005) [14]** utilized finite point multiquadric method of solving an elliptic linear boundary-value problem. **Ki-Hoon Shin (2006) [16]** introduced a method for FEA-based design of heterogeneous objects like FGM and optimized both geometry and material distribution over the design domain. **Hui Wang and Qing-Hua Qin (2008) [21]** developed the method of fundamental solutions coupling with radial basis functions (MFS–RBF) on the basis of analog equation theory, a meshless algorithm to simulate the static thermal stress distribution in two-dimensional (2D) functionally graded materials (FGMs). **Yasser M. Shabana and Naotake Noda (2008) [22]** used the homogenization method (HM) based on the finite element method (FEM) as it has advantages, such as it is appropriate for estimating the effective properties of composites with a given periodic fiber distribution and complicated geometries. **S.S. Alieldin, A.E. Alshorbagy and M. Shaat (2011) [28]** expressed the displacements and normal rotations at any point into a finite element in terms of the nodes of the element. **Kyung-Su Na, and Ji-Hwan Kim (2011) [29]** reported stress analysis of functionally graded composite plates composed of ceramic, functionally graded material and metal layers using finite element method. They selected 18-node solid element for more accurate modeling of material properties in the thickness direction. **H. Nguyen-Xuan, Loc V. Tran, Chien H. Thai and T. Nguyen Thoi (2012) [31]** presented an improved finite element approach in which a node-based strain smoothing is merged into shear-locking-free triangular plate elements. The formulation used only linear approximations. **A. E. Alshorbagy, S. S. Alieldin, M. Shaat, and F. F. Mahmoud (2013) [35]** using FEM developed different numerical simulations been developed to investigate the thermoplastic behavior of a FG plate.

IV. FINITE ELEMENT MODELING SOFTWARE'S PACKAGES

The varying nature of FGMs makes design and analysis more challenging compared to traditional materials. Two commonly used methods for modeling FGMs are: 1) the uncoupled approach, in which spatially varying effective material properties are employed, and 2) the coupled approach in which the materials microstructure is explicitly taken into account. The uncoupled approach is popular due to its simplicity and efficiency. One example of this approach involves analyzing a functionally graded (FG) two-phase plate using classical lamination theory, wherein each through-thickness layer is given a different volume fraction. In this analysis technique properties are employed for each layer, and thus the actual distribution of the phases does not explicitly affect the structural solution for the plate. FEA can be considerably more accurate. The availability of commercial FEA codes (e.g., ABAQUS, ANSYS, and NASTRAN) also makes FEA attractive for design/analysis of FGMs. ANSYS is a general-purpose finite-element modeling package for numerically solving a wide variety of mechanical problems. These problems include static/dynamic and structural analysis (both linear and nonlinear) problems. **Craig S. Collier, Phillip W. Yarrington, Jacob Aboudi, Marek-Jerzy Pindera, Steven M. Arnold, Brett A. Bednarcyk (2002) [8]** developed a new software package called Higher-Order Theory – Structural/Micro Analysis Code (HOT-SMAC) as an effective tool for design and analysis of functionally graded materials. They compared the results with the commercially available FEA codes (e.g., ABAQUS, ANSYS, NASTRAN) and concluded that the accuracy of the codes vary with the approaches. **L.F. Qian, R.C. Batra and L.M. Chen (2004) [12]** developed a computer code to analyze static deformations, and free and forced vibrations of a FG thick plate and validated by comparing computed results with the analytical solutions of the corresponding problems. **Shyang-Ho Chi, Yen-Ling Chung (2006) [17]** in part-II evaluated the numerical solutions directly from theoretical formulations and calculated by finite element method using MARC program. **Ki-Hoon Shin (2006) [16]** analyzed the FE models using ANSYS commercial software (ANSYS Inc.) and performed a linear elastic analysis. **Yasser M. Shabana and Naotake Noda (2008) [22]** performed all simulations by developing finite element FORTRAN programs in order to evaluate all of the thermally independent effective properties for each ceramic volume fraction and the meshes are generated by the commercial finite element package ABAQUS.

V. MATERIAL GRADIENT OF FGM

The functionally graded material (FGM) can be produced by continuously varying the constituents of multi-phase materials in a predetermined profile. The most distinct features of an FGM are the non-uniform microstructures with continuously graded macro properties. An FGM can be defined by the variation in the volume fractions. Most researchers use the power-law function, exponential function, or sigmoid function to describe the volume fractions.

