Fully Developed Flow of Two Viscous Immiscible Fluids Through a Channel with Heat Transfer

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Abstract

An analytical solution is presented for the problem of fully developed flow and heat transfer of two viscous, incompressible immiscible fluids through a channel taking into an account the viscous dissipation. The channel walls are maintained at two different and constant temperatures. The transport properties of both the fluids are assumed to be constant. Exact solutions are given for the equation governing the flow and heat transfer for both the region with suitable boundary and interface conditions. The variation of velocity and temperature profiles with respect to various parameters such as Prandtl number, Eckert number, ratio of viscosity and thermal conductivity are presented graphically and discussed.

Keywords– flow and heat transfer, channel flow, immiscible fluids.

1. Introduction

Recent years, the requirement of modern technology has stimulated interest in flow and heat transfer studies which involves interaction of several phenomena such as heat exchangers, transport of heat or cooled fluids, chemical processing equipment and micro-electronic cooling. The problem of flow and heat transfer was extensively investigate by many researcher with different hypothesis[1-7].

All the mentioned studies pertain to a single-fluid model. Most of the problems relating to the petroleum industry, plasma physics, magneto-fluid dynamics, *etc*., involve multi-fluid flow situations. In modeling such problems, the presence of a second immiscible fluid phase adds a number of complexities as to the nature of interacting transport phenomena and interface conditions between the phases. In general, multi-phase flows are driven by gravitational and viscous forces. There has been some theoretical and experimental work on stratified laminar flow of two immiscible fluids in a horizontal pipe [8-10] Loharsbi and Sahai [11] studied two-phase MHD flow and heat transfer in a parallel plate channel, with one of the fluids being electrically conducting. Fully developed flow and heat transfer in horizontal channel consisting of an electrically conducting fluid layer sandwiched between two fluids layers is studied analytically by Umavathi et al. [12] and also Umavathi et. al. [13-15] have presented analytical solutions for unsteady/oscillatory two-fluid flow and heat transfer in a horizontal channel. Hishyar and Abdullah [16] presented two immiscible fluids in contact with each other on a solid boundary, have been studied: The asymptotic solution near contact line is fined for different cases. Stamenković, M. Ž., *et. al*.[17] investigates the magnetohydrodynamic flow of two immiscible, electrically conducting fluids between isothermal and insulated moving plates in the presence of an applied electric and inclined magnetic field with the effects of induced magnetic field.

Keeping in view the wide area of practical importance of multi-fluid flows as mentioned, the objective of this study to investigate the viscous incompressible and immiscible fluids flow and heat transfer thorough a channel.

2. Mathematical Formulation

Consider viscous flow of two immiscible fluids in a horizontal channel. The region $0 \le y \le h$ (Region-I) is filled with a viscous fluid having density ρ_1, dynamic viscosity μ_1, specific heat at constant pressure C_{P_1} thermal conductivity k_1 and the region $-h \le y \le 0$ (Region-II) is filled with a different viscous fluid having density ρ_2, dynamic viscosity μ_2, specific heat at constant pressure C_{P_2} and thermal conductivity k_2.

The flow of both regions is assumed to be fully developed and fluid properties are constant and driven by a common pressure gradient ($-\frac{\partial p}{\partial x}$). The two plates are maintained at constant temperatures T_{w_1} at $y = h$ and T_{w_2} at $y = -h$

Under these assumptions and taking $\rho_1 = \rho_2 = \rho_0$ and $C_{p_1} = C_{p_2} = C_p$ the governing equations of motion and energy are given by:

Region-I

$$\mu_1 \frac{\partial^2 u_1}{\partial y^2} - \frac{\partial p}{\partial x} = 0 \qquad (2.1)$$

$$k_1 \frac{\partial^2 T_1}{\partial y^2} + \mu_1 \left(\frac{\partial u_1}{\partial y} \right)^2 = 0 \qquad (2.2)$$

Region-II

$$\mu_2 \frac{\partial^2 u_2}{\partial y^2} - \frac{\partial p}{\partial x} = 0 \qquad (2.3)$$

$$k_2 \frac{\partial^2 T_2}{\partial y^2} + \mu_2 \left(\frac{\partial u_2}{\partial y} \right)^2 = 0 \qquad (2.4)$$

where u is the x-component of fluid velocity and T is the fluid temperature. The subscripts 1 and 2 correspond to region-I and region-II, respectively. The boundary conditions on velocity are the no-slip boundary conditions which required that the x-component of velocity must vanish at the wall. The boundary conditions on temperature are isothermal conditions. We also assume the continuity of velocity, shear stress, temperature and heat flux at the interface between the two fluid layers at $y=0$.

