Frequency and Power Allocation for Low Complexity Energy Efficient OFDMA Systems with Proportional Rate Constraints

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Abstract —In this paper, we develop a low complexity energy efficient OFDMA system. Energy efficiency is one of the important facts in wireless communication. The goal of this paper is to maximize the energy efficiency of OFDMA systems. To obtain the solution for subchannel and power allocation a single non-linear equation has used. This maximizes the energy efficiency of the OFDMA systems from base station under proportional rate constraints and total power constraints. The convergence of the proposed algorithm is guaranteed, while in previous work is not guaranteed. The dynamic channel assignment (DCA) is used to improve energy efficiency and minimize the complexity of the algorithm. Simulation results show that the low complexity energy efficient OFDMA systems under proportional rate constraints.

Keywords— OFDMA, Energy efficiency, DCA

I. INTRODUCTION

Energy efficiency is becoming increasingly important for mobile communications. The energy efficiency of the transmissions from base station has the increasing demand of the data rates [1]-[3]. Our aim is to maximize the bits/Joule/Hz energy efficiency (EE) using the power and frequency allocation. But it is tough because the EE is not concave in powers and channel allocation in OFDMA is very demanding. Charnes-Cooper transformation, Parameterized convex programming, and bi-level optimization [3]-[6] have been suggested to compromise with power allocation. And the channel allocation strategy should propose the analytical approaches. Though, to solve the convergence problem [7] none of the algorithm is present. The energy efficiency problems have effective algorithms in medium access (MAC) layer [8], but the same algorithm is not used to maximize EE in the physical layer.

The proposed algorithm is used to solve the problem of convergence. Also it is used to obtain the solution for subchannel and power allocation. This is possible due to single non-linear equation. The proportional rate constraints are used with power allocation that maximizes the energy efficiency with low data rates [3]-[8] compared to minimum rate constraints. The dynamic channel assignment (DCA) is also used to maximize the EE of the OFDMA systems and because of this the complexity of the system is reduced.

In digital communication QPSK is a higher order modulation scheme. Because of its advanced noise immunity, bandwidth efficiency and simpler circuitry it has generally used in OFDM, Bluetooth etc. To generate orthogonal parallel channels (called subcarriers) in frequency the concept of OFDMA is used. Energy-efficient communication has important advantage of minimizing interference to other co-channel users as well as dropping environmental impacts, e.g. heat dissipation and electronic pollution. To give the best performance, subcarriers can be adaptively allotted to each user, meaning the minimum problems with fading and interference based on the location and propagation characteristics of each user [3]. In this paper we are considering the system model & detection schemes as proposed in the previous woks [1, 2].

Section II defines the optimization problem. The analysis and the solution are presented in Section III, the numerical results in Section IV, and the conclusion in Section V.

II. SYSTEM MODEL AND THE OPTIMIZATION PROBLEM

Suppose orthogonal sub channels K and N users for downlink of a single cell. For one user each sub channel is allowed completely. If the transmission power p_k, the noise spectral density, the channel gain on the kth sub channel is a_k and $h_k = a_k / \sigma$, then the system EE of the transmissions over all K sub channels in bits/Joule/Hz can be written as [1]:

$$EE = \frac{\sum_{k=1}^{K} log_2 (1+h_k p_k)}{p_c + \eta \sum_{k=1}^{K} p_k} \qquad (1)$$

Where p_c is the circuit power and η is the reciprocal of the efficiency [7] of the downlink transmitter.

Consider still being resolved, to assign the sub channels among the users the subchannel assignment protocol is used. Suppose a total of K_1 subchannels – sub channel 1 through sub channel k_1 – are assigned to User-1. A total of K_2 sub channels – sub channel k_1 +1 through sub channel k_2 – are assigned to User-2 and so on. A total of K_n sub channels – sub channel $k_{n-1} + 1$ through sub channel k_n – are assigned to User-n. Then the rate r_n of User-n can be written as [1]:

$$r_n = \sum_{k=k_{n-1}+1}^{k_n} log_2(1 + h_k p_k) \qquad (2)$$

where k_0=0. Consider the rate demand of User-n is α_n times that of User-1. In other words, the proportional rate demands are given by

$\alpha_{n+1} r_1 - r_{n+1} = 0$ for $n = 1, 2, \ldots, N\text{-}1$.   (3)

The target is to maximize the EE in (1) subject to (3) and a total power constraint P_T using a two step optimization procedure – first proposing a sub channel assignment protocol and then solving the following power allocation problem:

$$\max_{p_1, p_2, \ldots, p_K} EE$$

Subject to

C1: $\alpha_{n+1} r_1 - r_{n+1} = 0$ for $n = 1, 2, \ldots, N - 1$.   (4)

C2: $P_T - \sum_{k=1}^{K} p_k \geq 0$.

