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FRP Reinforcement, According To European Normatives, of Flexural R.C Elements under Shear Force

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Abstract

In this article is presented the reinforcement with fiber polymer of flexural reinforced concrete elements under the action of shear force. The reinforcement from shear force action is needed in those cases when the shear force from external actions exceeds the internal ultimate shear strength of the reinforced concrete members. Ultimate shear strength of r.c elements is a product of the contribution from both concrete and transversal reinforcement steel of the elements. Along the article are presented some reinforcement details of a member both in length and in member's cross section. Also, some design formulas of a fiber polymer reinforced member's shear strength are presented. Closing this article, to have a better idea how this reinforced member react from shear force action, a solved example of a beam under flexural condition is given.

Keywords-*reinforcement, FRP, shear, Eurocode 2*

1. Introduction

Design of a flexural element, for shear force, is an important aspect of the design procedure. For a certain case of loading, the shear strength of the element is product of concrete and transversal steel reinforcement strength. When the element is reinforced with FRP, which is the focus of this article, the shear strength is not only the product of concrete and steel reinforcement, but an important role plays also the FRP reinforcement. A great significance, to FRP shear strength, has the positioning of FRP in the element's length and in the cross section. Following the article, through the figures, some ways of positioning of FRP are presented. Each case of positioning is followed by the characteristic of implementation and with the difficulties that each case present.

2. FRP shear-strengthened of an element

Shear strengthening of a flexural RC element consist on setting FRP strips (action only in one direction) or a FRP mesh (action in both directions). The ideal case will be if the FRP strip follow the principal tensile stress, but is more practical the positioning of FRP strips perpendicular with the element's axis. In this case the strips must be positioned close or distant to each other. Some ways of positioning are given in Figure 1a,b,c and 2.

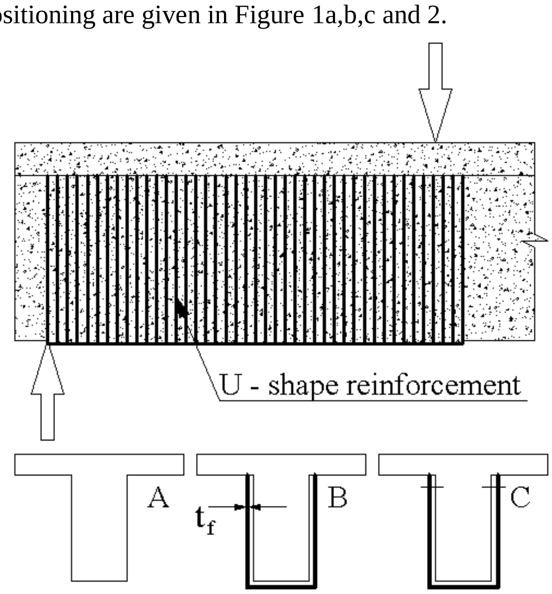

Fig. 1a.

