Free Convection over a Varying wall Vertical Cylinder embedded in a Porous medium with effect of Radiation, Variable Fluid Properties and Stratification.

T. Raja Rani

Military Technological College, Muscat, Oman

Abstract

The aim of the present paper is to study free convection boundary layer flows over a vertical cylinder which is embedded in a porous medium with effects of Radiation, variable viscosity, variable thermal conductivity and stratification are studied numerically. Temperature of the lateral surface of the cylinder as well as the ambient temperature is assumed to be linear functions of the axial coordinate. A similarity transformation is used to reduce partial differential equations governing the problem into ordinary differential equations and equations are solved numerically subject to boundary conditions by the use of Runge-Kutta-Gill method with shooting technique. Under the assumption of stable stratification similarity solutions are presented for the problem. Solutions for an unstratified medium are obtained as a special case by assigning zero value to the stratification parameter S. Qualitatively interesting graphs are discussed as functions of parameters of the problem.

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1. Introduction

The study of free convection from the outer surface of a heated vertical cylinder embedded in a fluid saturated porous medium has important geophysical and engineering applications (refer Nield and Bejan [7]). As quoted in the reference, such studies will aid in the assessment and evaluation of geothermal resources during geophysical exploration. Further, the heat transfer coefficients of such a study will be useful in estimating the cooling rate of intrusive bodies and consequently the life span of a geothermal reservoir. It will also be useful to calculate the heat loss from underground casing and piping systems for the optimum design of geothermal power plants.

In many practical situations the region of flow and the whole configuration will be confined to a finite region of space and so the heated boundary layer eventually hits the ceiling. This heated layer remains at the top of gradually colder layers and this leads to thermal stratification (refer Bejan[2]).

Minkowycz and Cheng [6] discussed free convection at a vertical cylinder embedded in a porous medium. Cheng and his co-workers, Cheng and Minkowycz [4], Cheng and Chang [3] have obtained similarity solutions for free convection in a porous medium adjacent to vertical and horizontal plates with wall temperatures being a power function of distance. Hossain and Nakayamma [5] discussed non- Darcy free convective flow along a vertical cylinder embedded in a porous medium with surface mass flux. However, in the above studies the effect of stable ambient stratification on heat and mass transfer was not discussed. It is known that stable ambient stratification phenomenon usually occurs in cooling ponds, lakes, solar ponds and atmosphere. In view of this, several researchers have discussed the effect of stratification on different heat transfer studies. Takhar.et.al.,[10] discussed natural convection at vertical cylinder in a thermally stratified high porosity medium. Saha and Hossain [9] discussed the natural convection flow with combined buoyancy effects due to thermal and mass diffusion in thermally stratified media. Angirasa and Srinivasan [1] studied the natural convection flows due to the combined buoyancy of heat and mass diffusion in a thermally stratified medium. Raja Rani and CNB. Rao [8] studied the effects of variable fluid properties on free Convection Flows and heat transfer at an isothermal vertical plate embedded in porous medium in the presence of magnetic field and radiation.

In the present paper the free convection flow about a vertical cylinder embedded in a saturated porous medium is discussed. Here the temperatures on the surface of the cylinder as well as the ambient temperature are assumed to vary linearly with *x*, where *x* is the distance from the leading edge of the cylinder. It is also assumed that ambient temperature is less than the temperature of the cylinder and, as a result, stable stratification occurs. However in an unstratified medium, the ambient temperature can be lower or higher than the temperature of the cylinder. In the present problem possible cases of hot cylinder and cold cylinder are considered in the case of an unstratified medium. The effects of radiation, stratification, variable viscosity and variable thermal conductivity on free convection at the vertical cylinder are discussed. Because of linear variation of temperatures of the cylinder and that of the ambient with the axial coordinate, similarity solutions exist for the problem.

