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Free Vibration Analysis Sandwich Plates Coupled with Fluid

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***Abstract*—This paper presents the free vibration analysis of composite thick rectangular plates coupled with fluid, The governing equations for a thick rectangular plate are analytically based on Reddy's higher order shear deformation theory (HSDT). The plate theory ensures a zero shear-stress condition at the top and bottom surfaces of the plate and do not requires a shear correction factor. Although the plate theory is quite attractive but it could not be used in the finite element analysis. This is due to the difficulties associated with the satisfaction of the C^1 continuity requirement. To overcome this problem associated with Reddy's HSDT, a new C^1 HSDT p-element with eight degrees of freedom per node is developed and used to find natural frequencies of thick composite plates**.

Whereas the velocity potential function and Bernoulli's equation are employed, to obtain the fluid pressure applied on the free surface of the plate. The simplifying hypothesis that the wet and dry mode shapes are the same, is not assumed in this paper.

A comparison is made with the published experimental and numerical results in the literature, showing an excellent agreement. natural frequencies of the plate are presented in graphical forms for different fluid levels, aspect ratios, thickness to length ratios and boundary conditions.

Keywords*—*Free vibration, Thick composites plates, Sandwich plate, hierarchical finite element method, C1 HSDT Fluid–structure interaction*, a*dded mass.

I. Introduction (*Heading 1*)

Systems of shells and plates subjected to flowing fluid are used extensively in modern engineering designs in a variety of industries. Some examples are; ship building, nuclear, aerospace and aeronautical industries, pipe line systems in petroleum and petrochemical industries and car manufacturing.

The simplifyng assumptions made in CPT and FSDT are reflected by the high percentage errors in the results of thick plates analysis. For these plates, higher-order shear deformation theories (HSDT) are required. The HSDT ensure a zero shear-stress condition on the top and bottom surfaces of the plate, and do not require a shear correction factor, which is a major fearture of these theories.
Nelson and Lorch [1], Lo et al. [2] presented a HSDT for laminated plates however the displacement field does satisfy the shear-stress free condition on the topand bottom surfaces of the plate. Lewinson [3], Murthy [4], and Reddy [5] presented a new higher order shear deformation theories considered as an extension of hencky's theory, which include a realistic displacement field satisfying the conditions of zero transverse shear-stress and/or strains, known as Reddy's third-order theory. This model requireds C^1 inter element continuity requirement. Phan and Reddy developed an non-conforming rectangular element with seven degrees of freedom per node, based on C^1 Reddy's third order theory to analyse laminated composites plates. Kant et al [6] investigate the free and transient vibration analysis of composites and sandwich plates based on a refined theory by using the finite element method and analytical solution. Nayak et al. [7,8] investigate the free vibration and transient response of composite sandwich plates by using two C^1 assumed strain finite element based on Reddy's third-order theory. Asadi and Fariborz [9] used a HSDT and the generalized differential quadrature method to analyse the free vibration of composite plates. Batra et al. [10] used a HSDT and the finite element method to analyse free vibrations and stress distribution in thick isotropic plate. Kulkarni and Kapuria [11] used a discrete Kirchoff quadrilateral element based on the third order theory for composite plates, Ambartsumian [12] proposed a higher-order transverse shear stress function to explain plate deformation. Soldatos and Timarci [13] suggested a similar approach for dynamic analysis of laminated plates. Various different functions were proposed by Reddy [14], Touratier [15], Karama et al. [16] and Soldatos [17]. The results of some of these methods were compared by Aydogdu [18]. Swaminathan and Patil [19] used a higher-order method for the free vibration analysis of antisymmetric angle-ply plates [20].

The litterature review clearly shows that very few conforming elements based on C^1 Reddy's third-order plate theory are developped. This is due to the difficulties associated with satisfaction of C^1 continuity requirement. To overcome this hindrance, the hierarchical finite element method can be used. In the hierarchical finite element method the mesh keeps unchanged and the polynomial degree of the shape functions is increased. See for instance Szabo and Sahrmann [21], Szabo and Babuska [22] and Hamza-Cherif [23]. In this paper we address these above-mentioned points. The new approach with hierarchical finite element method is formulated for thick plates vibration analysis. A new hieararchical p-element with eight degrees of freedom per node is developed, based on the C1 higher order shear deformation theory. The continuity along the inter-element boundary is not required in the model. To demonstrate the convergence and accuracy of the proposed method, present results are compared with existing data available from other analytical and numerical methods. The effects of core to face sheet thickness ratio, Young's modulus ratio, thickness ratio, and boundary conditions on the frequencies are presented in tabular form.

Systems of plates coupled with fluid are used extensively in modern engineering designs in a variety of industries. Some examples are; ship building, nuclear, aerospace and aeronautical industries, pipe line systems in petroleum and petrochemical industries and car manufacturing.

Many studies on the free and forced vibration analysis of plates, partially or totally submerged in the fluid, have been carried out.

