

Free Vibration Analysis of Laminated Sandwich Spherical Shell With Viscoelastic Core by using Ultraspherical Polynomial

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Abstract-In this paper, the free vibration behavior of laminated sandwich spherical shell with viscoelastic core are presented. Based on the first order shear deformation theory (FSDT), the theoretical formulations for analyzing the free vibration of the laminated sandwich spherical shell are established. The displacement components of the viscoelastic core are expressed as those of base and constraining layers by using continuity condition. The displacement fields of the laminated sandwich spherical shell are expanded by the Ultraspherical polynomials in meridional direction and Fourier series in circumferential directions. The accuracy and reliability of the presented method on the free vibration of laminated sandwich spherical shell with viscoelastic core are verified by comparing to the vibrational analysis results of published literatures and finite element software ABAQUS. Finally, the effects of some parameters on the frequency parameters and modal loss factors of the laminated sandwich spherical shell are presented, which contributes to the preliminary design of the laminated sandwich spherical shell with viscoelastic core.

Key Words: Laminated sandwich spherical shell, Ultraspherical polynomial, viscoelastic core, free vibration, Numerical solution

1. INTRODUCTION

Composite shells are widely used in various fields such as aerospace, ships and civil engineering due to their excellent properties. In particular, sandwich shell structures are widely used in various engineering applications due to their superior characteristics such as high vibration and noise reduction ability caused by the large shear deformation of the viscoelastic core [1, 2]. Therefore, the study on the dynamic characteristics of the sandwich structures has attracted a lot of interest of many researchers [3, 4].

Yang et al. [5] studied the vibration and damping characteristics of the sandwich conical shells and annular plates with arbitrary boundary conditions including classical and elastic ones by using a simple and efficient modified Fourier solution. Wang et al. [6] presented a semi-analytical method for the free vibration analysis of the functionally graded (FG) sandwich doubly-curved panels and shells of revolution with arbitrary boundary conditions. Bardell et al. [7] presented the vibration study of a general three-layer conical sandwich panel based on the h-p version of the finite element method. In their study, the h-p finite element formulation of sandwich panel was derived based on a set of trigonometric assumed displacement functions. Singha et al. [8] investigated the free vibration behavior of rotating

pretwisted sandwich conical shell panels with functionally graded graphene-reinforced composite (FG-GRC) face sheets and homogenous core using finite element method in conjunction with HSDT. Sofiyev and Osmancelebioglu [9] demonstrated the effectiveness of functionally graded coatings in the vibration of sandwich truncated conical shells. In their study, the governing equations were established by using FSDT and Donnell kinematics assumptions. Jin et al. [10] developed an accurate solution for the vibration and damping characteristics of a three-layered passive constrained layer damping (PCLD) cylindrical shell with general elastically restrained boundaries by means of the modified Fourier-Ritz method in conjunction with Donnell shell assumptions and linear viscoelastic theory. Sahu et al. [11] conducted the free vibration study of doubly curved sandwich shell panels having a core of viscoelastic material, constrained by a functionally graded material (FGM) layer by using finite element method (FEM) in framework of FSDT. Sobhani et al. [12] implemented the free vibration analysis of sandwich composite joined conical-cylindrical-conical shells made of three layers including two face sheets and one core using FSDT and Donnell shell assumption.

Meanwhile, researchers proposed different numerical methods such as FEM [11,13], differential quadrature

method [14-16], pb-2 Ritz method [17], Non-Uniform Rational B-Splines (NURBS) method [18-21], spectral-Tchebychev solution technique [22,23], dynamic stiffness method [24-26], meshfree method [27,28], Haar wavelet method(HWM) [29-32] for the dynamic analysis of composite shells and plates.

This study focuses on the free vibration and damping characteristics of laminated sandwich spherical shell with viscoelastic core by the Ritz method with Ultraspherical polynomial as displacement function. The theoretical formulations for vibration analysis are established based on the FSDT. Using the continuity condition in interface between the layers, the displacement components of the core are replaced by those of base and constraining layers. The displacement component of the sandwich shell are approximated by using Fourier series in conjunction with Ultraspherical polynomials. The accuracy and reliability of the proposed method are verified through the comparison with the results of published literature and finite element software ABAQUS. Finally, the effects of the parameters such as geometric dimension, boundary condition, lamination structures and material property on the vibration and modal loss factor of the laminated sandwich spherical shell with viscoelastic core are presented through some numerical examples.