2. Formulation and Solution

Let a vertical cylinder of radius *a* immersed in a saturated porous medium (the physical model is shown in fig.1). Origin is chosen at the centre of cylinder at its leading edge where the boundary layer thickness is zero. The axial coordinate (x) is taken vertically along the length of the cylinder and the radial coordinate (*r*) perpendicular to it. Temperature of the surface of the cylinder (T_w) is assumed to vary linearly with the distance *x* from its leading edge. The temperature of the ambient fluid (T_∞) is also assumed to vary as a linear function of *x*. Stable stratification occurs under the assumption that $T_w > T_\infty$ (hot plate). Temperature of the fluid at the leading edge is taken as T_0 which is the minimum temperature of the configuration. Viscosity of the fluid and effective thermal conductivity of the fluid saturated porous medium is assumed to vary as linear functions of temperature. Other fluid properties are assumed to be constant except for density which is taken to be a function of temperature only in the body force term one of the momentum equations.

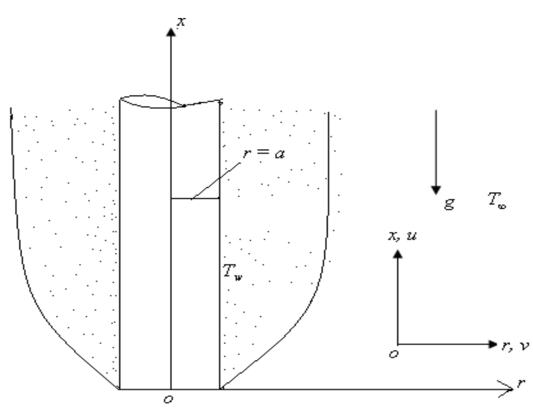

Fig .1 Physical model and coordinate system

The governing equations of the ***Darcy model*** boundary layer free convection flow in cylindrical coordinates are as follows:

$$\frac{\partial(rv)}{\partial r}+\frac{\partial(ru)}{\partial x}=0 \qquad (1)$$

$$\frac{\partial(P-P_s)}{\partial x}+(\rho-\rho_s)g+\frac{\mu u}{K}=0 \qquad (2)$$

$$\frac{\partial P}{\partial r}+\frac{\mu v}{K}=0 \qquad (3)$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}\right)=\left(\frac{1}{r}\frac{\partial}{\partial r}\left(k_m r\frac{\partial T}{\partial r}\right)\right)-\frac{\partial q_r}{\partial r} \qquad (4)$$

Boundary Conditions are:

$$\begin{aligned}&at \quad r=a,\ v=0,\ T=T_w=T_0+Ax,\ \ x\geq 0\\ &as \quad r\to\infty,\ u=0,\ T=T_\infty=T_0+Bx,\ \ x\geq 0\end{aligned} \qquad (5)$$

where u, v are fluid velocity components, *T* is fluid temperature, K is Permeability, k_m is effective thermal conductivity of the porous medium and μ is the dynamic viscosity, q_r is radiative heat flux and the Rosseland approximation used in the energy equation to describe the thermal radiative heat transfer.

In fact the radiative heat flux q_r is given by $q_r=-\dfrac{4\sigma_s}{3k_e}\dfrac{\partial T^4}{\partial y}$, where σ_s the Stefan-Boltzmann's constant and k_e is the mean absorption coefficient. Assuming temperature differences within the flow to be sufficiently small, the term $\dfrac{\partial q_r}{\partial y}$ of equation (4) is simplified by expanding T^4 into the Taylor series about T_∞, and neglecting higher order terms. It may be noted that because of the use of Rosseland approximation, the present analysis gets limited to optically thick fluids.

Taking $\rho=\rho_\infty[1-\beta(T-T_\infty)]$ in the body force term, and eliminating fluid pressure from equations (2) and (3), the governing equations are obtained as

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}\right)=\left(\frac{1}{r}\frac{\partial}{\partial r}\left(k_m r\frac{\partial T}{\partial r}\right)\right)-\frac{\partial q_r}{\partial r} \qquad (6)$$

A stream function ψ is introduced through the relations

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$$u = \frac{1}{r}\frac{\partial \psi}{\partial r} \quad \& \quad v = -\frac{1}{r}\frac{\partial \psi}{\partial x} \tag{7}$$

