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Free Vibration Analysis of Cracked Structure

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***Abstract*— Early detection of damage is of special concern for engineering structures. The traditional methods of damage detection include visual inspection or instrumental evaluation. For such visualisation and inspection, a lot of time is required and accuracy may also deflect. A crack in a structural member introduces local flexibility that would affect vibration response of the structure. This property may be used to detect existence of a crack together with its location and depth in the structural member. The presence of damage leads to changes in some of the lower natural frequencies and mode shapes. A comparatively recent development for the diagnosis of structural crack location and size by using the theoretical and finite element method has improved. A method based on measurement of natural frequencies is presented for detection of the location and size of a crack in a cantilever beam. Numerical calculations has been done by solving the Euler equation for un-crack beam and cracked beam to obtain first three natural frequencies of different modes of vibration considering various crack positions for the beam.**

Here ANSYS 12 software package have been used for finite element analysis of both crack and un-crack cantilever beam taking input file as a CAD design developed in CATIA. Total 10 models has been tested according to theoretical and software package. At the end of paper results has been discussed from the data available and method proposed.

Keywords*— Crack, Natural frequency, Finite element method.

I. Introduction

Any motion which repeats itself after a certain interval of time is called vibration. The swing of pendulum is a typical example of vibration. The theory of vibration deals with study of oscillatory motions of bodies and the forces associated with them. A vibration can caused due to external unbalanced force also. A vibratory system, in general, includes elastic member for storing potential energy, a mass or inertia member for storing kinetic energy and damper by which gradual loss of energy takes place. A simple pendulum as shown in figure 1 is an example of vibration system. Pendulum has a string for elastic nature, mass of bob acts as a means for kinetic energy. Like pendulum, spring-masssystem, vehicle suspension system, simply supported and cantilever beam, lateral vibrating string, vibration due to unbalance reciprocating or rotating force, etc. are the examples of vibrating system.

A. Lateral vibrations of a cantilever beam

The vibrations in which particles of the system vibrates in the direction perpendicular to the axis of system is known as lateral vibration. A large number of practical systems can be described using finite number of degree of freedom; such a system is shown in figure 1but some systems, especially those involving continuous elastic members, have an infinite number of degree of freedom. As an example, consider a cantilever beam shown in figure .2. Since the beam has infinite number of masses, we need infinite number of coordinates to specify the deflected configuration. Thus, cantilever beam is infinite degree of freedom system and it is necessary to study lateral vibrations of cantilever beam.

B. Crack identification in cantilever beam structure

Vibration-based methods have been proved as a fast and inexpensive means for crack identification. A crack in a structure induces a local flexibility which affects the dynamic behaviour of the whole structure to a considerable degree. It results in reduction of natural frequencies and changes in mode shapes. An analysis of these changes makes it possible to determine the position and depth of cracks. Most of the researches used in their studies are open crack models, that is, they assume that a crack remains always open during vibration. The assumption of an open crack leads to a constant shift of natural frequencies of vibration.

The decrease in experimental natural frequencies will lead to an underestimation of the crack depth. Harish and Parhi, 2009 have performed analytical studies on free vibrations of cracked beam models and obtains the appropriate results.

II. Literature Reviews

Different researchers have discussed damage detection of vibrating structures in various ways. They are summarized below.

IJERT

Free and forced vibration analyses of a cracked beam were performed by S Orhan et al. in order to identify the crack in a cantilever beam. Single- and two-edge cracks were evaluated. The study results suggest that free vibration analysis provides suitable information for the detection of single and two cracks, whereas forced vibration can detect only the single crack condition.