Lamb [23] calculated the first bending mode shape of a circular plate fixed at its circumference, in contact with water. Fu and Price [24] employed a finite element discretization to analyze the dry and wet dynamic characteristics of a vertical and horizontal cantilever plate. Robinson and Palmer [25] conducted a study on the modal analysis of a rectangular plate resting on an incompressible fluid. Kwak and Kim [26] studied on axisymmetric vibration of circular plates in the presence of fluid on the basis of the mixed boundary value problem. Free vibration of infinite elastic rectangular plate in contact with water was studied by Hagedorn [27]. Kwak [28] utilized a piecewise division to investigate the free vibrations of rectangular plates in contact with unbounded water on one side, while beam functions were used as admissible functions. Haddara and Cao [29] investigated dynamic responses of rectangular plates immersed in fluid. An approximate expression for the evaluation of the modal added mass was derived for cantilever and SFSF rectangular plates and the numerical results were verified by the experimental ones. The natural frequencies of annular plates in contact with a fluid on one side were theoretically obtained by Amabili et al. [25] using the added mass approach, whereas the coupled fluid–structure system was solved by adopting the Hankel transform. Meylan [26] employed an appropriate Green's function to study the forced vibration of an arbitrary thin plate floating on the surface of an infinite liquid. Cheung and Zhou [27] also studied the case of a horizontal rectangular plate composing the base of a rigid rectangular container. The dynamic characteristics of a vertical cantilever plate partially in contact with fluid were investigated by Ergin and Ugurlu [28]. Liang et al. [29] adopted an empirical added-mass formulation to determine the frequencies and mode shapes of submerged cantilevered plates. Based on a finite Fourier series expansion, Jeong et al. [30] studied the wet resonance frequencies and associated mode shapes of two identical rectangular plates coupled with a bounded fluid. Tayler and Ohkusu [31] suggested expressions for the free–free rectangular plates in terms of the sinusoidal eigen-modes of a pinned–pinned beam and rigid body modes. Zhou and Cheung [32] employed an analytical-Ritz method to investigate a rectangular plate in contact with water on one side.

Ugurlu et al. [33] investigated the effects of elastic foundation and fluid on the dynamic response characteristics of rectangular Kirchhoff plates using a boundary element method. Kerboua et al. [34] developed a combination of the finite element method and Sanders'shell theory to study the vibration analysis of rectangular plates in contact with fluid. Recently, Hosseini Hashemi et al. [35,36] presented a comprehensive investigation on hydroelastic vibration analysis of horizontal and vertical rectangular plates resting on Pasternak foundation for different boundary conditions. To analyze the interaction of the Mindlin plate with the elastic foundation and fluid system, three displacement components of the plate were expressed in the Ritz method by adopting a set of static Timoshenko beam functions satisfying geometric boundary conditions. In Hashemi et al [37] studied the free vibration of a horizontal rectangular plate, is immersed in the liquid or floating on the free surface. The governing equations for moderately thick rectangular plate are analytically based on the theory of Mindlin plates.

II. PLATE FORMULATION

A) Energy formulation

Consider a laminate composite thick plate of uniform thickness *h*, length *a* and width *b*, as shown on Fig. 1. The displacement of the plate are decomposed into three orthogonal components, *u*,*v* and *w* are the displacement components of middle plate in the x, y, and z directions, respectively.

In accordance with the higher-order shear deformable theory [9], the displacements can be expressed as

$$\begin{aligned} u &= u_0 + z\,\theta_x - f(z)\left(\frac{\partial w_0}{\partial x} + \theta_x\right) \\ v &= v_0 + z\,\theta_y - f(z)\left(\frac{\partial w_0}{\partial y} + \theta_y\right) \\ w &= w_0 \end{aligned} \tag{1}$$

in which

$$f(z) = \frac{4}{3h^2} z^3 \tag{2}$$

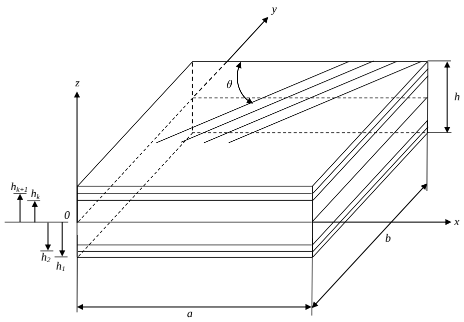

Fig.1.Laminate geometry with positive set of laminate reference axes, displacements and fiber orientation.

Where u_0,v_0, and w_0 are the displacements of the middle surface of the plate, θ_x and θ_y are rotations of transverse normal about y-axis and x-axis of the plate respectively.