III. ANALYSIS AND SOLUTION

An optimal channel assignment protocol has computational complexity would prevent it from being useful in practice. As a result of this, a low complexity channel assignment protocol present firstly and then we resolve the power allocation problem and propose a power allocation procedure.

A. Channel Assignment

In channel assignment strategy the EE of the single user system is directly proportional to its channel gain. From this we should assign each subchannel to the user with largest gain on that subchannel. There are two types of channel assignment strategy such as-

1) Fixed Channel Assignment
2) Dynamic Channel Assignment.

Here we used Dynamic Channel Assignment (DCA) strategy. In DCA voice channels are not allocated to cells permanently. The base station requests a channel from MSC on each call request. The DCA reduces call blocking that is to say, it increases the trunking capacity and it also increases the voice quality.

The Channel Assignment Protocol
1. Re-label the users in the descending order of their proportional rate demands.
2. **while** (there are subchannels left to assign)
3.   **for** each user
4.   assign the subchannel in which it has the largest gain.
5.   **end for**
6.   **for** each user
7.   Assign a subchannel if that user happens to have the largest gain on that sub channel.
8.   **end for**
9. **end while**

B. Power Allocation

The channel assignment protocol is used to assign the subchannels, in (4) we set out to solve the optimization problem. The objective function in (4) is not concave in the powers [1]. The transformation suggested by Charnes and Cooper [9] that can be used to transform a concave fractional

program into a concave program, though for the numerator is positive and concave, and the denominator is positive and affine.

Charnes-Cooper Transformation (CCT): The concave fractional program max $\{ N(x) / D(x) \mid M(x) \geq 0, L(x) = 0 \}$ reduces to the concave program max $\{ tN(y/t) \mid tM\left(\frac{y}{t}\right) \geq 0, L(x) = 0, tD\left(\frac{y}{t}\right) = 1, t > 0 \}$ under the transformation $t = 1/D(x)$, $x = y/t$.

The substitution $p_k = y_k/t$ for all k, $t = 1 / (p_c + \eta \sum_k p_k)$ reduces (4) to a standard concave maximization problem:

$$\max_{y,t} \ f(y,t) = t \sum_{k=1}^{K} \log_2(1 + h_k y_k / t)$$

subject to

C1: $\alpha_{n+1} \sum_{k=1}^{k_1} \log_2\left(1 + h_k y_k / t\right)$

$\quad - \sum_{k=k_n+1}^{k_n+1} \log_2\left(1 + h_k y_k / t\right) = 0,$

$\quad$ for $n = 1, 2, \ldots, N - 1$.

C2: $tP_T - \sum_{k=1}^{K} y_k \geq 0$ C3: $\eta \sum_{k=1}^{K} y_k + p_c t - 1 = 0, t > 0$

                                                  (5)

Hence, above first line indicates, the objective function will be referred to by *f*. The Lagrangian is:

$L(y, t, \lambda, \mu)$

$= f + \lambda \left[\eta \sum_{k=1}^{K} y_k + p_c t - 1\right] + \gamma \left[tP_T - \sum_{k=1}^{K} y_k\right]$

$\quad\quad\quad\quad + \sum_{n=1}^{N-1} \mu_n [\alpha_{n+1} r_1 - r_{n+1}],$

                                                  (6)

where λ, γ and μ_n are the dual variables. The KKT conditions [10] are significant and acceptable for optimality, since (5) is a standard concave problem. These conditions are listed below, where l is used to denote ln2 and q_k for $\frac{h_k p_k}{1 + h_k p_k}$. The first three equations result from the constraints and remaining from the derivatives.

$\alpha_{n+1} \sum_{k=1}^{k_1} \log_2(1 + h_k p_k) = \sum_{k=k_n+1}^{k_n+1} \log_2(1 + h_k p_k),$

                        for $n = 1, 2, 3, \ldots, N - 1$.