2. THEORETICAL FORMULATIONS

2.1. Model Description

Fig. 1 shows the diagram of a laminated sandwich spherical shell which is composed of the laminated base layer and constraining layer, the viscoelastic core. Orthogonal curvilinear coordinate systems (φ, θ, z) are set on the middle surfaces of each layer. R_ξ and h_ξ ($\xi=s, v, c$) mean the radius and thickness of each layer, in which the subscripts $s, v,$ and c denote the base layer, core layer and constraining layer, respectively.

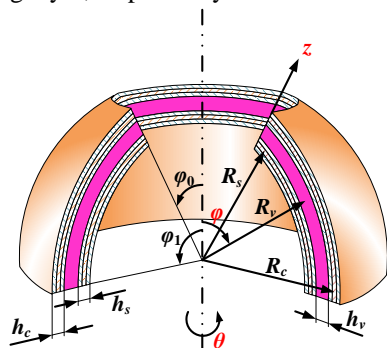


Fig. 1. Geometry and coordination of laminated sandwich spherical shell with viscoelastic core.

2.2 Governing equation

In this paper, the displacements at any point of each

layer are derived based on FSDT as follows [1, 2, 10];

$$\begin{aligned} u_\xi(\varphi, \theta, z, t) &= u_\xi^0(\varphi, \theta, t) + z\phi_{\varphi, \xi}(\varphi, \theta, t), \\ v_\xi(\varphi, \theta, z, t) &= v_\xi^0(\varphi, \theta, t) + z\phi_{\theta, \xi}(\varphi, \theta, t), \\ w_\xi(\varphi, \theta, z, t) &= w^0(\varphi, \theta, t) \end{aligned} \quad (1)$$

where u_ξ^0, v_ξ^0 and w^0 denote the displacements in middle surface of each layer in the φ, θ and z directions, respectively.

By using the displacement continuity between layers, the displacements of the core layer can be replaced by those of the base and constraining layers:

$$\begin{aligned} u_v &= \frac{1}{2} \left(u_s + u_c + \frac{h_s}{2} \psi_{\varphi s} - \frac{h_c}{2} \psi_{\varphi c} \right), \\ \psi_{\varphi v} &= \frac{1}{h_v} \left(u_c - u_s - \frac{h_s}{2} \psi_{\varphi s} - \frac{h_c}{2} \psi_{\varphi c} \right), \\ v_v &= \frac{1}{2} \left(v_s + v_c + \frac{h_s}{2} \psi_{\theta s} - \frac{h_c}{2} \psi_{\theta c} \right), \\ \psi_{\theta v} &= \frac{1}{h_v} \left(v_c - v_s - \frac{h_s}{2} \psi_{\theta s} - \frac{h_c}{2} \psi_{\theta c} \right) \end{aligned} \quad (2)$$

The strains at any point of each layer can be expressed as [1, 2]:

$$\begin{aligned} \varepsilon_{\varphi, \xi} &= \varepsilon_{\varphi, \xi}^0 + z\chi_{\varphi, \xi}, \quad \varepsilon_{\theta, \xi} = \varepsilon_{\theta, \xi}^0 + z\chi_{\theta, \xi} \\ \gamma_{\varphi\theta, \xi} &= \gamma_{\varphi\theta, \xi}^0 + z\chi_{\varphi\theta, \xi}, \quad \gamma_{\varphi z, \xi} = \gamma_{\varphi z, \xi}^0, \quad \gamma_{\theta z, \xi} = \gamma_{\theta z, \xi}^0 \end{aligned} \quad (3)$$

where $\varepsilon_{\varphi, \xi}, \varepsilon_{\theta, \xi}$ and $\gamma_{\varphi\theta, \xi}$ indicate the normal and shear strains at any point of each layer, $\gamma_{\varphi z, \xi}$ and $\gamma_{\theta z, \xi}$ denote transverse shear strains. In addition, $\varepsilon_{\varphi, \xi}^0, \varepsilon_{\theta, \xi}^0$ and $\gamma_{\varphi\theta, \xi}^0, \gamma_{\varphi z, \xi}^0, \gamma_{\theta z, \xi}^0$ denote the strains in the middle surface of each layer, $\chi_{\varphi, \xi}, \chi_{\theta, \xi}$ and $\chi_{\varphi\theta, \xi}$ denote the curvature and twist changes with respect to middle surface, respectively.