The linear strain-displacement relationships is given by

$$\{\varepsilon\}=\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\frac{\partial u_0}{\partial x}\\ \frac{\partial v_0}{\partial y}\\ \frac{\partial w_0}{\partial y}+\theta_y\\ \frac{\partial w_0}{\partial x}+\theta_x\\ \frac{\partial v_0}{\partial x}+\frac{\partial u_0}{\partial y}\end{Bmatrix}+z\begin{Bmatrix}\frac{\partial \theta_x}{\partial x}\\ \frac{\partial \theta_y}{\partial y}\\ 0\\ 0\\ \frac{\partial \theta_y}{\partial x}+\frac{\partial \theta_x}{\partial y}\end{Bmatrix}-\frac{\partial f(z)}{\partial z}\begin{Bmatrix}0\\ 0\\ \frac{\partial w_0}{\partial y}+\theta_y\\ \frac{\partial w_0}{\partial x}+\theta_x\\ 0\end{Bmatrix}-f(z)\begin{Bmatrix}\frac{\partial^2 w_0}{\partial x^2}+\frac{\partial \theta_x}{\partial x}\\ \frac{\partial^2 w_0}{\partial y^2}+\frac{\partial \theta_y}{\partial y}\\ 0\\ 0\\ \frac{\partial \theta_y}{\partial x}+\frac{\partial \theta_x}{\partial y}+2\frac{\partial^2 w_0}{\partial x\,\partial y}\end{Bmatrix} \quad (3)$$

The constitutive equations for a kth layer, in the orthotropic local coordinate derived from Hook's law for plane stress is given by

$$\{\sigma\}^k=[C]^k\{\varepsilon\}^k \quad (4)$$

In the case of plane stress the stress vector can be written as

$$\{\sigma\}^k=\{\sigma_{xx}\quad \sigma_{yy}\quad \tau_{yz}\quad \tau_{xz}\quad \tau_{xy}\}^k \quad (5)$$

The constitutive equations for a kth layer, in the orthotropic local coordinate derived from Hook's law for plane stress are given by

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_{12}\\ \sigma_{13}\\ \sigma_{23}\end{Bmatrix}^k=\begin{bmatrix}C_{11}&C_{12}&C_{13}&0&0\\ C_{12}&C_{22}&C_{23}&0&0\\ C_{13}&C_{23}&C_{33}&0&0\\ 0&0&0&C_{44}&0\\ 0&0&0&0&C_{55}\end{bmatrix}^k\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_{23}\\ \varepsilon_{13}\\ \varepsilon_{23}\end{Bmatrix}^k \quad (6)$$

Where the well-known engineering constants C_{ij} are given by

$$\begin{gathered}C_{11}=E_1/(1-\nu_{1,2}\nu_{2,1})\;C_{22}=E_2/(1-\nu_{1,2}\nu_{2,1})\\ C_{12}=\nu_{2,1}E_{12}/(1-\nu_{1,2}\nu_{2,1})\\ C_{21}=C_{2,1}\;C_{33}=G_{1,2}\;C_{44}=G_{1,3}\;C_{55}=G_{2,3}\end{gathered} \quad (7)$$

In which E_i, ν_{ij} and G_{ij} are the Young's modulus, Poisson's ratio and shear modulus of the lamina.
Where, 1 and 2 represent the directions parallel and perpendicular to the fibers direction. By performing a proper coordinate transformation, the stress-strain relationships of a single lamina in the oxyz co-ordinate system can be obtained.

The stress-strain relations in the global (*x, y, z*) coordinate system can be written as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\\ \sigma_{xz}\\ \sigma_{yz}\end{Bmatrix}^k=\begin{bmatrix}Q_{11}&Q_{12}&Q_{13}&0&0\\ Q_{12}&Q_{22}&Q_{23}&0&0\\ Q_{13}&Q_{23}&Q_{33}&0&0\\ 0&0&0&Q_{44}&Q_{45}\\ 0&0&0&Q_{45}&Q_{55}\end{bmatrix}^k\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{Bmatrix}^k \quad (8)$$

The kinetic energy of a vibrating composite thick plate is given by

$$Ec=\frac{1}{2}\int_0^1\int_0^1\left[\dot{u}_0^{\,2}+\dot{v}_0^{\,2}+\dot{w}_0^{\,2}\right]dxdy \quad (9)$$

Where ρ is the mass density per unit volume.

The strain energy of a thick plate is expressed as

$$Ed=\frac{1}{2}\int_0^1\int_0^1\left[\sigma_x^{\,k}\varepsilon_x^{\,k}+\sigma_y^{\,k}\varepsilon_y^{\,k}+\sigma_{xy}^{\,k}\gamma_{xy}^{\,k}+\sigma_{xz}^{\,k}\gamma_{xz}^{\,k}+\sigma_{yz}^{\,k}\gamma_{yz}^{\,k}\right]dxdy \quad (10)$$

B) Hierarchical finite element formulation

A four node rectangular hierarchical finite element with eight degrees of freedom per node (u_0,v_0, w_0, $\partial w_0/\partial x$, $\partial w_0/\partial y$, $\partial^2 w_0/\partial xy$, θ_x, θ_y) is developed on the basis of a third-order plate theory (See Fig. 2).Trigonometric hierarchical functions are used as shape functions. The model requires C^0 continuity for u_0, v_0, θ_x and θ_y and C^1 continuity for w_0.
The displacements and rotations of the rectangular plate p-element are expressed as