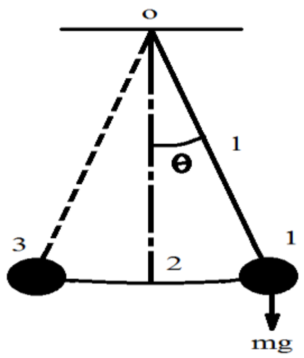

Figure 1. Vibrations of Simple Pendulum

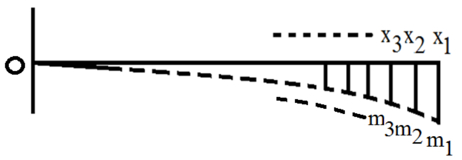

Figure2.Cantilever Beam asMulti Degree of Freedom Systems

However, dynamic response of the forced vibration better describes changes in crack depth and location than the free vibration in which the difference between natural frequencies corresponding to a change in crack depth and location only is a minor effect. The Euler– Bernoulli beam model was assumed. The crack is assumed to be an open crack and the damping has not been considered in this study.

F Leonard, J Lanteigne, S Lalonde and Y Turcotte et al. proposed a study based on cracks that occurred in metal beams obtained under controlled fatigue-crack propagation. Spectrograms of the free-decay responses showed a time drift of the frequency and damping: the usual hypothesis of constant modal parameters is no longer appropriate, since the latter are revealed to be a function of the amplitude.

An experimental investigation has been carried out byM. Karthikey and R. Tiwari et al. to establish an identification procedure for the detection, localization, and sizing of a flaw in a beam based on forced response measurements. The experimental setup consisted of a circular beam, which was supported by rolling bearings at both ends.

Sensibility analysis of the inverse problem of the crack parameters (location and depth) determined by M B Rosales, C P Filipich and F S Buezas et al. An efficient numerical technique is necessary to obtain significant results. Two approaches are herein presented: The solution of the inverse problem with a power series technique (PST) and the use of artificial neural networks (ANNs).

An analytical, as well as experimental approach by H. Nahvi and M.Jabbari et al. to the crack detection in cantilever beams by vibration analysis is established.

A model-based approach is developed by Zhigang Yu and Fulei Chu et al. Todetermine the location and size of an open edge crack in an FGM beam. The *p*-version of finiteelement method is employed to estimate the transverse vibration characteristics of a crackedFGM beam. A rational approximation function of the stress intensity factor (SIF) with crackdepth and material gradient as independent variables is presented in order to overcome thecumbersomeness and inaccurateness caused by the complicated expression of the analytical SIFsolution in crack modelling.

An analytical approach forcrack identification procedure in uniform beams with an open edge crack, based on bending vibration measurements, is developed by N. Khaji, M. Shafiei and M. Jabalpur et al. The method is based on the assumption that the equivalent spring stiffness does not depend on thefrequency of vibration, and may be obtained from fracture mechanics. The results provide simple expressions for the characteristic equations, which are functions of circular natural frequencies, crack location, and crack depth.

A new method for crack detection in beams based oninstantaneous frequency and empirical mode decomposition is proposed by S. Loutridis, E.Doukaand L.J. Hadjileontiadis et al. The dynamic behaviour of a cantilever beam with abreathing crack under harmonic excitation is investigated both theoretically and experimentally. The time-varying stiffness is modelled using a simple periodic function. It follows that theharmonic distortion increases with crack depth following definite trends and can be also used asan effective indicator for crack size.

The research work by W Zhang, Z Wang and H Ma et al. illustrates the crackidentification method combining wavelet analysis with transform matrix. Firstly, thefundamental vibration mode was applied to wavelet analysis. Secondly, based on the identifiedcrack locations, a simple transform matrix method requiring only the first two tested naturalfrequencies was used to further identify the crack depth.

Nonlinear vibration of beams made offunctionally graded materials (FGMs) containing an open edge crack is studied by S.Kitipornchai, L.L. Ke, J. Yang and Y. Xiang et al. based on Timoshenko beam theory andvon Kármán geometric nonlinearity. The Ritz method is employed to derive the governingEigenvalue equation which is then solved by a direct iterative method to obtain the nonlinearvibration frequencies of cracked FGM beams with different end supports.

An analysis of crackedbeam structure using impact echo method was proposed by E Çam, S Orhan and M Lüy et al. Here the vibrations as a result of impact shocks were analyzed. The signals obtained in defectfreeand cracked beams were compared in the frequency domain Experimental results andsimulations obtained by the software ANSYS.