According to the elastic theory, the relationships between strain and displacement in the middle surface are expressed as:

$$\begin{aligned} \varepsilon_{\varphi, \xi}^0 &= \frac{1}{R} \left(\frac{\partial u_\xi^0}{\partial \varphi} + w^0 \right), \\ \varepsilon_{\theta, \xi}^0 &= \frac{1}{R} \left(\frac{u_\xi^0}{\tan \varphi} + \frac{1}{\sin \varphi} \frac{\partial v_\xi^0}{\partial \theta} + w^0 \right) \\ \gamma_{\varphi\theta, \xi}^0 &= \frac{1}{R} \left(\frac{\partial v_\xi^0}{\partial \varphi} + \frac{1}{\sin \varphi} \frac{\partial u_\xi^0}{\partial \theta} - \frac{v_\xi^0}{\tan \varphi} \right), \\ \gamma_{\varphi z, \xi}^0 &= \frac{1}{R} \frac{\partial w_\xi^0}{\partial \varphi} - \frac{u_\xi^0}{R} + \phi_{\varphi, \xi}, \\ \gamma_{\theta z, \xi}^0 &= \frac{1}{R \sin \varphi} \frac{\partial w^0}{\partial \theta} - \frac{v_\xi^0}{R} + \phi_{\theta, \xi} \\ \chi_{\varphi, \xi} &= \frac{1}{R} \frac{\partial \phi_{\varphi, \xi}}{\partial \varphi}, \quad \chi_{\theta, \xi} = \frac{1}{R} \left(\frac{\partial \phi_{\theta, \xi}}{\sin \varphi \partial \theta} + \frac{\phi_{\varphi, \xi}}{\tan \varphi} \right), \\ \chi_{\varphi\theta, \xi} &= \frac{1}{R} \left(\frac{\partial \phi_{\theta, \xi}}{\partial \varphi} + \frac{\partial \phi_{\varphi, \xi}}{\sin \varphi \partial \theta} - \frac{\phi_{\theta, \xi}}{\tan \varphi} \right) \end{aligned} \quad (4)$$

The internal force and moment components of each layer are expressed by the strain components in the following relationship [1, 2]:

$$N_{\xi} = D_{\xi} \epsilon_{\xi}, \quad (5)$$

where N_{ξ} is the internal force and moment vector and ϵ_{ξ} is the strain vector, and they are expressed as follows, respectively.

$$N_{\xi} = [N_{\varphi, \xi}, N_{\theta, \xi}, N_{\varphi\theta, \xi}, M_{\varphi, \xi}, M_{\theta, \xi}, M_{\varphi\theta, \xi}, Q_{\varphi, \xi}, Q_{\theta, \xi}] \quad (6)$$

$$\epsilon_{\xi} = [\epsilon_{\varphi, \xi}^0, \epsilon_{\theta, \xi}^0, \gamma_{\varphi\theta, \xi}^0, \chi_{\varphi, \xi}, \chi_{\theta, \xi}, \chi_{\varphi\theta, \xi}, \gamma_{\varphi z, \xi}^0, \gamma_{\theta z, \xi}^0] \quad (7)$$

The stiffness coefficients matrix D_{ξ} ($\xi=s, c$) for the laminated base and constraining layer is expressed as follows.

$$D_{\xi} = \begin{bmatrix} A_{\xi 11} & A_{\xi 12} & A_{\xi 16} & B_{\xi 11} & B_{\xi 12} & B_{\xi 16} & 0 & 0 \\ A_{\xi 12} & A_{\xi 22} & A_{\xi 26} & B_{\xi 12} & B_{\xi 22} & B_{\xi 26} & 0 & 0 \\ A_{\xi 16} & A_{\xi 26} & A_{\xi 66} & B_{\xi 16} & B_{\xi 26} & B_{\xi 66} & 0 & 0 \\ B_{\xi 11} & B_{\xi 12} & B_{\xi 16} & D_{\xi 11} & D_{\xi 12} & D_{\xi 16} & 0 & 0 \\ B_{\xi 12} & B_{\xi 22} & B_{\xi 26} & D_{\xi 12} & D_{\xi 22} & D_{\xi 26} & 0 & 0 \\ B_{\xi 16} & B_{\xi 26} & B_{\xi 66} & D_{\xi 16} & D_{\xi 26} & D_{\xi 66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{\xi 44} & A_{\xi 45} \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{\xi 45} & A_{\xi 55} \end{bmatrix} \quad (8)$$

where $A_{\xi, ij}$, $B_{\xi, ij}$ and $D_{\xi, ij}$ are the stiffness coefficients of laminated composite material of base and constraining layers [1, 2]. In addition, stiffness coefficients matrix D_v for the viscoelastic core D_v is expressed as follows [5].

$$D_v = \begin{bmatrix} A_{v11} & A_{v12} & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{v12} & A_{v22} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{v66} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{v11} & D_{v12} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{v12} & D_{v22} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{v66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{v44} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{v55} \end{bmatrix} \quad (9)$$

The stiffness coefficients of the viscoelastic core are expressed as follows.

$$A_{v,ij} = h_v Q_{v,ij}, \quad D_{v,ij} = \frac{h_v^3}{12} Q_{v,ij} \quad i, j = 1 \sim 6 \quad (10)$$

The reduced stiffness coefficients $Q_{v,ij}$ of the isotropic material in the core layer are as

$$Q_{v11} = Q_{v22} = \frac{E_v}{1 - \mu_v^2}, \quad Q_{12} = \frac{\mu_v E_v}{1 - \mu_v^2}, \quad (11)$$

$$Q_{v44} = Q_{v55} = Q_{v66} = \frac{E_v}{2(1 + \mu_v)}$$

where E_v and μ_v are the Young's modulus and Poisson's ratios of the core material, respectively.