$$\begin{aligned}u_0(\xi,\eta,t)&=\sum_{m=1}^{Pu}\sum_{n=1}^{Pu}u_{0_{mn}}(t)f_m(\xi)f_n(\eta)\\ v_0(\xi,\eta,t)&=\sum_{m=1}^{Pu}\sum_{n=1}^{Pu}v_{0_{mn}}(t)f_m(\xi)f_n(\eta)\\ w_0(\xi,\eta,t)&=\sum_{m=1}^{Pw}\sum_{n=1}^{Pw}w_{mn}(t)g_m(\xi)g_n(\eta)\\ \theta_x(\xi,\eta,t)&=\sum_{m=1}^{P\theta}\sum_{n=1}^{P\theta}\theta_{x_{mn}}(t)f_m(\xi)f_n(\eta)\\ \theta_y(\xi,\eta,t)&=\sum_{m=1}^{P\theta}\sum_{n=1}^{P\theta}\theta_{y_{mn}}(t)f_m(\xi)f_n(\eta)\end{aligned} \quad (11)$$

Where P_u, P_W and P_θ are the number of shape functions used in the model.

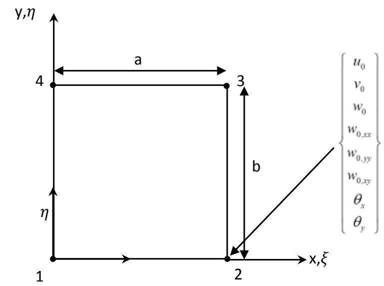

Fig. 2. Plate element coordinates and dimensions

The first shape functions f_1, f_2 and g_1 to g_4, are commonly used in the finite element method. The functions (f_{n+2} and g_{n+4}) are the trigonometric shape functions and lead to zero transverse displacement, and zero slope at each node. This feature is highly significant since these functions give additional freedom to the edges and the interior of the element.

The expressions of the trigonometric hierarchical shape functions $f_i(\xi)$ for C^0 continuity and $g_i(\xi)$ for C^1 are given by [41]

$$\begin{cases} f_1 = 1-\xi \\ f_2 = \xi \\ f_{n+2} = \sin(\delta r\,\xi) \\ \delta r = r\,\pi \\ r = 1,2,3,... \end{cases} \tag{12}$$

and

$$\begin{cases} g_1 = 1-3\xi^2+2\xi^3 \\ g_2 = \xi-2\xi^2+\xi^3 \\ g_3 = 3\xi^2-2\xi^3 \\ g_4 = -\xi^2+\xi^3 \\ g_{n+4} = \delta r\left[-\xi+\left(2+(-1)^r\right)\xi^2-\left(1+(-1)^r\right)\xi^3\right]+\sin(\delta r\,\xi) \\ \delta r = r\,\pi \\ r = 1,2,3,... \end{cases} \tag{13}$$

The displacements and rotations can be expressed in matrix form as

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta_x \\ \theta_y \end{Bmatrix} = [N]\{q\} \tag{14}$$

[N] is the matrix of shape functions, given by

$$[N] = \begin{bmatrix} [N_u] & 0 & 0 & 0 & 0 \\ 0 & [N_u] & 0 & 0 & 0 \\ 0 & 0 & [N_w] & 0 & 0 \\ 0 & 0 & 0 & [N_\theta] & 0 \\ 0 & 0 & 0 & 0 & [N_\theta] \end{bmatrix} \tag{15}$$

where

$$\{q\} = \begin{Bmatrix} q_u \\ q_v \\ q_w \\ q_{\theta_x} \\ q_{\theta_y} \end{Bmatrix} \tag{16}$$

In which q_u, q_v, q_w, $q_{\theta x}$, and $q_{\theta y}$ are the generalized displacements.

The matrices of shape functions are given by

$$[N_w] = \left[\left(g_1(\xi)\,g_1(\eta)\right)_1, \left(g_1(\xi)\,g_2(\eta)\right)_2, \ldots\left(g_k(\xi)\,g_l(\eta)\right)_r, \ldots\left(g_{P_w}(\xi)\,g_{P_w}(\eta)\right)_{P_wP_w}\right] \tag{17}$$

where $k=1,...,P_w$, $l=1,...,P_w$, and $r=j+(i-1)P_w$

and

$$[N_u] = [N_\theta] = \left[\left(f_1(\xi)\,f_1(\eta)\right)_1, \left(f_1(\xi)\,f_2(\eta)\right)_2, \ldots\left(f_i(\xi)\,f_j(\eta)\right)_m, \ldots\left(f_{P_\theta}(\xi)\,f_{P_\theta}(\eta)\right)_{P_\theta P_\theta}\right] \tag{18}$$

where $i=1,...,P_\theta$, $j=1,...,P_\theta$, and $m=j+(i-1)P_\theta$.