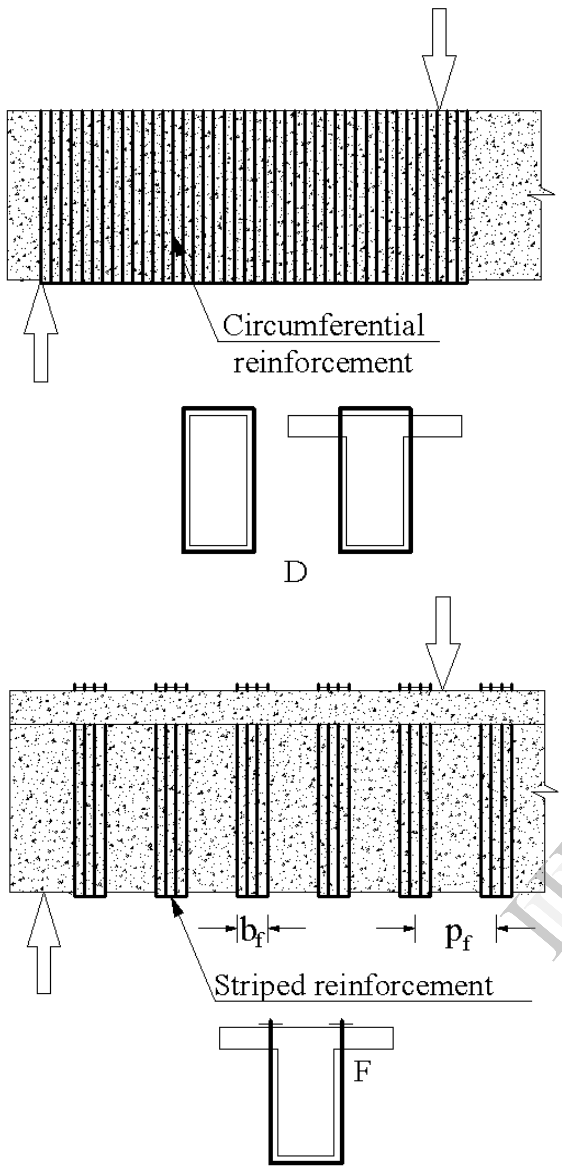

Fig. 1b.

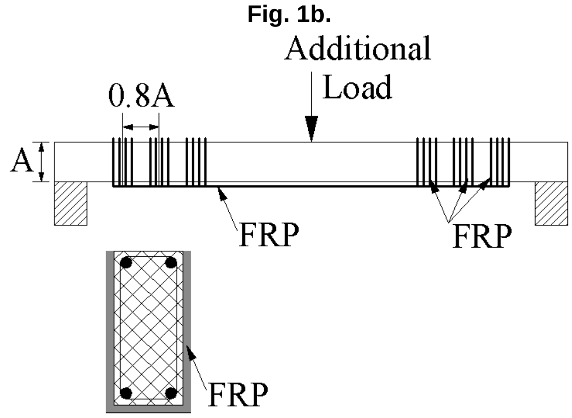

Fig. 2.

The role of the adhesive material in the shear strength is negligible so, only the FRP fibers contribute in the shear strength of the reinforcement. As is seen in Figure 3, the lateral FRP reinforcement is not preferred.

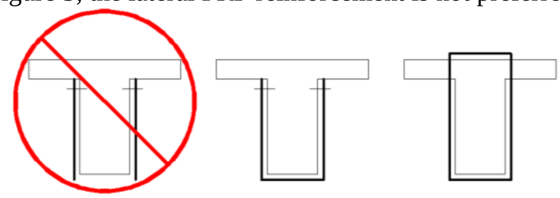

Fig. 3.

Always try to realize U-shape or circumferential reinforcement. Figure 4 shows the positioning of FRP in the element length.

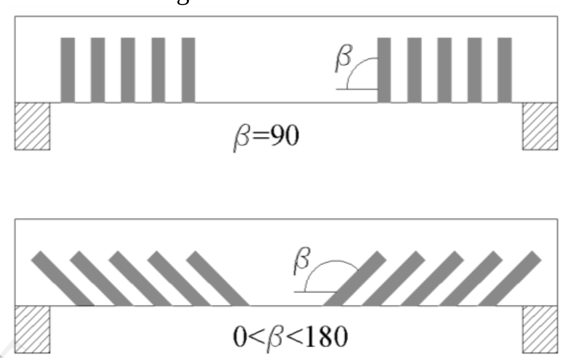

Fig. 4.

If a strip FRP reinforcement is used, the distance between two consecutive strips must not be great than 0.8A (see Figure 2), where A is the height of the element cross section, in this case every diagonal crack will be traversed but at least one strip. During site implementation, the FRP could be damaged and in this case is advisable to reduce the FRP modulus of elasticity by divide it with 1.2 for one way FRP, and by 1.5 for two ways FRP. Also, deformation values during design procedure must be 50% of failure deformation values. This depends from the reinforcement type (cutting, bending, compression or torsion)

3. Shear strength of a FRP reinforced element

Ultimate shear strength of a FRP reinforced element is given by equation (1):

$$V_{Rd} = \min\{V_{Rds} + V_{Rdf}, V_{Rd\max}\} \qquad (1)$$

V_{Rds} - Design value of shear force sustained by the RC element reinforced with transversal reinforcement, given by equation (2):

$$V_{Rds} = A_{sw} \cdot z \cdot f_{ywd} \cdot (\mathrm{ctg}\,\theta + \mathrm{ctg}\,\alpha) \cdot \frac{\sin\alpha}{s} \qquad (2)$$