In the case of viscoelastic core, the Young's modulus E_v of the viscoelastic material is composed of real and imaginary parts.

$$E_v = E_r + iE_i \quad (12)$$

where E_r and E_i denote the real part and imaginary part of the complex Young's modulus of the viscoelastic material layer, respectively.

The strain energies of the laminated sandwich spherical shell can be described as follows.

$$U = U_s + U_v + U_c$$

$$= \frac{R_s^2}{2} \int_0^{2\pi} \int_{\varphi_0}^{\varphi_1} \left[\begin{aligned} & N_{\varphi s} \epsilon_{\varphi s}^0 + N_{\theta s} \epsilon_{\theta s}^0 + N_{\varphi\theta s} \gamma_{\varphi\theta s}^0 \\ & + M_{\varphi s} \chi_{\varphi s} + M_{\theta s} \chi_{\theta s} + M_{\varphi\theta s} \chi_{\varphi\theta s} \\ & + Q_{\theta s} \gamma_{\theta z s}^0 + Q_{\varphi s} \gamma_{\varphi z s}^0 \end{aligned} \right] \sin \varphi d\varphi d\theta$$

$$+ \frac{R_v^2}{2} \int_0^{2\pi} \int_{\varphi_0}^{\varphi_1} \left[\begin{aligned} & N_{\varphi v} \epsilon_{\varphi v}^0 + N_{\theta v} \epsilon_{\theta v}^0 + N_{\varphi\theta v} \gamma_{\varphi\theta v}^0 \\ & + M_{\varphi v} \chi_{\varphi v} + M_{\theta v} \chi_{\theta v} + M_{\varphi\theta v} \chi_{\varphi\theta v} \\ & + Q_{\theta v} \gamma_{\theta z v}^0 + Q_{\varphi v} \gamma_{\varphi z v}^0 \end{aligned} \right] \sin \varphi d\varphi d\theta \quad (13)$$

$$+ \frac{R_c^2}{2} \int_0^{2\pi} \int_{\varphi_0}^{\varphi_1} \left[\begin{aligned} & N_{\varphi c} \epsilon_{\varphi c}^0 + N_{\theta c} \epsilon_{\theta c}^0 + N_{\varphi\theta c} \gamma_{\varphi\theta c}^0 \\ & + M_{\varphi c} \chi_{\varphi c} + M_{\theta c} \chi_{\theta c} + M_{\varphi\theta c} \chi_{\varphi\theta c} \\ & + Q_{\theta c} \gamma_{\theta z c}^0 + Q_{\varphi c} \gamma_{\varphi z c}^0 \end{aligned} \right] \sin \varphi d\varphi d\theta$$

The kinetic energies of the sandwich shell can be given as

$$T = T_s + T_v + T_c$$

$$= \frac{R_s^2}{2} \int_0^{2\pi} \int_{\varphi_0}^{\varphi_1} \left[\rho_s (\dot{u}_s^2 + \dot{v}_s^2 + \dot{w}_s^2) \right] \sin \varphi d\varphi d\theta$$

$$+ \frac{R_v^2}{2} \int_0^{2\pi} \int_{\varphi_0}^{\varphi_1} \left[\rho_v (\dot{u}_v^2 + \dot{v}_v^2 + \dot{w}_v^2) \right] \sin \varphi d\varphi d\theta \quad (14)$$

$$+ \frac{R_c^2}{2} \int_0^{2\pi} \int_{\varphi_0}^{\varphi_1} \left[\rho_c (\dot{u}_c^2 + \dot{v}_c^2 + \dot{w}_c^2) \right] \sin \varphi d\varphi d\theta$$