The equations of motion in the case of free vibration of composite plates can be expressed as

$$[M]\{\ddot{q}\}+[K]\{q\}=0 \tag{19}$$

Here [K] is called the stiffness matrix of the p-element, determined from the strain energy

$$[K] = \begin{bmatrix} [K_{uu}] & [K_{uv}] & [K_{uw}] & [K_{u\theta_x}] & [K_{u\theta_y}] \\ [K_{uv}]^T & [K_{vv}] & [K_{vw}] & [K_{v\theta_x}] & [K_{v\theta_y}] \\ [K_{uw}]^T & [K_{vw}] & [K_{ww}] & [K_{w\theta_x}] & [K_{w\theta_y}] \\ [K_{u\theta_x}]^T & [K_{v\theta_x}]^T & [K_{w\theta_x}]^T & [K_{\theta_x\theta_x}] & [K_{\theta_x\theta_y}] \\ [K_{u\theta_y}]^T & [K_{v\theta_y}]^T & [K_{w\theta_y}]^T & [K_{\theta_x\theta_y}]^T & [K_{\theta_y\theta_y}] \end{bmatrix} \tag{20}$$

and [M] is called the mass matrix of the p-element, given by the following relation

$$[M] = \begin{bmatrix} [M_{uu}] & 0 & [M_{uw}] & [M_{u\theta_x}] & 0 \\ 0 & [M_{vv}] & [M_{vw}] & 0 & [M_{v\theta_y}] \\ [M_{uw}]^T & [M_{vw}]^T & [M_{ww}] & [M_{w\theta_x}] & [M_{w\theta_y}] \\ [M_{u\theta_x}]^T & 0 & [M_{w\theta_x}]^T & [M_{\theta_x\theta_x}] & 0 \\ 0 & [M_{v\theta_y}]^T & [M_{w\theta_y}]^T & 0 & [M_{\theta_y\theta_y}] \end{bmatrix} \tag{21}$$

The sub-matrices of and are defined in appendix A.

III. FLUID FORMULATION

The following assumptions are made to model the dynamic fluid:

- The a small fluid motion with low vibration.
- The fluid is incompressible, non-viscous and irrotational (fluid flow is possible).

The potential function of the speed must satisfy Laplace's equation throughout the fluid area. This relationship is expressed in the Cartesian coordinate system as follows:

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2} \quad (22)$$

By using Bernoulli's equation and ignoring non-linear terms, the fluid pressure at the fluid interface plate (top and bottom surface of the plate) may be given by:

$$P_u = P_{|z=h/2} = -\rho_f \left.\frac{\partial\phi}{\partial t}\right|_{z=h/2} \quad (23)$$

$$P_L = P_{|z=-h/2} = -\rho_f \left.\frac{\partial\phi}{\partial t}\right|_{z=-h/2} \quad (24)$$

Where ρ_f is the density of the fluid per unit volume.

$$\frac{\partial\phi}{\partial z\big|_{z=h/2}} = \frac{\partial w}{\partial t} \quad (25)$$

$$\frac{\partial\phi}{\partial z\big|_{z=-h/2}} = \frac{\partial w}{\partial t} \quad (26)$$

The condition of the impermeability of the surface of the structure requires that the component of the fluid on the surface of the plate off the speed-up must correspond to the instantaneous rate of change of displacement of the plate in the transverse direction, this condition implies continuous contact between the surface of the plate and the device fluid layer, which is:

$$\phi(x,y,z,t) = F(z)S(x,y,t) \quad (27)$$

Where *F (z)* and *S (x, y, z)* are two distinct functions to be determined.

The following expression can be defined by introducing the equation (27) in (25, 26) and by replacing *S(x, y, z)*

$$\phi(x,y,z,t) = \frac{F(z)}{\left.\frac{\partial F}{\partial z}\right|_{z=h/2}}\frac{\partial w}{\partial t} \quad (28)$$

$$\phi(x,y,z,t) = \frac{F(z)}{\left.\frac{\partial F}{\partial z}\right|_{z=-h/2}}\frac{\partial w}{\partial t} \quad (29)$$

Substituting equation (29 and 28) in equation (22) the second order differential equation is obtained:

$$\frac{\partial^2 F}{\partial z^2} - \mu_f^{\,2} F(z) = 0 \quad (30)$$

Where μ_f is a plane wave number and a real constant that must be precise.

The general solution of equation (30) can be written:

$$F(z) = B_1 e^{\mu_f z} + B_2 e^{-\mu_f z} \quad (31)$$

B1 and *B2* are constants to be specified by introducing the equation (31) in (28 and 29), the following expressions are obtained for the potential function of speed:

$$\phi(x,y,z,t) = \frac{B_1 e^{\mu_f z} + B_2 e^{-\mu_f z}}{\left.\frac{\partial F}{\partial z}\right|_{z=h/2}}\frac{\partial w}{\partial t} \quad (32)$$

$$\phi(x,y,z,t) = \frac{B_1 e^{\mu_f z} + B_2 e^{-\mu_f}}{\left.\frac{\partial F}{\partial z}\right|_{z=-h/2}} \; (3.7) \quad (33)$$

A) Boundary conditions of a plate–fluid

The boundary conditions at the fluid-structure interface to the fluid end must be satisfied by adopting a potential function of appropriate speed. Fluid free surface, rigid wall and impermeability are generally taken into account. To achieve a good understanding of the problem, a flexible rectangular plate submerged in the liquid is studied, or the following conditions must be considered.