IJERT

III. THEORETICAL FORMULATIONS

A. *Crack theory*

Physical parameters affecting dynamic characteristics of cracked structures: Usually the physical dimensions, boundary conditions, the material properties of the structure play important role for the determination of its dynamic response. Their vibrations cause changes in dynamic characteristics of structures. In addition to this presence of a crack in structures modifies its dynamic behaviour. The following aspects of the crack greatly influence the dynamic response of the structure.

(i) The position of crack
(ii) The depth of crack
(iii) The orientation of crack
(iv) The number of cracks

Classification of cracks: Based on their geometries, cracks can be broadly classified as follows:

- Cracks perpendicular to the beam axis are known as "transverse cracks". These are the most common and most serious as they reduce the cross-section and thereby weaken the beam. They introduce a local flexibility in the stiffness of the beam due to strain energy concentration in the vicinity of the crack tip.
- Cracks parallel to the beam axis are known as "longitudinal cracks". They are not that common but they pose when the tensile load is applied is at right angles to the crack direction i.e. perpendicular to beam axis or the perpendicular to crack.
- "Slant cracks" (cracks at an angle to the beam axis) are also encountered, but are not very common. These influence the torsion behaviour of the beam. Their effect on lateral vibrations is less than that of transverse cracks of comparable severity.
- Cracks that open when the affected part of the material is subjected to tensile stresses and close when the stress is reversed are known as "breathing cracks". The stiffness of the component is most influenced when under tension. The breathing of the crack results in non-linearity's in the vibration behaviour of the beam. Cracks breathe whencrack sizes are small, running speeds are low and radial forces are large .Most theoretical research efforts are concentrated on "transverse breathing" cracks due to their direct practical relevance.
- Cracks that always remain open are known as "gaping cracks". They are more correctly called "notches". Gaping cracks are easy to mimic in a laboratory environment and hence most experimental work is focused on this particular crack type.
- Cracks that open on the surface are called "surface cracks". They can normally be detected by techniques such as dye-penetrates or visual inspection.
- Cracks that do not show on the surface are called "subsurface cracks". Special techniques such as ultrasonic, magnetic particle, radiography or shaft voltage drop are needed to detect them. Surface cracks have a greater effect than subsurfaceCracks on the vibration behaviour of shafts.

B. *Governing Equation of Free Vibration of Beam*

The beam with a transverse edge crack is clamped at left end, free at right end and has different cross section and same length like model in Figure 3 and 4. The Euler-Bernoulli beam model is assumed for the finite element formulation. The crack in this particular case is assumed to be an open crack and the damping is not being considered in this theory.

The free bending vibration of an Euler-Bernoulli beam of a constantrectangular cross section is given by the following differential equation as given in:

$$EI\frac{d^4y}{dx^4} - m\omega_i^2 y = 0 \quad \text{…………………. (1)}$$

Where 'm' is the mass of the beam per unit length (kg/m), 'ωi' is the natural frequency of the ithmode (rad/s), E is the modulus of elasticity (N/m^2) and I is the moment of inertia (m^4).

By defining $\lambda^4 = \frac{m\omega_i^2}{EI}$ equation is rearranged as a fourth-order differential equation as follows:

$$\frac{d^4y}{dx^4} - \lambda^4 y = 0 \quad \text{…….……..……… (2)}$$

The general solution to the equation is:

$$y = A\cos\lambda_i x + B\sin\lambda_i x + C\cosh\lambda_i x + D\sinh\lambda_i x \quad \text{.....(3)}$$

