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Free Vibration Analysis of Viscoelastic Sandwich Beam using Euler Bernoulli Theory

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***Abstract*—The wide variety of damping techniques are used to reduce the vibrations, out of them constrained layer damping (CLD) treatment is used for studying the free vibration behaviour of sandwich beams. The sandwich model is formed by placing the viscoelastic material bonded between the face and base layer. Viscoelastic sandwich beam consists of three layers with viscoelastic material as a core layer, the face layers are isotropic and linear elastic material. In this paper free vibrations of fixed free sandwich beam with different configurations are investigated analytically. Analytical solution is carried out using Euler-Bernoulli beam theory to find the natural frequencies with different configurations of the beam using MATLAB. The analytical results of the sandwich beams are to be compared with available literature values.**

Keywords — Free vibration; sandwich beam; natural frequency; CLD.

I. INTRODUCTION

A beam is a slender horizontal structural member that resists lateral loads by bending. This is an important element in engineering structures such as supporting members in high-rise buildings, railways, long-span bridges, flexible satellites, gun barrels, robot arms, airplane wings, etc. A sandwich beam is a layered assembly made from two thin, strong and stiff face sheets bonded to a lightweight core material to create light weight and strong structural element. The development of sandwich structures will continue to be demand for the need of light weight, to carry both in-plane and out-of-plane loads, good stability under compression, providing excellent stiffness to weight and strength to weight characteristics. Many types of damping mechanism have been developed to suppress the vibration level. Passive damping using viscoelastic materials is widely used in both commercial and aerospace applications. Viscoelastic materials are elastomeric whose long- chain molecules cause them to convert mechanical energy into heat when they are deformed. Banerjee [1] carried the free vibration analysis of three layered symmetric sandwich beams using the dynamic stiffness method. He used the Hamilton’s principle to derive the governing partial differential equations of motion in free natural vibration that are coupled both in axial and bending deformation. Compute the natural frequencies and mode shapes of a sandwich beam using Wittrick-Williams algorithm by him. Mohammad Sedighi and Hiwa Hosseini [2] investigated free vibration of stepped Timoshenko shaft with insignificant damping using FEM with uniform and non-uniform status. In case of non-uniform Timoshenko shaft the first and third part will be uniform while the middle part is in conical form. They calculated natural frequencies and corresponding mode shape for two cases of boundary conditions namely clamped free and simply supported. They compared the obtained values with those of classical Euler Bernoulli theory results. Mohammadi and Hassanirad [3] introduced two vibration measuring instruments that is Accelerometer and RFID. They recognized RFID is more easier and simple to analyze in order to measure free vibration of a cantilever beam. Dawid Cekus [4] used Lagrange multiplier method to find the solution of the free vibration problem of the cantilever tapered beam. He carried out sample numerical calculations for cantilever tapered beam and compared with experimental values. Mohammad Zannonc [5] studied vibration of thin film cantilever beams and derived equations of equilibrium. He derived equations of frequency and mode shapes to solve for different cantilever beams for various materials. A Mat-Lab programme is developed by him to find natural frequencies and corresponding mode shapes for one layer and two layer beams with different dimensions and properties. Rajesh and Suresh Kumar [6] studied the vibration analysis of fixed free structure with different core material. They found the dynamic effect of core material in different position of sandwich structure with same volume of viscoelastic and structure materials. They found that positioning of the core material at the center has a great influence in controlling vibration of structures. Abdel Salam and Bondok [7] developed sandwich beams to calculate the flexural rigidity and dynamic characteristics. They investigated different cases of sandwich beams with multi layer cores, multi cells, different shape and orientation of holes in its cores. Results of natural frequencies and mode shapes and the static deflection of the sandwich beams were obtained by them using ANSYS and compared with analytical values. Iiodees et al. [8] presented the methods to predict the natural frequencies and mode shapes of composite beams. They solved the equations of motion in two ways i.e., finite element method and exact integration method. Natural frequencies and mode shapes for a variety of ply layups are obtained by them. I. M. Daniel et al. [9] developed simple models in order to explain the behaviour of composite sandwich as a function of material, geometric and loading parameters. They used direct observation and nondestructive evaluation to study the mechanical characterization of the sandwich constituent materials. Linear variation in strain through thickness was determined by them in both core and skin thickness. Jinhee Lee [10] employed the pseudospectral method to non-ideal boundary Timoshenko beam model for free vibration analysis where as analytical method was carried out for Euler