J. N. Reddy (2000) [2] assumed that the material property gradation is through the thickness and the profile for volume fraction variation by power law. In power law material properties are dependent on the volume fraction V_f which obeys Equation (2.3)

$$V_f = (z/h+1/2)^n \quad (2.3)$$

Bhavani V. Sankar (2002) [9] assumed the thermo elastic constants of the beam and the temperature to vary exponentially through the thickness and obtained exact solutions. In the exponential function the material properties of FGMs are described as in Equation (2.4)

$$E(z) = E_2 e^{\frac{1}{h}\ln\left(\frac{E_1}{E_2}\right)(z+h/2)} \quad (2.4)$$

Shyang-Ho Chi, Yen-Ling Chung (2006) [18] assumed Young's moduli vary continuously throughout the thickness direction according to the volume fraction of constituents defined by power-law, sigmoid, or exponential function. The sigmoid functions are defined by Equations (2.5a) and (2.5b):

$$g_1(z) = 1 - \frac{1}{2}\left(\frac{\frac{h}{2} - z}{\frac{h}{2}}\right)^p \quad \text{for } 0 \le z \le h/2 \quad (2.5a)$$

$$g_2(z) = \frac{1}{2}\left(\frac{\frac{h}{2} + z}{\frac{h}{2}}\right)^p \quad \text{for } -h/2 \le z \le 0 \quad (2.5b)$$

VI. Boundary conditionS

The researchers have employed various boundary conditions which include simply supported, clamped, free and combinations of the aforementioned conditions. **G. N. Praveen and J. N. Reddy (1997) [1]** analyzed on a square plate which was simply supported at all edges. **Z.-Q. Cheng and R.C. Batra (1999) [4]** analyzed thermo mechanical deformations of a linear elastic functionally graded elliptic plate with rigidly clamped edges. **J. N. Reddy (2000) [5]** employed a square plate simply supported on all its edges. **Senthil S. Vel and R.C. Batra (2003) [10]** employed rectangular simply supported FG plate with the uniform temperature prescribed at the edges and subjected to either time-dependent temperature or heat flux on the top and the bottom surfaces. **L. F. Qian and R. C. Batra (2004) [12]** used thick rectangular functionally graded plate with edges held at a uniform temperature and either simply supported or clamped. The temperature prescribed on the top surface of the plate with the bottom surface of the plate kept thermally insulated. **A.J.M. Ferreira, R.C. Batra, C.M.C. Roque, L.F. Qian and P.A.L.S. Martins (2005) [14]** worked upon FG plate simply-supported at all its edges. **Shyang-Ho Chi, Yen-Ling Chung (2006) [17]** in part-I investigated an elastic, rectangular and simply supported, functionally graded material (FGM) plate of medium thickness subjected to transverse loading. **Hui Wang and Qing-Hua Qin (2008) [21]** analyzed hollow plate with hole, beam and a long cylinder made up of FGM and calculated static deformations. **M. Mahdavian (2009) [23]** examined the influence of different edge load distributions of a rectangular simply supported plate and analysis in this method has ability to spread solution for any loading and boundary conditions. **J. Suresh Kumar, B. Sidda Reddy, C. Eswara Reddy, K. Vijaya Kumar Reddy (2011) [27]** developed the Navier solutions are for rectangular plates with two sets of simply supported (SS) boundary conditions.

VII. FGM CONSTITUENT MATERIALS

A typical FGM, with a high bending strength is an inhomogeneous composite made from different phases of material constituents (usually ceramic and metal). Various combinations of ceramic and metals have been used and analyzed by the various researchers. The various FGMs' which have been worked upon in previous work is tabulated in Table 2.1.

Table 2.1. Various functionally graded materials

S.NO.	Research by	Year	Metal	Ceramic
1	G.N. Praveen	1997	Aluminum Aluminum	Zirconia Alumina
2	R.C. Batra	1999	Monel (70Ni–30Cu)	Zirconia
3	A.J.M. Ferreira,	2005	Aluminum	Silicon Carbide (SiC)
4	K.Y. Dai,	2005	Aluminum	Zirconia
5	Ki-Hoon Shin	2006	Aluminum	Alumina
6	Shyang-Ho Chi	2006	Used materials of various ratios of Young's modulus	
7	Yasser M. Shabana	2008	Ti-6Al-4V	Zirconia
8	Kyung-Su Na	2011	1. SUS304 2. Ni 3. Ti - 6Al - 4V	1. Si_3N_4 2. Al_2O_3 3. ZrO_2