                                            (7)

$\gamma \left[tP_T - \sum_{k=1}^{K} y_k\right] = 0, \quad tP_T - \sum_{k=1}^{K} y_k \geq 0, \quad \gamma \geq 0$

                                                  (8)

$\eta \sum_k y_k + p_c t - 1 = 0$

                                                  (9)

$\sum_{k=1}^{K} (l \log_2(1 + h_k p_k) - q_k) - \frac{1}{t} \sum_{n=1}^{N-1} \mu_n \alpha_{n+1} \sum_{k=1}^{k_1} q_k$

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$$+ \frac{\mu_1}{t} \sum_{k=k_1+1}^{k_2} q_k + \dots\dots \tag{10}$$

$$+ \frac{\mu_{N-1}}{t} \sum_{k=k_{N-1}+1}^{K} q_k + \lambda\, lp_c + \gamma\, l\, P_T = 0$$

$$\left(1 + \frac{1}{t} \sum_{n=1}^{N-1} \mu_n\, \alpha_{n+1}\right) \frac{q_k}{p_k} + \lambda\, \eta l - \gamma l = 0, \text{ for } k = 1, 2\dots, k_1 \tag{11a}$$

$$\left(1 - \frac{\mu_1}{t}\right) \frac{q_k}{p_k} + \lambda\, \eta l - \gamma l = 0, \text{ for } k = k_1 + 1, k_1 + 2\dots, k_2 \tag{11b}$$

$$\vdots$$

$$\left(1 - \frac{\mu_{N-1}}{t}\right) \frac{q_k}{p_k} + \lambda\, \eta l - \gamma l = 0, \tag{11c}$$

for $k = k_{N-1} + 1, k_{N-1} + 2\dots, N$

Two cases offered by the equation in (8): either $\gamma = 0$, or $tP_T - \sum_{k=1}^{K} y_k = 0$. The first mentions that the solution of the optimization problem lies inside the power constraint plane $tP_T - \sum_{k=1}^{K} y_k = 0$. The second mentions that the solution lies on the power constraint plane.

Case I: $\gamma = 0$

In equations (11a) - - (11c) show that all subcarriers allocated to a particular user where the quantity $p_k + 1/h_k$ remains constant. Here each user has its own water level. Indicating User-n's water level by ω_n,

$$p_k + 1/h_k = \omega_n, \text{ for } k = k_{n-1} + 1, k_{n-1} + 2, \dots n$$

$$\text{for } n = 1, 2, \dots N. \tag{12}$$

Equations (10) - - (12) can be used to show that

$$f = \frac{\sum_{n=1}^{N} \alpha_n}{\ln 2 \sum_{n=1}^{N} \alpha_n \omega_n} \tag{13}$$

Then,

$$\beta_{n+1} = \frac{\left[\prod_{k=1}^{k_1} h_k\right]^{\frac{\alpha_{n+1}}{K_n}}}{\left[\prod_{k=k_{n+1}+1}^{k_{n+1}} h_k\right]^{\frac{1}{K_n}}} \qquad \text{for } n = 1, 2, 3, \dots N - 1,$$

$$\gamma_{n+1} = \frac{K_1 \alpha_{n+1}}{K_n} \qquad \text{for } n = 1, 2, 3, \dots N - 1,$$

The relationship between the various water levels ordered by (7) can be briefly written as:

$$\omega_n = \beta_n\, \omega_1{}^{\gamma_n} \qquad \text{for } n = 2, 3, 4 \dots N. \tag{14}$$

After assigning $\beta_1 = \gamma_1 = 1$, (13) can be re-written applying ω_1 alone as:

$$F(\omega_1) = f(\omega_1, \beta_2 \omega_1{}^{\gamma_2}, \beta_3\, \omega_1{}^{\gamma_3}, \dots \beta_N \omega_1{}^{\gamma_N})$$

$$- \frac{\sum_{n=1}^{N} \alpha_n}{(\ln 2) \sum_{n=1}^{N} \alpha_n \beta_n \omega_1{}^{\gamma_n}} = 0 \tag{15}$$

The above non-linear equation $F(\omega_1) = 0$ has a unique solution that included by the fact that (5) is a concave optimization problem. However, it is feasible to prove the existence and uniqueness of the solution separately. An outline is as follows: Notice that $F(\omega_1)$ is continuous when $\omega_1 > 0$. It can be shown $\lim_{\omega_1 \to 0^+} F(\omega_1) = -\infty$ and $\lim_{\omega_1 \to \infty} F(\omega_1) = +\infty$. Hence, by Intermediate Value Theorem, $F(\omega_1) = 0$ has at least one positive solution. Suppose that $F(\omega_1) = 0$ has two positive solutions. Since $F(\omega)$ is differentiable for $\omega_1 > 0$, by Rolle's Theorem, $F'(\omega_1) = 0$ for some $\omega_1 > 0$. However, from (15) it can be shown that $F'(\omega_1) > 0$ for all $\omega_1 > 0$.