$V_{Rd\max}$ - design value of shear force which can be sustained by the element, limited by crushing of compression struts, given by equation (3):

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$$V_{Rd\max} = a_{cw} \cdot b \cdot z \cdot \nu_1 \cdot f_{cd} \cdot \frac{ctg\theta + ctg\alpha}{1 + ctg^2\theta} \quad (3)$$

Equation (1), (2) and (3) are according to Eurocode 2. In case of a RC element with rectangular or T cross section, reinforced with circumferential FRP shape, $V_{Rd,f}$ is given by equation (4):

$$V_{Rd,f} = \frac{1}{\gamma_{Rd}} \cdot 0.9 \cdot d \cdot f_{fed} \cdot 2 \cdot t_f \cdot (ctg\theta + ctg\beta) \cdot \frac{b_f}{p_f} \quad (4)$$

d – Cross section effective height

γ_{Rd} - Partial safety factor

f_{fed} - FRP effective design strength

t_f - FRP thick

b_f, p_f -strips width and strips distance between each other measured perpendicular with fiber direction. When the fiber are positioned close to each other, without distance between, or the FRP is in a two directional mesh, then $b_f/p_f = 1$. For more see Figure 5.

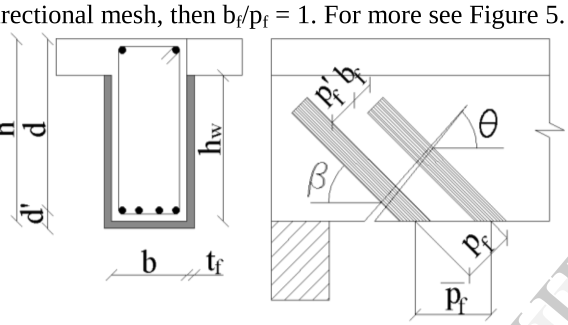

Fig. 5.

According to Figure 5, in equation (4), p_f can be replaced by $\overline{p_f} \cdot \sin\beta$. In this case $\overline{p_f}$ is the distance between two strips measured parallel to element axis. Distances between strips must be limited by:

$$50mm \le b_f \le 250mm \quad (5)$$

$$b_f \le p_f \le \min\{0.5d,\ 3b_f,\ b_f + 200mm\} \quad (6)$$

If min {0.5d, $3b_f$, b_f + 200mm} < b_f then a different type of FRP, with other geometry or physical parameters, should be used. In case of circular cross section RC elements, $V_{Rd,f}$ is given by equation (7):

$$V_{rd,f} = \frac{1}{\gamma_{Rd}} \cdot D \cdot f_{ed} \frac{\pi}{2} \cdot t_f \cdot ctg\theta \quad (7)$$

In this case, FRP is positioned perpendicular to element axis (β=90°) and all the element is cover by FRP positioned close to each other.

D - Cross section diameter.

Getting started from equation (1) we can determine the value of maximum shear force supported by FRP:

$$V_{Rdf,max} = V_{Rdmax} - V_{Rds} \quad (8)$$

FRP effective strength on a rectangular cross section reinforced with U-shape FRP is given by equation (9):

$$f_{ed} = f_{fdd} \cdot \left[1 - \frac{1}{3} \cdot \frac{l_{ed} \cdot \sin\beta}{\min\{0.9d, h_w\}}\right] \quad (9)$$

f_{fdd} -FRP maximal strength without disconnecting from the element, is given by equation (10):

$$f_{fdd} = \frac{1}{\gamma_{f,d}} \sqrt{\frac{2 \cdot E_f \cdot \Gamma_{Fd}}{t_f}} \quad (10)$$

γ_{fd} - Coefficient accepted 1.2 - 1.5

l_{ed} - Optimal design anchorage length given by:

$$l_{ed} = \min\left\{\frac{1}{\gamma_{Rd} \cdot f_{bd}} \sqrt{\frac{\pi^2 \cdot E_f \cdot t_f \cdot \Gamma_{Fd}}{2}}, 200mm\right\} \quad (11)$$

E_f - FRP modulus of elasticity

t_f - FRP thick

Γ_{Fd} - Specific energy of disconnection

$f_{bd} = 2\Gamma_{Fd}/s_u$ - Disconnection tangential stress between FRP and the RC element.

s_u =0.25 mm

$\gamma_{Rd} = 1.25$ - Correctional factor.