The elastic energies stored in distributed springs of each layers can be given as

$$U_{BC} = \frac{R_s \sin \varphi_0}{2} \int_0^{2\pi} \left[k_{u0} u_s^2 + k_{v0} v_s^2 + k_{w0} w_s^2 + k_{\varphi 0} \phi_{\varphi s}^2 + k_{\theta 0} \phi_{\theta s}^2 \right] d\theta$$

$$+ \frac{R_s \sin \varphi_1}{2} \int_0^{2\pi} \left[k_{u1} u_s^2 + k_{v1} v_s^2 + k_{w1} w_s^2 + k_{\varphi 1} \phi_{\varphi s}^2 + k_{\theta 1} \phi_{\theta s}^2 \right] d\theta$$

$$+ \frac{R_v \sin \varphi_0}{2} \int_0^{2\pi} \left[k_{u0} u_v^2 + k_{v0} v_v^2 + k_{w0} w_v^2 + k_{\varphi 0} \phi_{\varphi v}^2 + k_{\theta 0} \phi_{\theta v}^2 \right] d\theta$$

$$+ \frac{R_v \sin \varphi_1}{2} \int_0^{2\pi} \left[k_{u1} u_v^2 + k_{v1} v_v^2 + k_{w1} w_v^2 + k_{\varphi 1} \phi_{\varphi v}^2 + k_{\theta 1} \phi_{\theta v}^2 \right] d\theta \quad (15)$$

$$+ \frac{R_c \sin \varphi_0}{2} \int_0^{2\pi} \left[k_{u0} u_c^2 + k_{v0} v_c^2 + k_{w0} w_c^2 + k_{\varphi 0} \phi_{\varphi c}^2 + k_{\theta 0} \phi_{\theta c}^2 \right] d\theta$$

$$+ \frac{R_c \sin \varphi_1}{2} \int_0^{2\pi} \left[k_{u1} u_c^2 + k_{v1} v_c^2 + k_{w1} w_c^2 + k_{\varphi 1} \phi_{\varphi c}^2 + k_{\theta 1} \phi_{\theta c}^2 \right] d\theta$$

By introducing the Ultraspherical polynomial, the displacement components can be expanded as follows.

$$u_{\xi}^0 = \sum_{m=1}^N \sum_{n=0}^M U_{mn}^{\xi} \psi_m(\varphi) \cos(n\theta) e^{i\omega t}$$

$$v_{\xi}^0 = \sum_{m=1}^N \sum_{n=0}^M V_{mn}^{\xi} \psi_m(\varphi) \sin(n\theta) e^{i\omega t}$$

$$\phi_{\varphi \xi} = \sum_{m=1}^N \sum_{n=0}^M \Phi_{\varphi mn}^{\xi} \psi_m(\varphi) \cos(n\theta) e^{i\omega t} \quad (16)$$

$$\phi_{\theta \xi} = \sum_{m=1}^N \sum_{n=0}^M \Phi_{\theta mn}^{\xi} \psi_m(\varphi) \cos(n\theta) e^{i\omega t}$$

$$w = \sum_{m=1}^N \sum_{n=0}^M W_{mn} \psi_m(\varphi) \cos(n\theta) e^{i\omega t}$$

where $\psi_m(\varphi)$ is the Ultraspherical polynomial of degree m for the displacement, M and N denote the maximum order of Ultraspherical polynomial and circumferential wave number, respectively. The unknown coefficients $U_{mn}^s, V_{mn}^s, \Phi_{\varphi mn}^s, \Phi_{\theta mn}^s$ and W_{mn} are variables to be determined. And ω is the vibration angular frequency, and the non-negative integer n is the circumferential wave number.

The total Lagrangian energy function of the laminated sandwich spherical shell can be written as follows.

$$L = T - U - U_{BC} \quad (17)$$

Minimizing the above Lagrangian energy function with respect to the unknown coefficients,

$$\frac{\partial L}{\partial q} = 0, \quad (18)$$

$$q = U_{mn}^s, V_{mn}^s, \Phi_{\varphi mn}^s, \Phi_{\theta mn}^s, U_{mn}^c, V_{mn}^c, \Phi_{\varphi mn}^c, \Phi_{\theta mn}^c, W_{mn}$$

Substituting Eq. (17) into Eq.(18), following governing equations are obtained.

$$(\mathbf{K} - \omega^2 \mathbf{M}) \mathbf{A}_s = \mathbf{0} \quad (19)$$

where \mathbf{U}_s is the displacement vector, \mathbf{K} and \mathbf{M} are the stiffness and mass matrices.