After finding the optimum water level $\omega_1{}^*$ for User-1 by solving (15) for ω_1, optimum water levels $\omega_n{}^*$ for the other users can be calculated using (14). From the water levels, the power levels $p_k{}^*$ that maximize the total EE can be calculated using (12). This will form a legitimate solution, only if the power level $p_k{}^*$ satisfies the inequality in (8). If the $p_k{}^*$ values obtained do not satisfy (8), we go to case II.

Case II: $tP_T - \sum_{k=1}^{K} y_k = 0$

This is the case where the solution lies on the plane $\sum_{k=1}^{K} p_k = P_T$. This makes the denominator of the objective function a constant and hence, our optimization problem reduces to the following.

$$\max_{p} \sum_k \log_2(1 + h_k p_k)$$
$$\text{subject to}$$
$$\sum_k p_k = P_T$$

This is the familiar OFDMA throughput maximization problem, for which we know the solution:

$$p_k = p_k{}^{**} = \left[\omega^{**} - \frac{1}{h_k}\right]^+, \text{ where } \omega^{**} = \frac{P_T}{K} + \frac{1}{K} \sum_k \frac{1}{h_k} \tag{16}$$

The results of the analysis are now summarized into a power allocation procedure.

Power Allocation Procedure
1) Solve (15) and obtain User-1's water level $\omega_1{}^*$, and then $\omega_n{}^*$ and $p_k{}^*$ from (14) and (12).
2) If $\sum_{k=1}^{K} p_k{}^* \leq P_T$ then $p_k = p_k{}^*$ for all k.
   Else $p_k = p_k{}^{**}$ for all k, where $p_k{}^{**}$ is given by (16).

C. Convergence and Complexity

We will need the well known result that sorting an array of n elements takes, at lowest, a time in order of $n \log_2 n$. For sorting the users according to their rate requirements and then sorting each row and column of the channel matrix according to the gain using the subchannel assignment protocol. This would take a time in the order of $N \log_2 N + N K \log_2 K + K N \log_2 N$. We will need K/N iterations of the outer loop to assign all subchannels in the worst case scenario. This shows that the algorithm used for the subchannel assignment protocol will converge and the complexity is $NK \log_2 K + (K +1) N \log_2 N + K/N$. Since $K \gg N$, this reduces to $O (NK \log_2 K)$.

In the literature, generally the problem with the convergence of the algorithm comes from the power allocation part [7]. To solve a single non-linear equation the power allocation procedure is needed, regardless of the number of users in the system. In other words with the help of constant time complexity and there are extremely useful solvers that achieve this in few iterations [10]. Hence, the resource allocation algorithm converges and has complexity $O (N K \log_2 K)$. It should be recognized that the algorithm in [7] also has the same complexity as ours and that algorithm uses the minimum rate constraints as opposed to our proportional rate constraints. The algorithm in [7] uses a similar estimate of the EE, and depended upon bi-level optimization, and more essentially, the convergence of that algorithm is not proven.

IV. NUMERICAL RESULTS

Fig. 1 shows the details of the simulation parameters. The power solutions came from these simulations due to respective maximum total power constraints. In other words, the solutions obtained from case-I of the last section. The solution of the equation in (15) was found to be $\omega_1^* = 2.5$ Milli Watts, it is for one realization of the first scenario. This produces a maximum system EE of 267.5 bits/Joule/Hz.

SCENARIO 1 No. of Users (N) = 4 Subchannel (K) = 32 Radius (R) in meters = 400m	Proportional Rate Requirements $r_1:r_2:r_3:r_4 = 1.0 : 0.8 : 0.6 : 0.4$
SCENARIO 2 No. of Users (N) = 6 Subchannel (K) = 60 Radius (R) in meters = 300m − 1000m	No rate requirements
SCENARIO 3 Subchannel (K) = 32, 64, 128 Radius (R) in meters = 400m	N=4 $r_1:r_2:r_3:r_4 = 1.0 : 0.8 : 0.6 : 0.4$ N=8 $r_1:r_2:r_3:r_4:r_5:r_6:r_7:r_8 = 1:1:0.8:0.8:0.6:0.6:0.4:0.4$ N=12 $r_1:r_2:r_3:r_4:r_5:r_6:r_7:r_8:r_9:r_{10}:r_{11}:r_{12} =1:1:1:0.8:0.8:0.8:0.6:0.6:0.6:0.4:0.4:0.4$ N=16 $r_1:r_2:r_3:r_4:r_5:r_6:r_7:r_8:r_9:r_{10}:r_{11}:r_{12} :r_{13}:r_{14}:r_{15}:r_{16} =$ 1:1:1:1:0.8:0.8:0.8:0.8:0.6:0.6:0.6:0.6:0.4:0.4:0.4:0.4