Where A, B, C, D are constants and 'λ_i' is a frequency parameter. Adopting Hermitian shape functions, the stiffness matrix of the twonodedbeam element without a crack is obtained using the standardintegration based on the variation in flexural rigidity. The element stiffness matrix of the un-cracked beam is given as:

$$[K^e] = \int [B(x)]^T EI[B(x)]dx$$

$$[B(x)] = \{H_1(x) H_2(x) H_3(x) H_4(x)\} \quad \text{…… (4)}$$

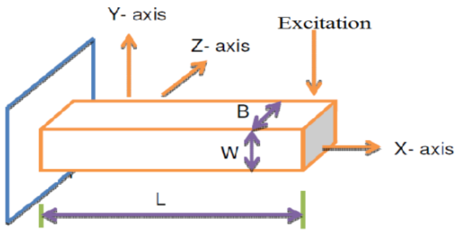

Figure 3 Uncracked Cantilever Beam Model

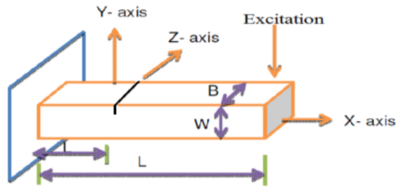

Figure4Crack Cantilever Beam Model

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Where the Hermitian shape functions defined as:

$$H_1(x) = 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3} \quad \text{.........................……. (5)}$$

$$H_2(x) = x - \frac{2x^2}{l} + \frac{x^3}{l^2} \quad \text{……………….......…. (6)}$$

$$H_3(x) = \frac{3x^2}{l^2} - \frac{2x^3}{l^3} \quad \text{................….. (7)}$$

$$H_4(x) = -\frac{x^2}{l} + \frac{x^3}{l^2} \quad \text{....………………. (8)}$$

Assuming the beam rigidity EI is constant and is given by EI_0 within the element, and then the element stiffness is:

$$[K^e] = \frac{EI_0}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \quad \text{……(9)}$$

$$[K_c^e] = [K^e] - [K_c] \quad \text{……....…….. (10)}$$

Here,

$[K_c^e]$ is the Stiffness matrix of the cracked element,

$[K^e]$ is the Element stiffness matrix,

$[K_c]$ is the Reduction in stiffness matrix due to the crack.

According to Peng et al. (2007), the matrix [Kc] is:

$$[K_c] = \begin{bmatrix} K_{11} & K_{12} & -K_{11} & K_{14} \\ K_{12} & K_{22} & -K_{12} & K_{24} \\ -K_{11} & -K_{12} & K_{11} & -K_{14} \\ K_{14} & K_{24} & -K_{14} & K_{44} \end{bmatrix} \quad \text{......….. (11)}$$

Where:

$$K_{11} = \frac{12E(I_0 - I_c)}{L^4}\left[\frac{2l_c^3}{L^2} + 3l_c\left(\frac{2L_1}{L^2} - 1\right)^2\right] \quad \text{…….. (11a)}$$

$$K_{12} = \frac{12E(I_0 - I_c)}{L^3}\left[\frac{l_c^3}{L^2} + l_c\left(2 - \frac{7L_1}{L} + \frac{6L_1^2}{L^2}\right)^2\right] \quad \text{.. (11b)}$$

$$K_{14} = \frac{12E(I_0 - I_c)}{L^3}\left[\frac{l_c^3}{L^2} + l_c\left(2 - \frac{5L_1}{L} + \frac{6L_1^2}{L^2}\right)^2\right] \quad \text{... (11c)}$$

$$K_{22} = \frac{12E(I_0 - I_c)}{L^3}\left[\frac{3l_c^3}{L^2} + 2l_c\left(\frac{3L_1}{L} - 2\right)^2\right] \quad \text{... (11d)}$$

$$K_{24} = \frac{12E(I_0 - I_c)}{L^2}\left[\frac{3l_c^3}{L^2} + 2l_c\left(2 - \frac{9L_1}{L} + \frac{9L_1^2}{L^2}\right)^2\right] \quad \text{.... (11e)}$$

$$K_{44} = \frac{12E(I_0 - I_c)}{L^2}\left[\frac{3l_c^3}{L^2} + 2l_c\left(\frac{3L_1}{L} - 1\right)\right] \quad \text{…... (11f)}$$