If the core layer is made of viscoelastic material, the complex eigenvalues will be obtained from the above Eq. (19). The real part of the natural frequency ω of the laminated sandwich shell and the modal loss factor η are defined as follows.

$$f = \frac{\sqrt{\text{Re}(\omega^2)}}{2\pi} \quad (20)$$

$$\eta = \frac{\text{Im}(\omega^2)}{\text{Re}(\omega^2)} \quad (21)$$

3. NUMERICAL RESULTS AND DISCUSSION

In this section, some numerical examples are presented to verify the accuracy and reliability of the proposed method for the vibration analysis of laminated sandwich spherical shell. Unless otherwise stated, the material properties of the base and constraining layers of sandwich shell under consideration are selected to following: $E_{i1}=150\text{GPa}$, $E_{i2}=10\text{GPa}$, $G_{i12}=G_{i13}=6\text{GPa}$, $G_{i23}=5\text{GPa}$, $\mu_{i12}=0.25$ and $\rho_i=1500\text{kg/m}^3$ ($i=s, c$).

3.1 Convergence and verification

From the theoretical formulation, it can be seen that the solution accuracy of the proposed method is determined according to the maximum order and parameters of Ultraspherical polynomial. Therefore, it needs the convergence study to determine these parameters. Fig. 2 shows the variation characteristics of frequency parameter

of the laminated sandwich spherical shell with viscoelastic core according to the increase of Ultraspherical polynomial maximum order M .

The material properties of the viscoelastic core and the geometries of the laminated sandwich spherical shell are given as: $E_v=(23+i7.82)\text{MPa}$, $\mu_v=0.34$, $\rho_v=1340\text{kg/m}^3$, $R_s=1\text{m}$, $h_s=h_c=h_v=0.02\text{m}$, $\varphi_0=\pi/6$, $\varphi_1=\pi/2$. The longitudinal mode number of the considered frequencies is as $m=1$. As shown in Fig. 2, the present method can be ensured stable convergence for predicting the frequency parameters and modal loss factors of the laminated sandwich spherical shell with viscoelastic core.

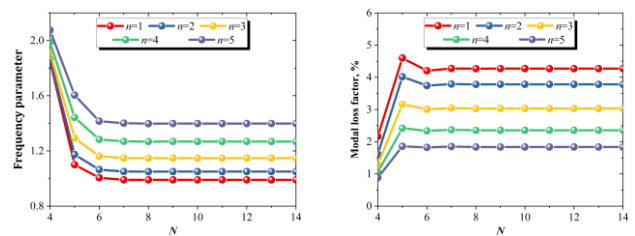


Fig.2. Convergence of frequency parameters and modal loss factor according to the increase of Ultraspherical polynomial maximum order M .

The boundary conditions are generalized by the introduction of a virtual stiffness technique, and the type of boundary conditions is selected according to the spring stiffness. The effects of spring stiffness values of elastic boundary on the frequency parameters of laminated sandwich spherical shell with viscoelastic core ($[90^\circ/\text{core}/90^\circ]$) are investigated in Fig. 3. The material properties of the viscoelastic core and the geometries of the sandwich shell are same as in Fig. 2. The left bound of the shell is chosen as elastic boundary and the right bound is completely clamped. It can be seen from Fig. 3 that frequency parameters increase rapidly within the stiffness value range of 10^6 to 10^{12} .

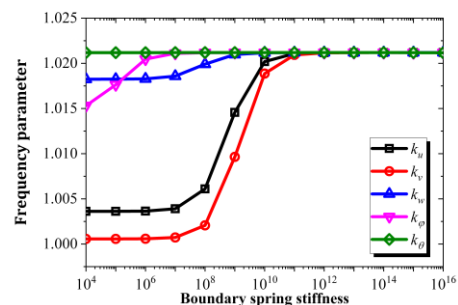


Fig.3. Convergence of frequency parameters for a laminated sandwich spherical shell with viscoelastic core according to different boundary spring stiffness.

Based on the above study, the spring stiffness values for different boundary conditions considered in this study

are set as shown in Table 1, in which the symbols F, C, S and SD mean free, clamped, simply supported and shear diaphragm boundary conditions, respectively.

Table 1. Stiffness values for boundary springs.

B.Cs	Boundary spring stiffness value				
	k_u	k_v	k_w	k_ϕ	k_θ
F	0	0	0	0	0
C	10^{14}	10^{14}	10^{14}	10^{14}	10^{14}
S	10^{14}	10^{14}	10^{14}	0	10^{14}
SD	0	10^{14}	10^{14}	0	0

Next, the free vibration results of a laminated sandwich spherical shell are compared with those of literature to verify the accuracy of the proposed method. The material properties of isotropic core and the geometric dimensions of sandwich shell are given as: $E_v=70\text{GPa}$, $\mu_v=0.3$, $\rho_r=2700\text{kg/m}^3$, $R_s=1\text{m}$, $h_s=h_c=0.03\text{m}$, $h_v=0.02\text{m}$, $\phi_0=\pi/12$ and $\phi_1=\pi/2$. The frequencies by FEM are calculated using the finite element software ABAQUS, in which S4R elements are used for analysis of the spherical shell. As observed from Tables 2, the frequency results obtained by the proposed method agree well with those of the literature and ABAQUS.