1) Plate–fluid model with free surface

At the liquid free surface, the following condition may be applied to the speed potential (Fig.3), provided that the free movement of the liquid surface creates substantial disturbances.

$$\frac{\partial\phi}{\partial z\big|_{z=h_1+h/2}} = -\frac{1}{g}\left.\frac{\partial^2\phi}{\partial t^2}\right|_{z=h_1+h/2} \quad (34)$$

Where 'g' is acceleration due to gravity. The introduction of Eq. (32-33) simultaneously into relation (34) and (25-26), results in the following expression for the potential function

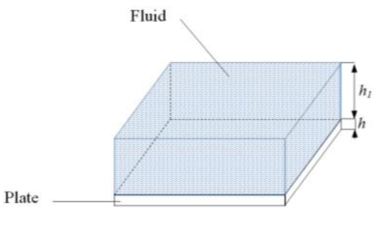

Fig.3. plate-fluid model with free surface

$$\phi = \frac{e^{\mu_f z} + C_1 e^{\mu_f(z-2h_1)}}{\mu_f\left(1-C_1 e^{2\mu_f h_1}\right)}\frac{\partial w}{\partial t} \quad (35)$$

where

$$C_1 = \frac{\mu_f g - \omega^2}{\mu_f g + \omega^2},\; \mu_f = \pi\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} \quad (36)$$

The application of a fluid pressure on the upper surface of the plate is obtained by introducing the above relationship in the Bernoulli equation:

$$P_U = \frac{-\rho_f}{\mu_f}\left[\frac{1+C_1 e^{2\mu_f h_1}}{1-C_1 e^{2\mu_f h_1}}\right]\frac{\partial^2 w}{\partial t^2} = Zf_1 \frac{\partial^2 w}{\partial t^2} \qquad (37)$$

2) Plate-fluid model bounded by a rigid wall

The boundary condition on the wall, shown in (Fig.4), was studied by Lamb [24] and called state of zero frequency. This condition limited the wall stiffness is expressed by:

$$\frac{\partial \phi}{\partial z}\bigg|_{z=-h_2} = 0 \qquad (38)$$

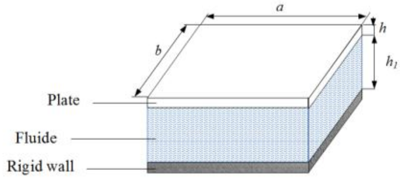

Fig.4. plate-fluid model bounded by a rigid wall a floating plate

By introducing the equation (33) in (37):

$$\phi = \frac{e^{\mu_f z} + C_2 e^{-\mu_f z}}{\mu_f\left(e^{\mu_f h/2} - C_2 e^{\mu_f h/2}\right)}\frac{\partial w}{\partial t} \qquad (39)$$

If the dynamic pressure (lower surface of the plate) is determined by:

$$P_L = -\frac{\rho_f}{\mu_f}\left[\frac{e^{-2\mu_f h2}+1}{e^{-2\mu_f h2}-1}\right]\frac{\partial^2 w}{\partial t^2} = Zf_2 \frac{\partial^2 w}{\partial t^2} \qquad (40)$$

In the case where the plate is fully immersed, as shown in Figure 5, the total dynamic pressure will be a combination of pressure corresponding to the conditions to the fluid limits on both upper and lower surfaces of the plate:

$$P = -\frac{\rho_f}{\mu_f}\left[\frac{1+C_2 e^{\mu_f h1}}{1-C_2 e^{2\mu_f h1}} + \frac{e^{\mu_f h2}+1}{e^{2\mu_f h2}-1}\right]\frac{\partial^2 w}{\partial t^2} = Zf_3 \frac{\partial^2 w}{\partial t^2} \qquad (41)$$

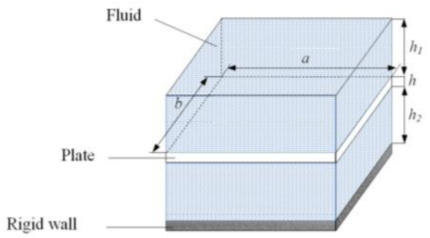

Fig.5. plate-fluid model bounded by a rigid wall a submerged plate

B) Modeling fluid by p-element

Using the procedure of the hierarchical finite elements, the fluid force vector f can be expressed by a finite element using the following relationship*:*

$$\{fp\} = \int [N_w]^T \{\mathrm{Pr}\} dxdy \qquad (42)$$

Where $[N_w]$ is the matrix of functions shape, {Pr} is a vector expressing the pressure exerted by the fluid on the plate (Eq. 38, 40,41).

The dynamic pressure is then defined by:

$$[M_f] = \{fp\} = Zf_i \dot{w}_0^2 d\xi d\eta \qquad (43)$$

The rectangular plate is modeled by a quadrilateral hierarchical finite element (Fig. 2).

IV. EQUATIONS OF MOTION OF FLUID-STRUCTURE

The global system of equations of motion of a rectangular plate interacting with a fluid can be represented as follows:

$$([M_s]-[M_f])\{\ddot{q}\} + [K_s]\{q\} = 0 \qquad (44)$$

Where the subscripts *f* and *s* refer to the vacuum plate and in fluid respectively. [Ms] and [Ks] is the mass matrix and stiffness of the vacuum plate. [Mf] is the fluid inertia; {q} is the displacement vector.