Specific energy of disconnection Γ_{Fd} is given by equation (11):

$$\Gamma_{Fd} = \frac{k_b \cdot k_G}{Fs} \sqrt{f_{cm} \cdot f_{ctm}} \quad (12)$$

f_{cm} - Average concrete compressive strength

f_{ctm} - Average concrete tensile strength

F_s - Safety factor

k_b - Correctional factor given by equation (13):

$$k_b = \sqrt{\frac{2 - b_f/b}{1 + b_f/b}} \ge 1 \quad (13)$$

To clarify b_f and b see Figure 6.

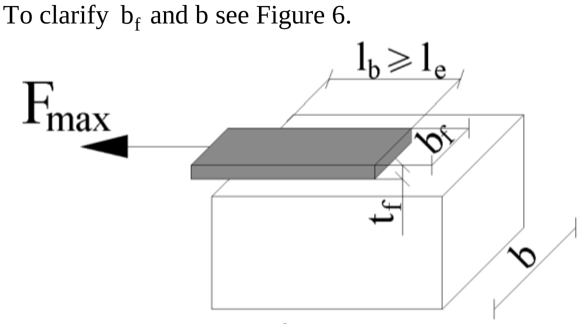

Fig. 6.

Equation (12) is valid for $b_f/b \ge 0.25$. If $b_f/b < 0.25$ than k_b=1.18 corresponding to b_f/b=0.25.

k_G - Correctional coefficient determined experimentally. For precast FRP k_G=0.023mm and for FRP prepared in site k_G=0.037mm. If an element is strengthened with the help of many FRP strips, each strips with a width b_f, k_b can be calculated with

equation (12) accepting b values as the distance between to nearby strips axes.
β- Angle between FRP strips and element's axes. See Figure 5.
h_w - See Figure 6.
For a rectangular section strengthened with circumferential FRP:

$$f_{fed} = f_{fdd} \cdot \left[1 - \frac{1}{6} \cdot \frac{l_e \cdot \sin\beta}{\min\{0.9d, h_w\}}\right] + \frac{1}{2}\left(\Phi_R \cdot f_{fd} - f_{fdd}\right) \cdot \left[1 - \frac{l_e \cdot \sin\beta}{\min\{0.9d, h_w\}}\right] \quad (14)$$

$$\Phi_R = 0.2 + 1.6 r_c/b \quad (15)$$

$$0 \le r_c/b \le 0.5 \quad (16)$$

r_c - Rounding radius of FRP at elements edges.
b – Cross section width
On equation (14), the second part is taken in consideration only if it's positive. In case of circular cross section with diameter D and FRP covers all the element and β=90:

$$f_{fed} = E_f \cdot \varepsilon_{f\,max} \quad (17)$$

E_f - FRP modulus of elasticity
$\varepsilon_{f\,max}$ - FRP relative deformation accepted $5 \cdot 10^{-3}$.

4. Solved Case

Let study a RC element (beam) under flexural conditions. Shear force acting on the element V_{Ed} = 12000daN; rectangular cross section with width b=30cm and height h=50cm; C25/30 concrete with f_{cd} = 141.7daN/cm^{2,} f_{ctd} =12daN/cm^2; S-500 steel with f_{yd} = 4348daN/cm2; FRP modulus of elasticity E_f = 2350000daN/cm2; A_s = 4.62cm^2; 8mm stirrups every 10cm. Calculate FRP area needed to reinforce the element if the acting shear force is doubled. A'_s on the compression zone is accepted equal to 0 and there is no inclined steel reinforcement on the element. In the beginning lets calculate $V_{Rd,c}$ which is the ultimate design shear force sustained by the element without transversal reinforcement:

$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100\rho_1 f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}\right] \cdot b \cdot d \quad (18)$$

f_{ck} – cylindrical strength of concrete f_{ck}=25N/mm^2 and $C_{Rd,c} = 0.18/\gamma_c = 0.18/1.5 = 0.12$

$$k = 1 + \sqrt{\frac{200}{d}} \le 2 \quad (19)$$

d – Effective depth of the cross section in mm

$$k = 1 + \sqrt{\frac{200}{465}} = 1.655 < 2 \quad (20)$$

b – Cross section width

$$\rho_1 = \frac{A_{sl}}{b \cdot d} = \frac{4.62}{30 \cdot 46.5} = 0.00331 \quad (21)$$

A_{sl} – Longitudinal reinforcement area A_{sl}=4.62 cm^2

$$\sigma_{cp} = \frac{N_{Ed}}{A_c} \quad (22)$$

N_{Ed} – Axial force acting on the element, compression or tensile, in this case N_{Ed}=0 so σ_{cp} = 0 and k_1 = 0.15, so:

$$V_{Rd,c} = [0.12 \cdot 1.655 \cdot (100 \cdot 0.00331 \cdot 25)^{1/3} + 0.15 \cdot 0] \cdot 300 \cdot 465 = 56000N = 5600daN$$

Because V_{Ed} = 12000daN > $V_{Rd,c}$ = 5600daN than the steel transversal reinforcement is needed so 8mm diameter stirrups every 10cm are used. Using equation (2) lets calculate V_{Rds}:

$$V_{Rds} = A_{sw} \cdot z \cdot f_{ywd} \cdot (ctg\theta + ctg\alpha) \cdot \frac{\sin\alpha}{s} \quad (2)$$

A_{sw} – lateral reinforcement area A_{sw}=2·0.5=1cm^2
s – Stirrups distance, s=10cm
z=0.9d=0.9·46.5=41.85cm
f_{ywd} – Design yield strength of shear reinforcement f_{ywd}= f_{yd}=4348 daN/cm^2. Is recommended that 1 ≤ cotθ ≤ 2.5, cotθ = 1 is accepted. For α = 90°, cotgα = 0 and sinα = 1, so:

$$V_{Rd,s} = 1 \cdot 41.85 \cdot 4348 \cdot (1+0) \cdot 1/10 = 18196daN$$

$$V_{Ed} = 12000daN < V_{Rd,s} = 18196daN$$

Also, we need to control if V_{Ed} is less than V_{Rdmax} the design value of shear force which can be sustained by the element, limited by crushing of compression struts, given by:

$$V_{Rd\,max} = a_{cw} \cdot b \cdot z \cdot \nu_1 \cdot f_{cd} \cdot \frac{ctg\theta + ctg\alpha}{1 + ctg^2\theta} \quad (3)$$

When σ_{cp} = 0 the recommended value for α_{cw} is α_{cw}=1. ν_1 = 0.6·(1-f_{ck}/250) = 0.6·(1-25/250) = 0.54. So ν_1 = 0.54 for f_{ck} = 25Mpa. Also cotgθ = tanθ = 1 is accepted. For α = 90°, ctgα = 0. So:

$$V_{Rd,max} = 1 \cdot 30 \cdot 41.85 \cdot 0.54 \cdot 141.7 \cdot (1+0)/(1+1) = 48033daN$$

$$V_{Ed} = 12000daN < V_{Rd,max} = 48033daN$$

The element is safe against shear force. Let suppose that the shear force value is double, so V_{Ed} = 24000daN. In this case:

$$V_{Ed} = 24000daN < V_{Rd,s} = 18196daN$$

So the element must be reinforced with FRP because there is a risk of failure. Let suppose that a striped reinforcement, like in Figure (4), is used. To calculate the shear force sustained by FRP equation (4) is used:

$$V_{Rd,f} = \frac{1}{\gamma_{Rd}} \cdot 0.9 \cdot d \cdot f_{fed} \cdot 2 \cdot t_f \cdot (ctg\theta + ctg\beta) \cdot \frac{b_f}{p_f} \quad (4)$$