Introducing non- dimensional variables η, f, θ and parameters ξ_o through the relations

$$\left.\begin{aligned}
\eta &= \left(\frac{K\rho g \beta A}{\mu \alpha_m}\right)^{\frac{1}{2}} \left(\frac{r^2 - a^2}{2a}\right)^{\frac{1}{2}} \\
\psi &= \left(\frac{\alpha_m K \rho_\infty g \beta A}{\mu_f}\right)^{\frac{1}{2}} ax\, f(\eta) \\
\xi_0 &= \frac{2}{a}\left(\frac{\mu_f \alpha_m}{K\rho_\infty g \beta A}\right)^{\frac{1}{2}} \\
\theta &= \frac{T - T_\infty}{T_w - T_0} \\
F &= \frac{k_f k_e}{4\sigma_s T_\infty^3} \\
Rd &= \frac{4\sigma_s T_\infty^3}{kk_e}
\end{aligned}\right\} \tag{8}$$

The governing equations in non-dimensional form are obtained as

$$\left[1 + \gamma_\mu\left(\theta - \frac{1}{2}\right)\right] f'' + \gamma_\mu f'\theta' = \theta' \tag{9}$$

$$\left[\left[1 + \gamma_k\left(\theta - \frac{1}{2}\right)\right](\xi_0\,\eta + 1) + \frac{4}{3} Rd\,(\xi_0\,\eta + 1)\right]\theta'' + \gamma_k(\eta\,\xi_0 + 1)\theta'^2 + \left[\left[1 + \gamma_k\left(\theta - \frac{1}{2}\right)\right]\xi_0 + \frac{2}{3}(Rd)\xi_0 + f\right]\theta' - f'\theta = 0 \tag{10}$$

Boundary conditions in terms of f and θ are:

$$\begin{aligned} &at\ \eta = 0,\ f = 0, \theta = 1 - S \\ &as\ \eta \to \infty,\ f' = 0, \theta = 0 \end{aligned} \tag{11}$$

where $S = 1 - \frac{B}{A}$ is the stratification parameter. Because of the assumption that $T_w > T_\infty$, it follows that $0 < S < 1$ and the fluid saturated porous medium becomes a stably stratified medium. However *S* =0 corresponds to an unstratified medium in which case, T_w need not be greater than T_∞.

The case when the parameters γ_μ, γ_k take non-zero values is referred to as the variable fluid property case, or in short VFP case. The case when the parameters γ_μ, γ_k take zero values is referred as the constant fluid properties case, or in short CFP case. When γ_μ, γ_k take non-zero values, four cases arise (i) $\gamma_\mu < 0, \gamma_k < 0$ (ii) $\gamma_\mu > 0, \gamma_k > 0$ (iii) $\gamma_\mu < 0, \gamma_k > 0$ (iv) $\gamma_\mu > 0, \gamma_k < 0$ (Refer [8]). As a result, depending on the numerical values of γ_μ and γ_k, the results become applicable to certain fluids in specific ranges of temperatures.

3. SOLUTION OF THE PROBLEM

3.1.1 Parameters of the problem

The parameters of the present problem are *S*, *Rd*, γ_μ, γ_k, ξ_0. The parameter *S* is the stratification parameter. As pointed out earlier, this parameter assumes values between 0 &1 and hence the medium will be a stably stratified medium. *S*=0 corresponds to an unstratified medium. *Rd* is the radiation parameter. Zero value for the parameter *Rd* corresponds to the case when the heat energy through radiation from the fluid is neglected. Positive numerical values of *Rd* correspond to intensity of thermal radiation from the fluid. Solutions are found for the values of 0, 0.5 and 10 of the parameter *Rd*. Thermal radiation can causes thickening of the thermal boundary layer and hence increasing values of the parameter Rd can increase thermal boundary layer thickness. In this paper solutions are found for the values of -1, 0 and 1 of both γ_μ, γ_k. Calculations are done for the value 0 & 0.5 for S. Here ξ_0 is the curvature parameter. The value infinity corresponds to flat plate case. Calculations are done for the values 0.25, 0.5 and 1 of ξ_0.