Bernoulli beam. He found that weighing factor increases as natural frequency decreases for non-ideal at one end and free boundary conditions at the other end. Natural frequencies of a Timoshenko beam are computed for various thickness to length ratio with the non-ideal clamped boundary condition at the left end and ideal free boundary condition at the right end of the beam. Baki Ozturk and Safa Bozkurt Coskun [11] obtained the analytical results for free vibration analysis of beam on elastic foundation for different support conditions They validated the analytical solutions with the variational iteration method and homotopy perturbation method results and found in close agreement. Zhifeng Liu et al. [12] studied differential transform simulation method to solve the free vibration problems of uniform Euler Bernoulli beam. They conducted mode experimental method to obtain experimental data for analysis by signal processing. From results they found modified differential transform method can achieve good values in predicting solution of sandwich problems. Prokic et al. [13] presented a numerical method for solution of free vibration of beams governed by a set of second-order ordinary differential equations with different boundary conditions. They demonstrated the accuracy of proposed method by comparing the calculated results with literature values. They showed that good accuracy values can be obtained for relatively small number of nodes. Ahmed [14] investigated the flexural vibration characteristics of curved sandwich beams using finite element displacement method. He studied various displacement models to investigate their effect on the natural frequencies of curved fixed-fixed sandwich beams. The effect of shear deformation on the first three modes of the fixed-fixed curved sandwich beams is very small was found by him.

II. MATHEMATICAL MODELLING

A solid elastic fixed free beam with uniform cross section of length (L), width (b), thickness (h), mass density (ρ) and modulus of elasticity (E) is considered for analysis. The equation of motion for the transverse vibration of a beam with uniform cross section and homogeneous material at any point x and time t is denoted by $w(x,t)$ and transverse force per unit length by $f(x,t)$ using Euler-Bernoulli theory and the rotation of the element is insignificant. Euler-Bernoulli theory is valid if the ratio between the length of the beam and its height is relatively large (more than 10). The effect of shear deformation and rotary inertia are ignored.

The differential equation of motion of the beam is [12]:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = f(x,t) \qquad (1)$$

Assuming the external excitation is zero for free vibration i.e., $f(x,t)=0$, the Eq. (2) can be rewritten as

$$\frac{\partial^4 w}{\partial x^4} + \frac{\rho A}{EI}\frac{\partial^2 w}{\partial t^2} = 0 \qquad (2)$$

Where, I is the area moment of inertia of the beam cross section, the mass per unit length is ρA, flexural rigidity is EI and beam finite element of length dx.

The solution of the Eq. (2) is solved by the method of separation of variable, one is depending on the position and the other is depending on time, as follows

$$w(x,t) = X(x)T(t) \qquad (3)$$

Where, $X(x)$ *and* $T(t)$ are independent of time t and position x, respectively.

Substituting Eq. (3) into Eq. (2)

$$\frac{X^{IV}}{X} = -\frac{\rho A}{EI}\frac{T^{II}}{T} \qquad (4)$$

Variables have been separated and each side of above equation must equal to a constant β^4.

$$\frac{X^{IV}}{X} = -\frac{\rho A}{EI}\frac{T^{II}}{T} = \beta^4 \qquad (5)$$

If the time variable 't' is separated from Eq. (5)

$$T^{II} + \omega^2 T = 0 \qquad (6)$$

$$\beta^2 = \omega\sqrt{\frac{\rho A}{EI}} \qquad (7)$$

The general solution of the Eq. (6) is

$$T(t) = C_5 \, Sin\,\omega t + C_6 \, Cos\,\omega t \qquad (8)$$

where C_5 and C_6 are constants.