V. RESULTS AND DISCUSSIONS

In this section the results obtained by this method are compared with those in the literature, in Table 1 a comparison is made with those Kerboua et al [40] who used the finite element method and experimental approach presented by Haddara and Cao [30], the plate used in this example is isotropic totally submerged in the fluid in Figure 5 of a length $a = 0.20165$ m, width $b = 0.655$ m and a thickness $h = 9.63\ 10^{-3}$ m, the conditions for the fluid-structure limits $h1 = h2 = 0.4- (h / 2)$.

Material properties of the steel plate

$E_1 = 207$ Gpa, $\nu = 0.3$, $\rho = 1500\ kg/m^3$

Fluid Property $\rho_f = 1000\ kg/m^3$

Table 2 compares the results of the present study and those of Kerboua et al [40] and Fu and Price [44] that have used the finite element method and a study experimental presented by Lindholm et al. [45]

TABLE 1: Comparison of the first five frequencies (rad/s) a plate ALAL, submerged in water,

Mode	Present	Experimental [30]	KERBOUA et al [40]
1	32.41	28.72	31.28
2	130.70	117.13	126.40
3	145.51	154.51	141.78
4	295.95	281.79	285.98
5	312.61	335.04	304.57

TABLE 2: Comparison of the first three frequencies (rad/s) a cantilever square plate submerged in water as function of fluid level (h1 variable and h2 >>a)

Mode	In vacuo			*h1/a=0.05*			*h1/a=0.5*			
	Present	KERBOUA et al [40]	Fu et Price [44]	Present	KERBOUA et al [40]	Fu et Price [44]	Present	KERBOUA et al [40]	Fu et Price [44]	Lindholm et al [45]
1	12.82	12.93	12.94	8.20	8.60	8.95	7.82	7.00	7.35	6.56
2	31.31	31.69	31.69	20.05	21.09	23.1	19.11	17.16	20.19	19.66
3	78.39	79.37	79.37	50.20	52.92	55.7	47.86	42.98	50.11	45.32

A very good agreement in the results obtained in this study is the references mentioned in Tables 1 and 2.

Material properties of plate (Graphite-Epoxy)

E_1= 128 Gpa, E_2 = 11 Gpa, G_{12} =4.48 Gpa, ν_{12} = 0.078, ρ = 1500 kg/m^3

Fluid density $\rho_f = 1500\ kg/m^3$

In the next example of fluid interaction validation - a composite laminated plate structure with eight layers is considered, Table 3 represent the first natural frequencies for two situations; a cantilevered plate the free surface of the fluid and a plate cantilevered totally immersed in the liquid. It should be mentioned that the plate is embedded on the shorter side. It can be seen that the results of this are very close to those of Alizera [47], Pal et al [46], noted that in these two references, they used the finite element method combined with theories CPT and FSDT respectively.

TABLE 3: Comparison frequency (Hz) of a basic rectangular composite plate (0.152m 0.076m) and a / h = 0.00104 m, FFFE graphite / [45 / -45 / -45 / 45] sym

fundamental (Hz) Frequency	Present	Pal et al [40]	Alireza [39]
Plate with free surface (CL1)	8.38	8.13	8.35
Plate totally submerged (CL2)	6.02	5.94	6.01

Figures 6-9 showing the variation of frequency as a function of the fluid height h1, in this example fixed by the height h2 to actually vary the ratio h1 / a (Figure 5), takes as an example of a study square sandwich plate has five layer symmetrical 90 / -60 / 30 / core / 90 / -60 / 30 with a thickness h = 0.2 m, the ratio of the thickness of the core to that of the skin hc / hf = 16, by notes that the frequency decreases according to the report, the frequencies begin to stabilized from the ratio h1 / a = 0.8 is that for different cases to limit condition of the structure.

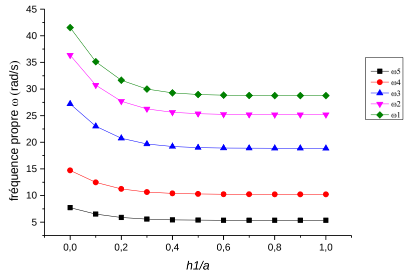

Fig. 6. variation of the natural frequency (rd / s) depending on the height of fluid in a sandwich plate CFFF (h1 variable and h2>>a) with hc / hf = 16

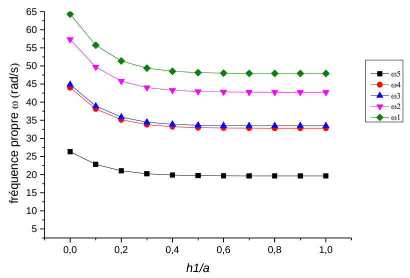

Fig. 7. variation of the natural frequency (rd / s) depending on the height of fluid in a sandwich plate SSSS (h1 variable and h2>>a) with hc / hf = 16