The hydrodynamic boundary and interface conditions for the two fluids can then be written as

$$u_1(h) = 0$$
$$u_2(-h) = 0$$
$$u_1(0) = u_2(0) \qquad (2.5)$$
$$\mu_1 \frac{\partial u_1}{\partial y} = \mu_2 \frac{\partial u_2}{\partial y} \quad at\ y = 0$$

The thermal boundary and interface conditions on temperature for both fluids are given by

$$T_1(h) = T_{w1}$$
$$T_2(-h) = T_{w2}$$
$$T_1(0) = T_2(0) \qquad (2.6)$$
$$k_1 \frac{\partial T_1}{\partial y} = k_2 \frac{\partial T_2}{\partial y} \quad at\ y = 0$$

To make equations dimensionless, we use the following quantities

$$u_i = U_0 u_i^* \quad y = h y^* \quad \theta_i = \frac{T_{wi} - T_{w2}}{T_{w1} - T_{w2}}$$

$$P = \frac{h^2}{\mu_1 U_0} \left(-\frac{\partial P}{\partial x} \right) \qquad \Pr = \frac{\mu_1 C_p}{k_1}$$

$$Ec = \frac{U_0^2}{C_p (T_{w1} - T_{w2})}$$

(2.7)

Region-I

$$\frac{d^2 u_1}{dy^2} + P = 0 \qquad (2.8)$$

$$\frac{d^2 \theta_1}{dy^2} + Ec \Pr \left(\frac{du_1}{dy} \right)^2 = 0 \qquad (2.9)$$

Region-II

$$\frac{d^2 u_2}{dy^2} + \frac{P}{\alpha} = 0 \qquad (2.10)$$

$$K_1 \frac{\partial^2 T_1}{\partial y^2} + \mu_1 \left(\frac{\partial u_1}{\partial y} \right)^2 = 0 \qquad (2)$$

$$\frac{d^2 \theta_2}{dy^2} + \frac{Ec \Pr \alpha}{\beta} \left(\frac{du_2}{dy} \right)^2 = 0 \qquad (2.11)$$

where $\alpha = \frac{\mu_2}{\mu_1}$ is the ratio of viscosities and $\beta = \frac{k_2}{k_1}$ is the ratio of thermal conductivities.

The hydrodynamic and thermal boundary and interface conditions for both fluids in non-dimensional form become

$$u_1(h) = 0$$
$$u_2(-h) = 0$$
$$u_1(0) = u_2(0) \qquad (2.12)$$
$$\frac{\partial u_1}{\partial y} = \alpha \frac{\partial u_2}{\partial y} \quad at\ y = 0$$
$$\theta_1(h) = 1$$
$$\theta_2(-h) = 0$$
$$\theta_1(0) = \theta_2(0) \qquad (2.13)$$
$$\frac{\partial \theta_1}{\partial y} = \beta \frac{\partial \theta_2}{\partial y} \quad at\ y = 0$$

3. Solution

Equations (2.8) to (2.11) are solved exactly for $u_1, u_2, \theta_1\ and\ \theta_2$ using the conditions (2.12) and (2.13). The solutions of velocity and temperature for both the regions are

$$u_1 = C_1 + C_2 y - \frac{P}{2} y^2$$
$$u_2 = C_3 + C_4 y - \frac{P}{2\alpha} y^2$$
$$\theta_1 = D_1 + D_2 y + l_1 y^2 + l_2 y^3 + l_3 y^4$$
$$\theta_2 = D_3 + D_4 y + l_4 y^2 + l_5 y^3 + l_6 y^4$$

The constants appearing in the above solutions are defined in the Appendix section.

4. Results and Discussions

In this section representative flow and heat transfer of two immiscible fluids through a horizontal channel are presented and discussed for various parametric conditions. Exact solutions are obtained for the governing equations. The solutions are depicted graphically in Figs. 1 to 5 for different values of viscosity ratio on the flow and thermal conductivity ratio, Prandtl number and Eckert number on temperature field. The parameters are fixed as 1 except the varying one, Pr=0.7, Ec=0.2.

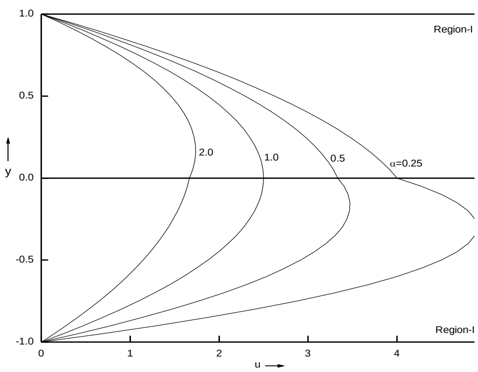

Fig.1 Velocity profile for different viscosity ratio α

Figure 1 show that velocity profiles are suppressed for large values of viscosity ratios. The flow profile is large in region-I compare to region-II for values of viscosity ratios less than one. The flow profile is large in region-II compare to region-I for values of viscosity ratio greater than one. The flow profiles almost remain the same in both the regions for equal values of viscosity ratios and similar effects on the temperature field as shown in Fig 2.