Similarly if the position variable 'x' is separated from Eq. (5)

$$X^{IV} - \beta^4 X = 0 \qquad (9)$$

The general solution of the Eq. (9) is

$$w(x) = C_1 Cosh\beta_1 x + C_2 Sinh\beta_1 x + C_3 Cos\beta_2 x + C_4 Sin\beta_2 x \qquad (10)$$

where C_1 . . . . C_4 are constants, *Sinh and Cosh* are hyperbolic functions.

By multiplying Eq. (8) with Eq. (10) upon solving and substituting initial and boundary conditions, six combined constants are obtained.

The natural frequency of the beam is calculated from Eq. (7) as,

$$f_n = \frac{\omega}{2\pi} \qquad (11)$$

On substituting the fixed free boundary conditions in Eq. (10), the following equations are obtained and rewritten in matrix form as:

$$\begin{bmatrix} Sinh\beta l + Sin\beta l & Cosh\beta l + Cos\beta l \\ Cosh\beta l + Cos\beta l & Sinh\beta l - Sin\beta l \end{bmatrix}\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (12)$$

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The non-trivial solution of the determinant of the coefficient matrix is as follows:

$$Cos\beta l \, Cosh\beta l = -1 \tag{13}$$

which is the characteristic equation of a fixed free Euler-Bernoulli solid beam.

The Eigen values are obtained by calculating the first three roots of Eq. (13) using Newton-Raphson method.

$$\beta_n l = \left(\frac{2n-1}{2}\right)\pi \quad where\ n=1,2,3... \tag{14}$$

$$\omega_n = \left(\frac{2n-1}{2}\pi\right)^2 \frac{1}{l^2}\sqrt{\frac{EI}{\rho A}} \quad where\ n=1,2,3... \tag{15}$$

For a symmetric sandwich structure, thickness and material properties of all layers are same as shown in Figure 1.

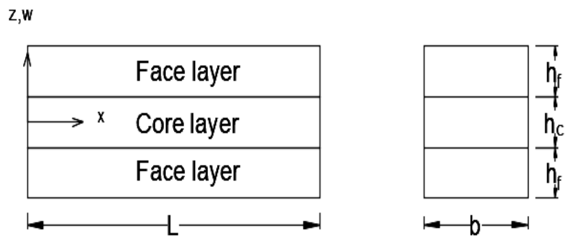

Fig. 1: Sandwich Beam

The flexural rigidity of sandwich structure with single core is given as [7]:

$$D = 2\frac{E_f b h_f^3}{12} + \frac{E_c b h_c^3}{12} + \frac{E_f b h_f}{2}(h_f + h_c)^2 \tag{16}$$

where, D is the bending stiffness (or) flexural rigidity,

E_f and E_c is the young's modulus of face material and core material respectively,

h_f and h_c are the thickness of face and core material respectively,

b is the width of structure,

h = Total beam height $(2h_f + h_c)$

Dynamic characteristics of a fixed free beam are expresses as [7]:

i. The equivalent mass,

$$m_e = b\,(2h_f\rho_f + h_c\rho_c) \tag{17}$$

ii. The equivalent stiffness, $K_e = \frac{3*D}{L^3}$ (18)

iii. The equivalent natural frequency,

$$f_n = \frac{1}{2\pi}\left[\frac{K_e}{(.25m_e)}\right]^{0.5} \tag{19}$$

The frequency of different modes can be calculated according to the following equation

$$f_i = \frac{(\beta_i)^2}{2\pi}\left[\frac{K_e}{(.25m_e)}\right]^{0.5} \tag{20}$$

Where $(\beta_i)^2$ is a constant depend on the boundary conditions and i = 1,2,3,..........,n are the frequency corresponds to each node.

III. RESULTS AND DISCUSSION

The materials considered for sandwich beam are Aluminium as face materials and natural rubber as core material is considered for free vibration analysis. The frequencies of various sandwich beams which are made with different face and core materials are calculated using Mat lab. Various investigations are carried out for free-vibration behaviors of rectangular cross-sectioned aluminium sand beams with different geometric characteristics under fixed free boundary condition.