VIII. THERMOMECHANICAL ANALYSIS

A lot of work has been carried out for the stress and deformation analyses for statically and dynamically loaded FGM structures. The studies considered in this section are concerned with stress, deformation and stability problems of FGM, plates accounting for various effects, such as geometric and physical nonlinearity and transverse shear deformability. **G. N. Praveen and J. N. Reddy (1997) [1]** examined the thermoelastostatic and thermoelastodynamic response of plates subjected to pressure loading and thickness varying temperature fields. They concluded that the basic response of the plates that correspond to properties intermediate to that of the metal and the ceramic, does not necessarily lie in between that of the ceramic and metal. **Z.-Q. Cheng and R.C. Batra (2000) [4]** computed deformations due to thermal and mechanical loads applied to the top and bottom surfaces of the plate separately. **J. N. Reddy (2000) [5]** concluded that all the plates with intermediate properties undergo corresponding intermediate values of center deflection because the metallic plate is the one with the lower stiffness than the ceramic plate. **J.N. Reddy and Zhen-Qiang Cheng (2001) [7]** found that the assumption of a constant through-thickness deflection usually made by two-dimensional plate theories is invalid for the case of the thermal load, but it is a good approximation for the case of the mechanical load. **Bhavani V. Sankar (2002) [9]** concluded that for the case of nearly uniform temperature along the length of the beam, beam theory is adequate in predicting thermal residual stresses. **Craig S. Collier, Phillip W. Yarrington, Jacob Aboudi, Marek-Jerzy Pindera, Steven M. Arnold, Brett A. Bednarcyk (2002) [8]** solved the thermal problem for the temperature distribution in the FGM and then solved the mechanical problem based on this temperature distribution. **Senthil S. Vel and R.C. Batra (2003) [10]** found that for the case of rapid time-dependent surface temperature, the transient longitudinal stress is nearly 8 times its steady state value and for the case of transient prescribed heat flux, the transient stresses are less than their respective steady state values. They also found that with the passage of time, longitudinal stresses at a point change from compressive to tensile and the transverse shear stresses change sign too. **L.F. Qian, R.C. Batra and L.M. Chen (2004) [12]** found that both for static and dynamic loads, the centroidal deflection of a FG plates between those for a pure ceramic and a pure metallic plate. **K.Y. Dai, G.R. Liu, X. Han, K.M. Lim (2005) [15]** analyzed the plate under the mechanical loading as well as thermal gradient and found that the relations between the deflection and the volume fraction exponent are quite different under the two loadings. **A.J.M. Ferreira, R.C. Batra, C.M.C. Roque, L.F. Qian and P.A.L.S. Martins (2005) [14]** found that the CPU time required to solve the problem with the collocation method is considerably less than that needed for the MLPG code mainly because no numerical integration is needed in the collocation scheme. **Victor Birman, Larry W. Byrd (2007) reviewed [20]** the principal developments in functionally graded materials (FGMs) with an emphasis on the recent work published since 2000. Diverse areas relevant to various aspects of theory and applications of FGM were reflected in this paper. They included homogenization of particulate FGM, heat transfer issues, stress, stability and dynamic analyses, testing, manufacturing and design, applications, and fracture. The critical areas where further research is needed for a successful implementation of FGM in design are outlined in the conclusions. **Hui Wang and Qing-Hua Qin (2008) [21]** concluded that the appropriate graded parameter can lead to low stress concentration and little change in the distribution of stress fields. **Yasser M. Shabana and Naotake Noda (2008) [22]** concluded that all of the stiffness matrix coefficients are logically increasing with the ceramic volume fraction and the coefficient of thermal expansion logically decreases with the ceramic volume fraction. **M. Mahdavian (2009) [23]** found that the critical buckling coefficients for FGM plates are considerably higher than isotropic plates. **S.S. Alieldin, A.E. Alshorbagy and M. Shaat (2011) [28]** proposed three approaches to determine the property details of an FG plate equivalent to the original laminated composite plate. The computational effort of the electrostatic analysis of that FG plate is much less than that required for the analysis of the original laminated one. **Kyung-Su Na, and Ji-Hwan Kim (2011) [29]** Numerical results were compared for three types of materials. It was found that the minimum compressive stress ratio are observed for the fully FGM plate with largest volume fraction index. The tensile stress ratio has the smallest value for the fully FGM plate with smallest volume fraction index. The stress ratio distribution shows the smoothest response for the fully FGM plate with smallest volume fraction index. **J. Suresh Kumar, B. Sidda Reddy, C. Eswara Reddy, K. Vijaya Kumar Reddy (2011) [27]** studied the effect of shear deformation and nonlinearity response of functionally graded material plate and concluded that the effect of nonlinearity in functionally graded composite plates is to decrease the central deflections with increase of side to thickness ratio. This effect is found to be more predominant in decreasing the deflections in thin plates for side to thickness ratio of 10. The effect of geometric nonlinearity is to decrease the transverse shear stresses, transverse normal stresses with increase in side to thickness ratio. **Mohammad Talha and B N Singh (2012) [33]** obtained the numerical results for different thickness ratios, aspect ratios, volume fraction index and temperature rise with different loading and boundary conditions. **H. Nguyen-Xuan, LocV.Tran, ChienH.Thai and T.Nguyen-Thoi (2012) [31]** applied the method for static, free vibration and mechanical/thermal buckling problems of functionally graded material (FGM) plates. They analyzed the behavior of FGM plates under mechanical and thermal loads numerically in detail through a list of benchmark problems.**D.K. Jha, Tarun Kant and R.K. Singh (2012) [32]** reported a critical **review** of the reported studies in the area of thermo-elastic and vibration analyses of functionally graded (FG) plates since 1998. They reviewed various areas of work for FGM and their application. They found that for analysis of FGM use of improved 2D theoretical models which are now seem to provide accuracy as good as the 3D models should be pursued in the interest of computational cost and efficient analyses. **A. E. Alshorbagy, S. S. Alieldin, M. Shaat, and F. F. Mahmoud (2013) [35]** concluded that FG plates provide a high ability to withstand thermal stresses, which reflects its ability to operate at elevated temperatures, the FGMs are more sensitive to the variation of the intensity of the heat flow, in or out of the structure, than that may be happened in the case of the isotropic material structures, due to the continuity of the material properties distribution along the thickness of the plates, the strains and stresses are varied smoothly without any sort of singularities. **Mostapha Raki, Reza Alipour and Amirabbas Kamanbedast (2012) [34]** carried out a buckling analysis of a functionally graded plate less than one type of thermal loads and derived the buckling temperatures considering closed form solutions, uniform temperature rise and gradient through the thickness.