Here, lc=1.5W, L=Total length of the beam, L1=Distance between the left node and crack, I_0=Moment of inertia of the beam cross section, I_c= Moment of inertia of the beam with crack. It is supposed that the crack does not affect the mass distribution of the beam. Therefore, the consistent mass matrix of the beam element can be formulated directly as:

$$[M^e] = \int_0^l \rho A[H(x)]^T[H(x)]dx \quad \text{.........……. (12)}$$

$$[M^e] = \frac{\rho A l}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \quad \text{………. (13)}$$

The natural frequency then can be calculated from the relation:

$$[-\omega^2[M] + [K]]\{q\} = 0 \quad \text{................….. (14)}$$

Where: q=displacement vector of the beam.

The table 1 shows the computational values of natural frequencies for different beams model having different cross sections, cracks depth and location from above theory. The cross section dimensions for the beam are 10*10 mm and of length 600 mm. The crack depth are 1mm, 2mm, 3mm at positions of 150mm, 300mm, 450mm from the fixed support of the beam.

Table 1 Theoretical Natural Frequency, Crack Depth and Location

Sr. No.	Crack depth (mm)	Crack location (mm)	Fnf (Hz)	Snf (Hz)	Tnf (Hz)
1	0	0	22.660	142.440	397.376
2	1	150	22.651	142.433	397.375
3	2	150	22.566	141.839	395.711
4	3	150	22.353	40.506	391.981
5	1	300	22.651	142.433	397.375
6	2	300	22.566	141.839	395.711
7	3	300	22.353	40.506	391.981
8	1	450	22.651	142.433	397.375
9	2	450	22.566	141.839	395.711
10	3	450	22.353	40.506	391.981

The theoretical formula for natural frequency can be obtained by Euler's Beam theory as,

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$$\omega_n = C^* \sqrt{\frac{EI}{mL^4}}$$

Where C is Constant,
C = 0.56 for First mode, 3.51 for Second, 9.82 for Third mode for Cantilever Beam.

IV. FINITE ELEMENT FORMULATIONS

Finite element analysis has been carried out by ANSYS12 software. ANSYS is a general-purpose finite-element modelling package for numericallysolving a wide variety of mechanical problems. These problems include static/dynamic, structural analysis (both linear and nonlinear), heat transfer, and fluidproblems, as well as acoustic and electromagnetic problems.

In general, a finite-element solution may be broken into the following three stages.

(1) **Pre-processing**: defining the problem

The major steps in pre-processing are (i) define key points/lines/areas/volumes,
(ii) Define element type and material/geometric properties, and (iii) meshlines/areas/ volumes as required.

The amount of detail required will depend on the dimensionality of the analysis,i.e., 1D, 2D, axisymmetric, and 3D.

(2) **Solution**: assigning loads, constraints, and solving. Here, it is necessary to specify the loads (point or pressure), constraints(translational and rotational), and finally solve the resulting set of equations.

(3) **Post processing**: Further processing and viewing of the results

In this stage one may wish to see (i) lists of nodal displacements, (ii) elementforces and moments, (iii) deflection plots, and (iv) Frequencies and temperature maps.

Following steps show the guidelines for carrying out Modal analysis.

Define Materials

1. Set preferences. (Structural)
2. Define constant material properties.

Model the Geometry

3. Follow bottom up modelling and create/import the geometry

Generate Mesh

4. Define element type.
5. Mesh the area.

Apply Boundary Conditions

6. Apply constraints to the model.

Obtain Solution

7. Specify analysis types and options.
8. Solve.

The ANSYS 12 finite element program was used for free vibration of thecracked beams. For this purpose, the total 10 models are created at various crack positions in CAD software (CATIA) and imported in ANSYS (.stp file). The beammodel wasdiscretised into no. of elements with N nodes. Cantilever boundary conditions can also be modelled by constraining all degrees offreedoms of the nodes located on the left end of the beam. The subspace mode extractionmethod was used to calculate the natural frequencies of the beam.

For the model creation different dimensions taken as follows:
Beam width- 10mm
Beam height- 10mm
Beam length- 600mm
Crack depth- 1mm,2mm, and 3mm
Crack location- 150mm,300mm, and 450mm.