Table 2. Comparison of first six natural frequencies for a laminated sandwich spherical shell with isotropic core.

B.Cs	ω	Lamination					
		[0°/90°/0°/core/0°/90°/0°]			[0°/45°/0°/core/0°/45°/0°]		
		FEM	Present	Diff,%	FEM	Present	Diff,%
C-C	1	862.30	860.34	0.2273	904.98	901.99	0.3304
	2	872.93	871.36	0.1799	906.75	903.04	0.4092
	3	911.26	909.89	0.1503	964.00	962.23	0.1836
	4	976.42	975.61	0.0830	1040.7	1039.8	0.0865
	5	1037.4	1034.3	0.2988	1056.5	1054.0	0.2366
	6	1037.6	1034.4	0.3084	1082.4	1080.1	0.2125
F-C	1	438.84	438.42	0.0957	417.21	416.95	0.0623
	2	446.10	448.07	-0.4416	483.95	483.48	0.0971
	3	779.14	777.93	0.1553	666.92	665.97	0.1424
	4	798.10	797.71	0.0489	754.71	753.18	0.2027
	5	822.09	822.09	0.0000	801.92	800.83	0.1359
	6	850.77	851.75	-0.1152	898.80	896.98	0.2025

3.2 Parametric study

Based on the verification study of the proposed method, the effect of some parameters on the frequency and the modal loss factor of the laminated sandwich spherical shell with viscoelastic core are investigated. First, the effect of lamination schemes of base and constraining layers on the frequency parameter $\Omega=\omega R_s(\rho_s/E_{s2})^{0.5}$ of a laminated sandwich spherical shell with viscoelastic core is shown in Table 3. The material properties of the viscoelastic core are

same as in Fig. 2 and the geometric dimensions of the shell are given as: $R_s=1\text{m}$, $h_s=h_c=2h_v=0.02\text{m}$, $\phi_0=\pi/10$ and $\phi_1=\pi/2$.

Table 3. Frequency parameters for a laminated sandwich spherical shell with various lamination schemes.

Lamination	n	B.Cs				
		C-C	C-S	S-S	F-C	F-S
[0°/core/0°]	1	1.0875	1.0033	0.9557	0.6697	0.6364
	2	1.1688	1.0960	1.0644	0.3641	0.3567
	3	1.3029	1.2383	1.2287	0.5921	0.5922
	4	1.4532	1.3883	1.3886	0.9195	0.9190
[45°/core/45°]	1	1.0519	1.0135	0.9903	0.7642	0.7314
	2	1.0993	1.0658	1.0498	0.4415	0.4336
	3	1.1621	1.1319	1.1216	0.7838	0.7814
	4	1.2469	1.2180	1.2114	1.1157	1.1053
[0°/90°/core/0°/90°]	1	1.5973	1.5889	1.5836	0.7710	0.7684
	2	1.4163	1.4020	1.3765	0.4052	0.4033
	3	1.4146	1.3956	1.3786	0.7658	0.7658
	4	1.4771	1.4476	1.4460	1.2187	1.2177
[0°/45°/core/0°/45°]	1	1.1308	1.0917	1.0848	0.8276	0.7750
	2	1.2034	1.1711	1.1651	0.4081	0.3948
	3	1.2926	1.2644	1.2562	0.7137	0.7136
	4	1.3944	1.3674	1.3595	1.1054	1.1037

In addition, the effect of fiber angle α of base and constraining layers on the dimensionless frequency and modal loss factor of a laminated sandwich spherical shell with viscoelastic core ([0°/ α /0°/core/0°/ α /0°]) is illustrated in Fig. 4. The material properties of the sandwich shell are same as in Table 4, and the geometric dimensions are given as: $R_s=1\text{m}$, $h_s=h_c=0.03\text{m}$, $h_v=0.02\text{m}$, $\phi_0=\pi/12$ and $\phi_1=\pi/2$. As shown in Fig. 4, the curves of all frequencies are symmetric with respect to $\theta=90^\circ$. In addition, the dimensionless frequencies of the sandwich spherical shells are minimum at $\alpha=0^\circ$, while the modal loss factors are maximum at $\alpha=0^\circ$ or $\alpha=90^\circ$.

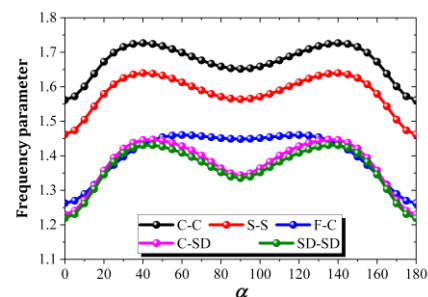


Fig.4. Variation of frequency parameters for a laminated sandwich spherical shell with various fiber angles.