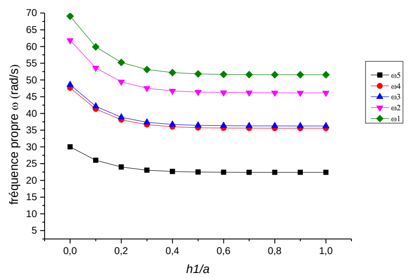

Fig. 8. variation of the natural frequency (rd / s) depending on the height of fluid in a sandwich plate CCCC (h1 variable and h2>>a) with hc / hf = 16

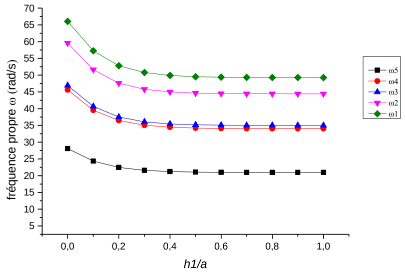

Fig. 9. variation of the natural frequency (rd / s) depending on the height of fluid in a sandwich plate SCSC (h1 variable and h2>>a) with hc / hf = 16

Figures 10-13 showing the variation of frequency as a function of the fluid height h2, in this example the height h2 = 0, and indeed varying the ratio h1 / a (Figure 3), takes as an example of study square sandwich plate has five layers symmetrical 90 / -60 / 30 / soul / 90 / -60 / 30 with thickness h = 0.2 m, the ratio of the thickness of the core to the skin hc / hf = 16 in remarks that the frequency decrease with the report, the frequencies begin to stabilized from the ratio h2 / a = 0.8 is that for different cases to limit condition of the structure.

The properties of the materials and the fluid in the two previous examples are:

Properties for face layers: glass polyester resins

$E_1 = 24.51Gpa,\ E_2 = 7.77Gpa,$

$G_{12} = 3.34Gpa,\ G_{13} = 3.34Gpa G_{23} = 1.34Gpa,$

$\nu_{12} = 0.078,\quad \nu_{21} = 0.24$

$\rho = 1800\ kg/m^3$

Properties for core layer: *HEREX C70.130*

$E_c = 103.63Mpa,\ G_c = 50Mpa, \nu_{12} = 0.32,\ \rho_c = 130\ kg/m^3$

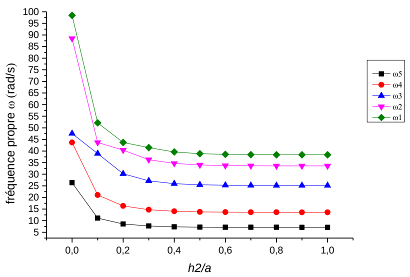

Fig. 10. variation of the natural frequency (rd / s) depending on the height of fluid in a sandwich plate CFFF (h1 = 0 and h2 variable) with hc / hf = 16

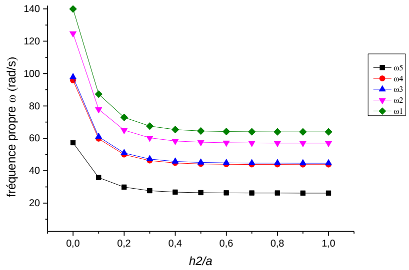

Fig. 11. variation of the natural frequency (rd / s) depending on the height of fluid in a sandwich plate SSSS (h1 = 0 and h2 variable) with hc / hf = 16

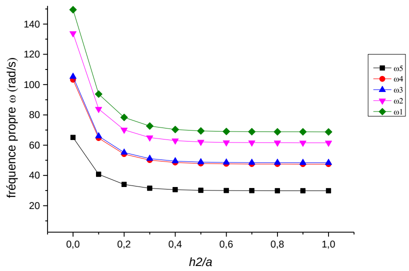

Fig. 12. variation of the natural frequency (rd / s) depending on the height of fluid in a sandwich plate CCCC (h1 = 0 and h2 variable) with hc / hf = 16

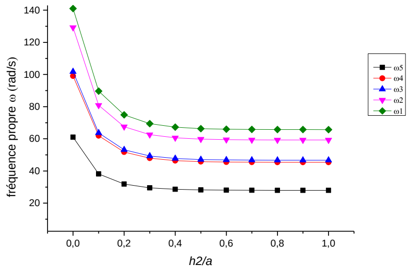

Fig. 13. variation of the natural frequency (rd / s) depending on the height of fluid in a sandwich plate SCSC (h1 = 0 and h2 variable) with hc / hf = 16

VII. CONCLUSION

A new C1 HSDT p-element with eight degrees of freedom per node has been developed and used to find natural frequencies of sandwich thick plates totally or partially submerged in conjunction with Reddy's higher-order shear deformation theory. Potential fluid flow induced pressure on the structure. To define this pressure as a function of transverse displacement and velocity, Bernoulli equations and the impermeability condition were used. The mass, stiffness matrices were defined and relations for fluid-solid-interactions were developed by integration for p-element. Then, a detailed parametric study was conducted to show the influence of different fluid depths, aspect ratios and thickness to length ratios for four combinations of boundary conditions. Based on these observations the element can be recommended for free vibration analysis of composite plate structures emerged in fluid with sufficient accuracy.

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