The results of finite element analysis for the beams have first natural frequencies are shown below:

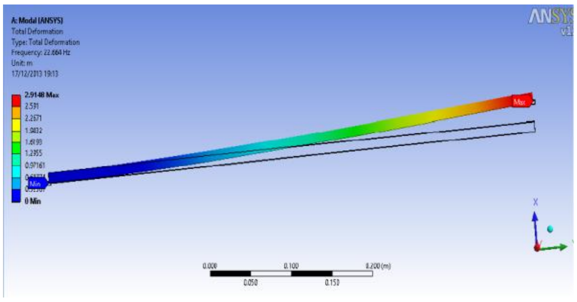

Figure 5. Finite element result of beam model 1

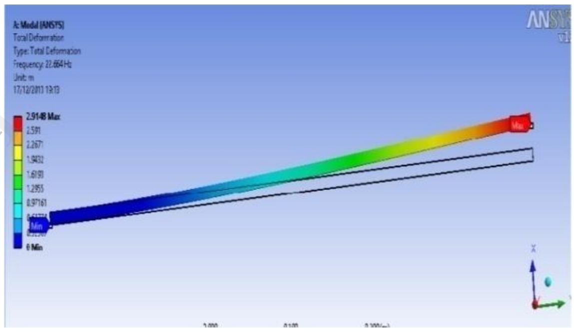

Figure 6. Finite element result of beam model 2

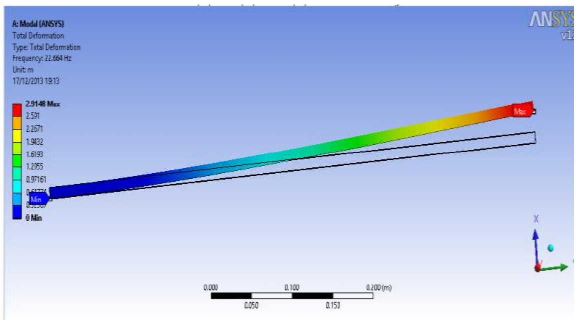

Figure 7. Finite Element Result of Beam Model 3

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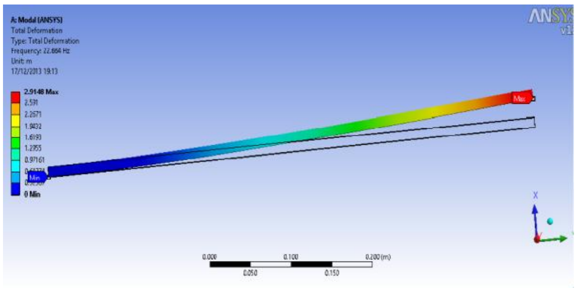

Figure 8. Finite Element Result of Beam Model 4

Similarly all the beams have tested by finite element modelling. The results are tabulated in the table 2.

Table 2 Finite Element Natural Frequency, Crack Depth and Location

Sr. No.	Crack depth (mm)	Crack location (mm)	Fnf (Hz)	Snf (Hz)	Tnf (Hz)
1	0	0	22.663	141.84	396.34
2	1	150	22.648	141.84	395.82
3	2	150	22.613	141.82	394.41
4	3	150	22.561	141.77	395.16
5	1	300	22.641	141.75	396.34
6	2	300	22.623	140.78	396.33
7	3	300	22.572	141.17	396.33
8	1	450	22.665	141.73	395.41
9	2	450	22.666	141.46	395.38
10	3	450	22.665	141.59	394.33

V. Results and Discussion

Discussion based on the output generated by Theoretical analysis and the information supplemented by FEA analysis in ANSYS is as follows:

It is already known that the natural frequency decreases as the crack depth increases in a structural part. Firstly determination of natural frequency of different modes of vibration is done for un-cracked beam numerically (solving Euler's Equation for Beam in vibration analysis), and then FEA analysis in ANSYS. Here total 10 models have been used taking different combinations of relative crack location and relative crack depth. Several steps have been shown to develop a natural frequency modal based on FEA which is explained through an example and all the frequencies values are tabulated in table 2.