Subsequently, the effect of thickness of viscoelastic layer on the dimensionless frequencies and modal loss factors of the sandwich shell is investigated. Table 4 shows the frequency parameters of a sandwich [0°/core/0°] spherical shell with various thickness ratio h_v/h ($h=h_s+h_c+h_v$). The total thickness of the shell is $h=0.1\text{m}$, and

the thicknesses of the base layer and constraining layer are assumed equal. The material properties are same as in Fig. 2, and the other geometric dimensions of the sandwich shell are same as in given as: $R_s=1m$, $\varphi_0=\pi/12$ and $\varphi_1=\pi/2$. It is obvious that thickness ratio has a remarkable influence on the Frequency parameters of the sandwich spherical shell.

Table. 4. Frequency parameters for a sandwich $[0^\circ/\text{core}/0^\circ]$ spherical shell with various thickness ratios.

h_v/h	n	B.Cs.				
		C-C	C-S	S-S	F-C	F-S
0.1	1	1.5136	1.3332	1.2340	0.7271	0.6708
	2	1.6137	1.4391	1.3583	0.5489	0.5363
	3	1.7775	1.6137	1.5601	1.1169	1.1102
	4	1.9740	1.8182	1.7890	1.6510	1.5923
0.3	1	1.1914	1.0604	0.9880	0.6327	0.5925
	2	1.2768	1.1548	1.0995	0.4318	0.4231
	3	1.4176	1.3051	1.2740	0.8676	0.8634
	4	1.5810	1.4736	1.4615	1.2934	1.2637
0.5	1	0.8807	0.8051	0.7633	0.5265	0.5004
	2	0.9551	0.8881	0.8607	0.3251	0.3186
	3	1.0742	1.0134	1.0045	0.6320	0.6313
	4	1.2048	1.1435	1.1433	0.9562	0.9515

In Fig. 5, the variations of frequency parameters of laminated sandwich spherical shell according to the circumferential wave number are investigated. The geometric dimensions of the shells are given as: $R_s=1m$, $h_s=h_c=0.02m$, $h_v=0.01m$, $\varphi_0=\pi/6$ and $\varphi_1=\pi/2$. It is obvious from the Fig. 5 that the frequency parameters of the laminated sandwich spherical shell with viscoelastic core decreases as the circumferential wave number increases, regardless of lamination structure. For the sandwich shell with angle-ply lamination structure of base and constraining layers, the frequency parameters increases as the circumferential wave number increases. However, for the sandwich shell with cross-ply lamination structure of base and constraining layers, the lowest frequencies of the shell exists in $n=3$.

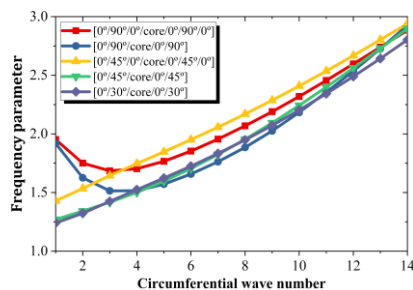


Fig. 5. Variation of frequency parameters for a laminated sandwich spherical shell with viscoelastic core according to circumferential wave number.

4. CONCLUSION

In this study, a method for analyzing the vibration characteristics of the laminated sandwich spherical shell with viscoelastic core using an Ultraspherical polynomial as a displacement function is presented. The theoretical formulations of the structural model are established by using the energy principle in framework of FSDT. The energy of the sandwich shell is composed of that of base layer, core layer and constraining layer. The displacement components at any point of the sandwich shell are expanded by the Ultraspherical polynomial in the meridional direction and Fourier series in the circumferential direction. The continuity condition is applied for replacing the displacements of the core layer by those of base and constraining layers. Numerical examples for free vibration and damping analyses of laminated sandwich spherical shell with viscoelastic core are presented to verify the reliability and accuracy of the presented method. First, the free vibration analysis results of the laminated sandwich spherical shell with viscoelastic core obtained by the proposed method are compared with those of published literature and finite element software ABAQUS. Finally, the effects of several parameters such as geometric dimension, material properties, lamination scheme of base and constraining layers and boundary condition on the frequency parameter and modal loss factor of the laminated sandwich spherical shell with viscoelastic core are investigated.

ACKNOWLEDGEMENT

The author would like to extend their gratitude towards Pyongyang University of Mechanical Engineering of DPRK. In addition, the authors would like to thank the anonymous Editors and reviewers for carefully reading the paper and their very valuable comments.

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