Material properties of aluminium and polyurethane rigid foam are [1]:

Modulus of elasticity of face material (E_f)= 6.89×10^{10} N/m^2 ; modulus of elasticity (E_c)=0.00952 Gpa; thickness of faces (h_f) = 0.4527 mm ; thickness of core h_c = 12.7 mm ; density of core material (ρ_c)= 32.8 kg/m^3 ; density of face material (ρ_f)= 2680 kg/m^3 ; Width of beam b = 1.0099 mm; Length of the beam L = 0.9144 m.

Material properties of aluminium and natural rubber are [6]:

Modulus of elasticity of face material (E_f)= 6.89×10^{10} N/m^2 ; modulus of elasticity (E_c)=0.00154 Gpa; thickness of faces (h_f) =2 mm ; thickness of core h_c = 1 mm ; density of core material (ρ_c)= 950 kg/m^3 ; density of face material (ρ_f)= 2680 kg/m^3 ; Width of beam b = 30 mm; Length of the beam L = 300 mm.

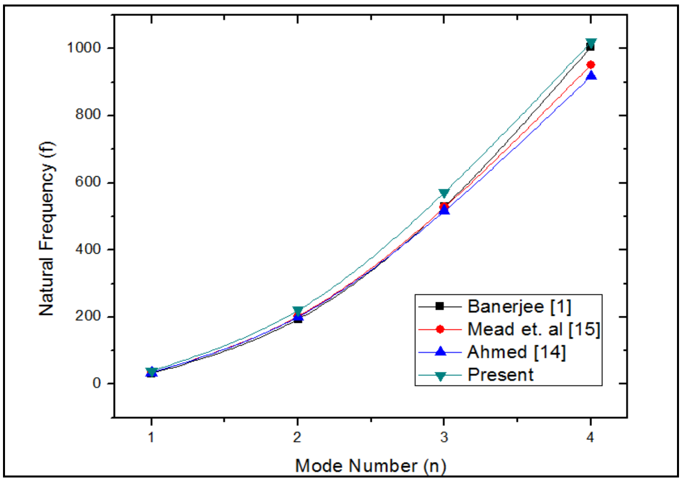

Fig.2: Natural frequencies of fixed free sandwich beam with face material as aluminium and core as polyurethane rigid foam

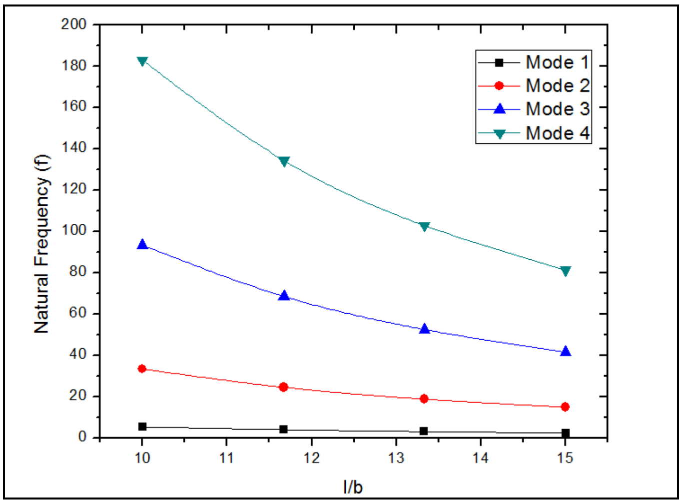

Fig.3: Natural frequencies of fixed free sandwich beam with face material as aluminium and core as natural rubber with respect to length to width ratio