Fig. 1. Simulation details

Recognize that all the EE values reported here are the systems EE over all the subchannels of a particular scenario per unit bandwidth of each subchannel. The total transmission rate of each user over all the subchannels allocated to it and the total power expenditure of each user at the maximum EE are shown in Table I. The easiest form of PSK is BPSK. Generally it used for high speed data transfer application also it is widely used as non-linear modulation scheme. It has small error rates than other systems. Another modulation scheme QPSK also used here. QPSK is able to send twice data in the same bandwidth. The advantage of QPSK is that, it provides twice the spectral efficiency with same energy efficiency. It is a highly bandwidth efficient digital modulation technique. The transmission rates as ratios of User-1's rate show the proportional rate demands being satisfied. Since our algorithm use different types of rate constraints and the bi-level optimization in [7], a direct comparison is not feasible. We modified these two algorithms to avoid the rate demands and used the second scenario in Fig. 1 to compare them.

TABLE I.
SCENARIO 1: POWER LEVELS AND TRANSMISSION RATES AT THE MAXIMUM EE.

User Number	1	2	3	4
Power level (Milli Watts)	15.3	7.3	6.9	3.7
Transmission rate (bits/s/Hz)	15.68	12.54	9.41	6.27
Transmission rate/ User-1's rate	1.00	0.80	0.60	0.40

Fig. 2 shows our algorithm out-performing the one in [7]. For each cell radius, the EE values shown are the averages from 50 random placements of the users. We increased the energy efficiency as the cell radius kept constant. Scenario-3 is used to examine the variation of the system EE with the number of users when the number of subchannels is fixed at three different values.
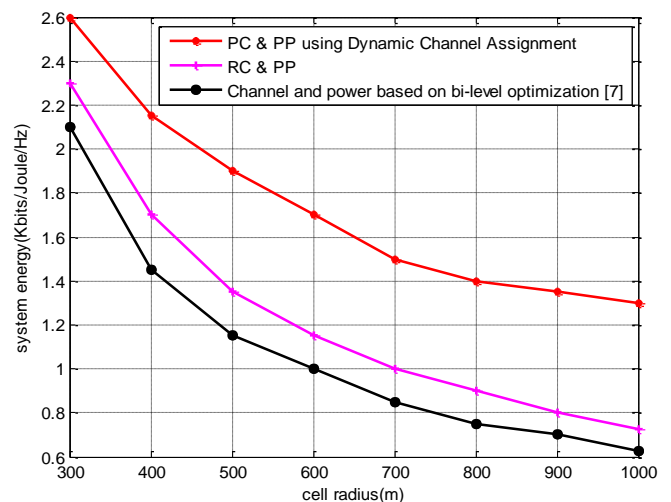

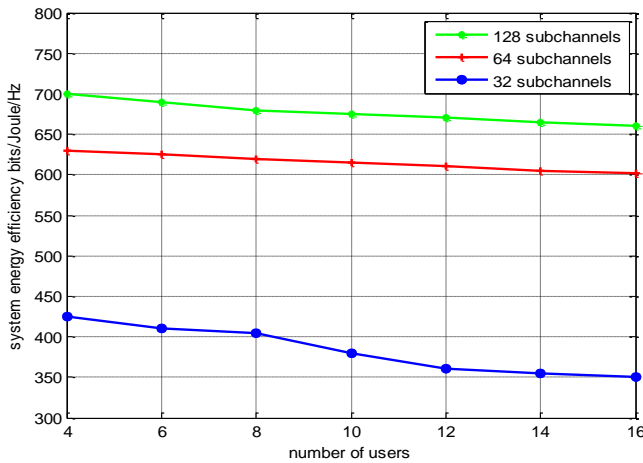Fig. 2. Scenario-2: Variation of the maximum EE with cell radius.

Fig. 3. Scenario-3: Variation of the maximum EE with number of users.

The outputs are shown in Fig. 3. Each point in graph is obtained by averaging the EE values from 50 random placements. The system energy efficiency is increased with number of users kept constant.

V. CONCLUSION

The proposed work called Frequency and power allocation for low complexity energy efficient OFDMA systems with proportional rate constraints. To solve problem of convergence the power and frequency allocation is used. It has the two step solution that maximizes the bits/Joule/Hz energy efficiency of the OFDMA systems based upon the downlink transmissions from a base station in the presence of proportional rate constraints. It has been shown via numerical calculations on MATLAB platform that proposed work provides significant energy efficiency of the OFDMA systems.

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