IX. NON-DIMENSIONLESS PARAMETERS

Dimensionless numbers occur in several contexts. Dimensionless expressions are the required tool to compare data from different experiments leading to the recommendation that all data should be plotted in dimensionless form. The meaningful dimensionless numbers are ratios of terms in various equations, measuring their relative importance. This can be used to approximate the equations rationally. The dimensionless numbers which have been used in previous work is tabulated in Table 2.2.

Table 2.2. Non-Dimensional parameters

S.NO.	Research by	Year	Non Dimensional Max. Deflection	Non Dimensional Stress
1	G.N. Praveen	1997	w/h	$\sigma h^2/p_o a^2$
2	J. N. Reddy	2000	$w_o E_t h^3/(p_o a^4)$	$\sigma h^2/p_o a^2$
3	Zhen-Qiang Cheng	2001	$w_o/(\alpha Ta)$	$\sigma h/(\alpha TK)$.
4	Senthil S. Vel	2003	$w_o/(\alpha TL)$	$\sigma/(\alpha TE)$
5	L.F. Qian	2004	$100 w_o E_t h^3/ (1-\nu^2)(12 p_o a^4)$	$\sigma h^2/p_o a^2$
6	A.J.M. Ferreira	2005	w_o/h	σ/q
8	S.S. Alieldin,	2011	w_o/h	$\sigma h^4/(p_o a^4)$
9	Tahar Hassaine Daouadji1	2012	$10 w_o E_t h^3/(p_o a^4)$	$\sigma h/(p_o a)$

X. CONCLUSION

(1) Material gradient of FGM have been given by Power law function, sigmoid function and Exponential function. In both power-law and exponential functions, the stress concentrations appear in one of the interfaces in which the material is continuous but rapidly changing. In Sigmoid FGM, which is composed of two power-law functions there is a gradual change in volume fraction as compared to Power law and Exponential function**.** Power law function has been

applied to many of the FGMs' but sigmoid function has not been applied to much of the FGMs.

(2) Power law function and Exponential function have been used in various research works separately but the comparisons of the three FGM laws, Ceramic and metal have not been reported. Volume fraction exponent plays an important role in determination of the FGM properties. A very few values of volume fraction exponent have been considered in previous research works for the analysis.

(3) In most of the research works FGM plate with edges simply supported have been considered. Plate subjected to other boundary conditions e.g. clamped, free and combination is also useful but they have not been reported. FGM plate subjected to thermomechanical loading with various boundary conditions has not been reported.

(4) FGM are very much useful in thermomechanical environment. Though many of the work has been reported where the FGM plates have been subjected to mechanical loading in constant thermal environment but behavior of FGM with variation in temperature with and without mechanical loading have not been reported.

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