γ_{Rd} – partial safety factor accepted 1.2
d = 46.5cm
t_f = 1.2mm = 0.12cm

$ctg\theta = 1$
$\beta = 90°$, $ctg\beta = 0$.
$b_f = 5cm$
$p_f = 10cm$
To calculate f_{fed}, equations (9) and (10) are used:

$$f_{ed} = f_{fdd} \cdot \left[1 - \frac{1}{3} \cdot \frac{l_{ed} \cdot \sin\beta}{\min\{0.9d, h_w\}}\right] \quad (9)$$

$$f_{fdd} = \frac{1}{\gamma_{f,d}} \sqrt{\frac{2 \cdot E_f \cdot \Gamma_{Fd}}{t_f}} \quad (10)$$

$\gamma_{f,d}$ – coefficient, with value 1.2 – 1.5. Is accepted 1.35.

$$\Gamma_{Fd} = \frac{k_b \cdot k_G}{Fs} \sqrt{f_{cm} \cdot f_{ctm}} \quad (12)$$

f_{cm} - Average concrete compressive strength $f_{cm} = 33 N/mm^2$

f_{ctm} - Average concrete tensile strength $f_{ctm} = 2.6$ N/mm^2

F_s - Safety factor $F_s = 1.5$

k_b - Correctional factor given by equation (13):

$$k_b = \sqrt{\frac{2 - b_f / b}{1 + b_f / b}} \geq 1 \quad (13)$$

Because $b_f/b = 5/30 = 0.166 < 0.25$ than $b_f/b = 0.25$ is accepted to calculate k_b, so:

$$k_b = \sqrt{\frac{2 - 0.25}{1 + 0.25}} = 1.183 \geq 1$$

k_G - Correctional coefficient determined experimentally. For precast FRP k_G=0.023mm and for FRP prepared in site k_G=0.037mm.

$$\Gamma_{Fd} = \frac{1.183 \cdot 0.037}{1.5} \sqrt{33 \cdot 2.6} = 0.270 \frac{N}{mm}$$

$$f_{fdd} = \frac{1}{1.35} \sqrt{\frac{2 \cdot 235000 \cdot 0.270}{1.2}} = 240.8 \frac{N}{mm^2}$$

$$l_{ed} = \min\left\{\frac{1}{\gamma_{Rd} \cdot f_{bd}} \sqrt{\frac{\pi^2 \cdot E_f \cdot t_f \cdot \Gamma_{Fd}}{2}}, 200mm\right\}$$

$\gamma_{Rd} = 1.25$ - Correctional factor.

$f_{bd} = 2\Gamma_{Fd} / s_u$ - Disconnection tangential stress between FRP and the RC element.

$s_u = 0.25$ mm.

$$f_{bd} = 2 \cdot 0.270/0.25 = 2.16 N/mm^2$$

$$l_{ed} = \min\left\{\frac{1}{1.25 \cdot 2.16} \sqrt{\frac{3.141^2 \cdot 235000 \cdot 1.2 \cdot 0.270}{2}}, 200mm\right\}$$

$$= \min\{227mm, 200mm\}$$

l_{ed}=200 mm is accepted.

$$f_{Ed} = 240.8 \cdot \left[1 - \frac{1}{3} \cdot \frac{200 \cdot \sin 90}{0.9 \cdot 465}\right] = 202.4 \frac{N}{mm^2}$$

$$V_{Rd,f} = \frac{1}{1.2} \cdot 0.9 \cdot 465 \cdot 202.4 \cdot 2 \cdot 1.2 \cdot (1+0) \cdot \frac{50}{100} =$$

$$= 84704N = 8470.4daN$$

Finally:

$$V_{Ed} = 24000daN < V_{Rd,s} + V_{Rd,f} = 18196 + 8470 = 26666daN$$

$$V_{Ed} = 24000daN < V_{Rd,max} = 48033daN$$

$$V_{Rdf,max} = V_{Rdmax} - V_{Rds} = 48033 - 18196 = 29837daN > V_{Rd,f} = 8470daN$$

All conditions are fulfilled and the element is safe against shear force.

5. Conclusion

The use of FRP is a safe method for reinforcing flexural elements against the shear force. Till now, the basis of FRP positioning in element length and in its cross section together with some design formulas given above, are well known. As was seen by the solved case, FRP reinforcement plays an important role on reinforcing the element against shear force, increasing significantly the element strength. With all that, FRP is a relatively new material in material construction field and for these reason there is a great need for studies and experiments to fully understand this material in order that the structural engineer make use of them, not only in separate members but in all structure.

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