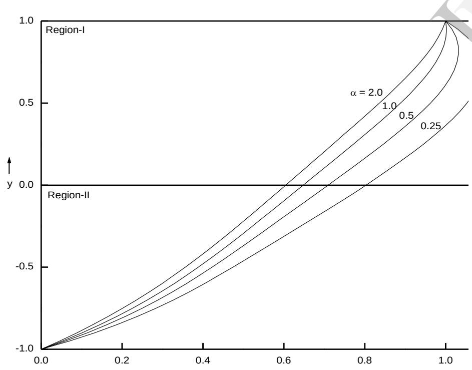

Fig. 2 Temperature profile for different viscosity ratio α

Fig. 3 depicts the effect of thermal conductivity ratio, as the ratio increases the magnitude of suppression is large in region-II compared to region-I. This is obvious because the upper plate is maintained at a low temperature compared to region-I.

Figures 4 and 5 display the effect of Prandtl number and Eckert number respectively on temperature filed. It is seen that temperature is increases with increase in Prandtl number as well as Eckert number. Since the values of Prandtl number are very small for liquid and metals and it is very high for highly viscous fluid. Thus one can conclude that the flow can be controlled by considering different fluids having different properties.

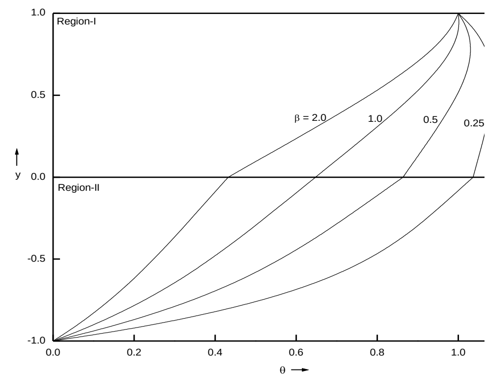

Fig.3 Temperature profile for different conductivity ratio β

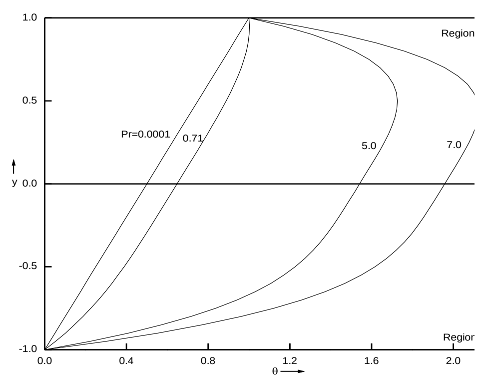

Fig. 4 Temperature profile for different Prandtl number Pr

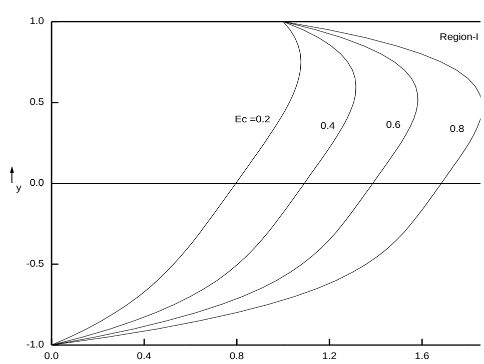

Fig.5 Temperature porfile for different values of Eckert number Ec

Acknoelegment

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Nomenclature:

C_P	specific heat at constant pressure
k	thermal conductivity
P	pressure
Pr	Prandtl number
T	temperature
T_w	wall temperature
t	time
U_0	average velocity

Greek letters

ρ	fluid density
μ	viscosity of fluid
θ	non-dimensional temperature

Subscripts

1,2 quantities for region-I and region-II respectively.

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Appendix

$$C_1 = C_3; C_2 = \frac{ph}{2} - \frac{C_1}{h}; C_3 = hC_4 + \frac{ph^2}{2\alpha}$$

$$C_4 = \frac{ph(\alpha - 1)}{2\alpha(\alpha + 1)}; D_1 = -hD_2 - k_7; D_2 = \frac{\beta(l_8 - l_7)}{h(1+\beta)}$$

$$D_3 = D_1; D_4 = \frac{D_3 + l_8}{h}; l_1 = \frac{-Ec \Pr C_2^2}{2};$$

$$l_2 = \frac{Ec \Pr PC^2}{3}; l_3 = \frac{-Ec \Pr P^2}{12}; l_4 = \frac{-Ec \Pr \alpha C_4^2}{2\beta}$$

$$l_5 = \frac{Ec \Pr PC_4}{3\beta}; l_6 = \frac{-Ec \Pr P^2}{12\alpha\beta}$$

$$l_7 = l_1 h^2 + l_2 h^3 + l_3 h^4 - 1$$

$$l_8 = l_4 h^2 - l_5 h^3 + l_6 h^4$$