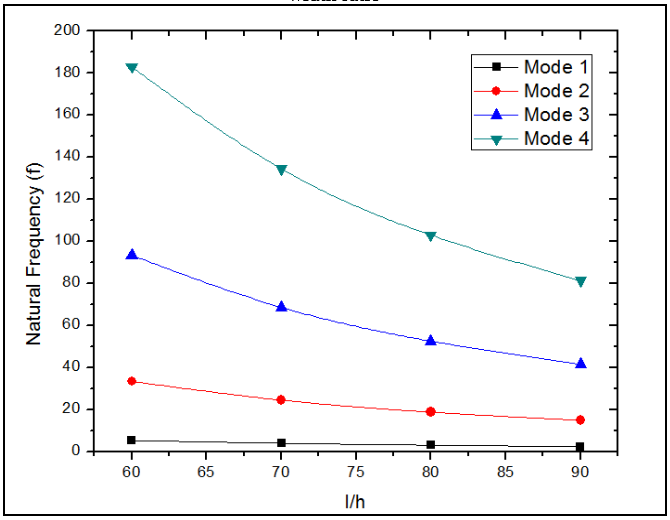

Fig.4: Natural frequencies of fixed free sandwich beam with face material as aluminium and core as natural rubber with respect to length to thickness ratio

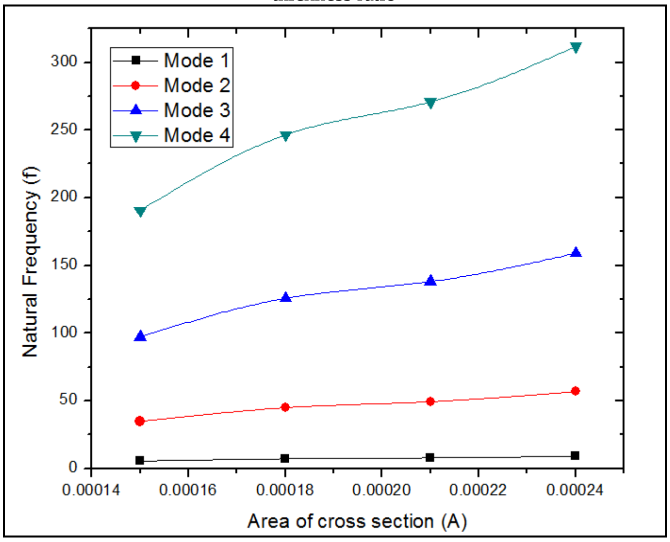

Fig.5: Natural frequencies of fixed free sandwich beam with face material as aluminium and core as natural rubber with respect to area of cross section

Natural frequencies of fixed free sandwich beam with face material as aluminium and core as polyurethane rigid foam is shown in Fig.2. The material properties of aluminium and polyurethane rigid foam are considered for free vibration behaviour of sandwich beam. From this figure, it is noticed that as mode number is increasing natural frequencies are increasing. It is due to non dimensional number increases for corresponding mode number increase. Fig. 3 represents the natural frequencies of a rectangular cross section sandwich beam with face as aluminium and core as natural rubber for various lengths to width ratios. From this figure, it is observed as l/b ratio increases natural frequency decreases for various mode numbers. This may be because of natural frequency is inversely proportional to the length parameter. Natural frequencies of a rectangular cross section sandwich beam with face as aluminium and core as natural rubber against length to thickness ratios is shown in Fig. 4. From this figure, it is noticed as l/h ratio increases natural frequency decreases for various mode numbers. This may be because of natural frequency is directly proportional to thickness and inversely proportional to the length parameter. Fig. 5 gives the relationship between natural frequencies to various areas of cross section of sandwich beam. The material properties of aluminium and natural rubber are considered for studying the free vibration behaviour of sandwich beam. It is seen from this figure as the cross section of sandwich beam varies natural frequency increases due to thickness increases.

IV. CONCLUSIONS

In this work various studies are performed to investigate the free-vibration behaviour of rectangular cross-sectioned aluminium sandwich beams with different geometric characteristics under fixed free boundary condition. The conventional Euler-Bernoulli beam theory was considered for modelling the various sandwich beams, i.e., aluminium, polyurethane rigid foam as face and core material respectively in first sandwich beam and aluminium , natural rubber as face and core material respectively in second sandwich beam. The natural frequencies are obtained and discussed for the first four modes including the effects of length to thickness, length to width and area of cross section of the sandwich beam with fixed free boundary condition. The analytical results obtained using Mat lab are compared with literature values and found in good agreement.

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