Figure 9 shows the Variation of First Natural Frequency with Crack Depth. It is clear from the Figure 9 that the natural frequency of beam structure decreases with increase of crack depth. Also from figure 10 it is clear that the natural frequency varies with crack location. It is clear from analysis that the natural frequency of different modes of vibration can be precisely obtained from this method.

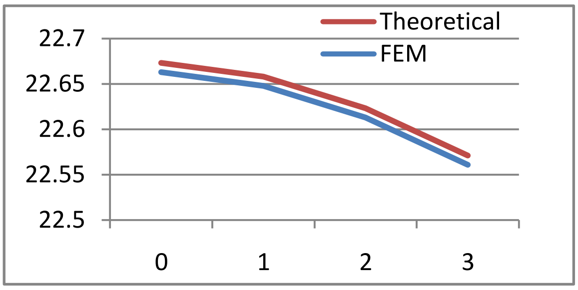

Figure 9 Variation of First Natural Frequency with Crack Depth

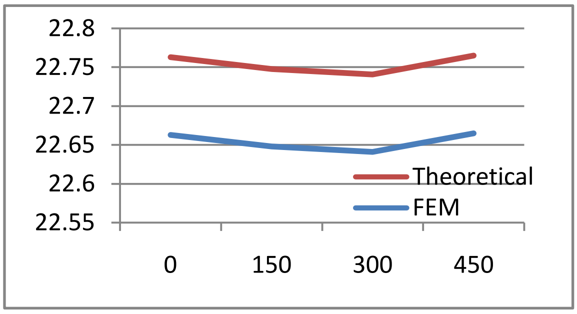

Figure 10 Variation of First Natural Frequency with Crack Location

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VI. Conclusions

The present investigation based on the theoretical Analysis and the FEA Analysis draws the following conclusions.

- Inputs for FEA are crack location and crack depth and outputs are natural frequency for different modes of vibration.
- The results show that the values of natural frequencies by theory and ANSYS are close to the agreement.
- Significant changes in natural frequency observed at the vicinity of crack location.
- When the crack location is constant but the crack depth increases, the natural frequency of the beam decreases.
- When the crack depth is constant and crack location from the cantilever end varied, Natural frequencies of first, second and third modes are also varies.

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REFERENCES

1. Amiya Kumar Dash, 'Analysis of adaptive fuzzy technique for multiple crack diagnosis of faulty beam using vibration signatures', Journal of Advances in Fuzzy Systems, Vol. 1, 2013, pp. 16 pages.
2. Dayal R. Parhi And SasankaChoudhary,'Smart crack detection of a cracked cantilever beam using fuzzy logic technology with hybrid membership functions', Journal of Engineering And Technology Research, Vol. 3(8), Aug. 2011, pp. 270-300.
3. MeysamSiyahMansoory, 'Automatic crack detection in eggshell based on SUSAN Edge Detector using Fuzzy Thresholding', Journal of Modern Applied Science, Vol.5 No.6, Dec 2011, pp. 1-9.
4. Mohammad R. Jahanshahi, 'An innovative methodology for detection and quantification of cracks through incorporation of depth perception', Journal of Machine Vision and Applications, Feb 2011, pp. 3-11.
5. N. V. NarasimhaRao, 'Fault diagnosis of dynamic cracked structure using fuzzy interface system', International Journal of Emerging Trends and Engineering Development, Vol.5, Jul 2012, pp. 1-10.
6. SasankaChoudhury and Dayal Parhi R, 'Crack detection of a cantilever beam using Kohonen network techniques', Global Advance Research Journal of Engineering, Technology and Innovation, Vol. 2(5), June 2013, pp. 153-157.
7. ShashankKamthan, 'Fuzzy logic approach for impact source identification in ceramic plates', US Army RDECOM-TARDEC, Survivability Research, Warren, pp. 1-8.

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