3.1.2 Numerical Solution

Equations (9) and (10) are solved subject to the conditions (11) by the use of Runge-Kutta –Gill method together with a shooting technique. Appropriate results of the present analysis are

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compared with the results of Mynkowycz & Cheng [6]. Some of the qualitatively and quantitatively interesting results are presented through figures 2 to 10 and through table 1.

3.1.3 DISCUSSION OF THE RESULTS

Variations in skin friction $f''(0)$ are presented in figures 2 and 3; corresponding variations in wall heat transfer rate $(-\theta'(0))$ are presented in figures 4 and 5 respectively. Skin friction assumes negative values for all values of the parameters when $\xi_0 = 0.25$. As the radiation parameter takes increasing values, absolute values of the skin friction can be seen to diminish. The diminishing trend is more pronounced in an unstratified medium rather than in a stratified medium. For fluids like Methyl chloride absolute value of skin friction in the variable fluid property (VFP) case will be larger than that in the constant fluid property (CFP) case. However for fluids like Dichloro Difluro methane, opposite is the behaviour of skin friction.

From figure 3, it may be noted that skin friction doesn't vary with ξ_0 when radiation parameter takes a constant value. The values however change with changing values of *Rd*. The variation in skin friction with changing values of the stratification parameter and with changing values of γ_μ, γ_k is as pointed out earlier.

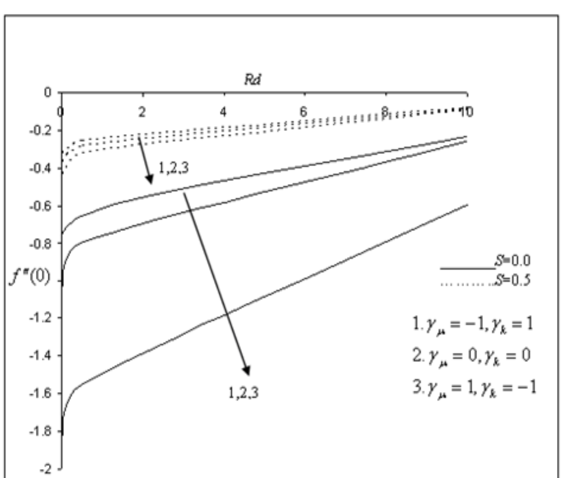

Fig.2 Variations of $f''(0)$ with *Rd* for $\xi_0 = 0.25$

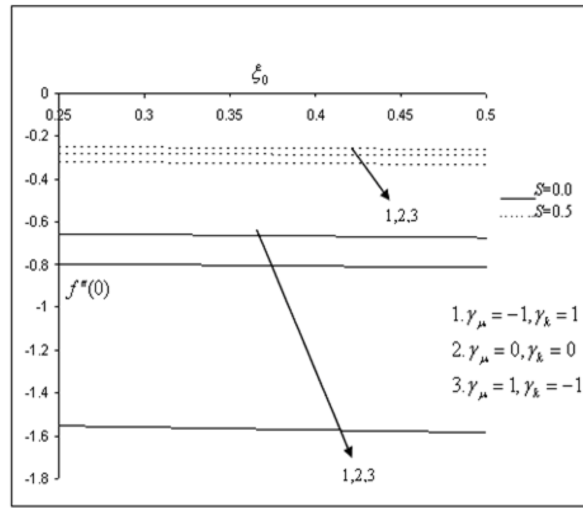

Fig 3 Variations of $f''(0)$ with ξ_0 for $Rd = 0.5$

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From figures 4 and 5, wall heat transfer rate can be observed to take positive values for all values of the parameters. '$-\theta'(0)$' assumes larger numerical values in the absence of stratification than in its presence. '$-\theta'(0)$' Diminish with increasing values of the radiation parameter *Rd*. For small values of *Rd*, for fluids like Dichloro Difluro methane the heat transfer coefficient in the VFP case assumes smaller values than in CFP case. Opposite is the trend for fluids like Methyl chloride.

From Figure 5, it may be noted that, like skin friction, the heat transfer coefficient also doesn't change with changing values of ξ_0 for a fixed value of *Rd*.

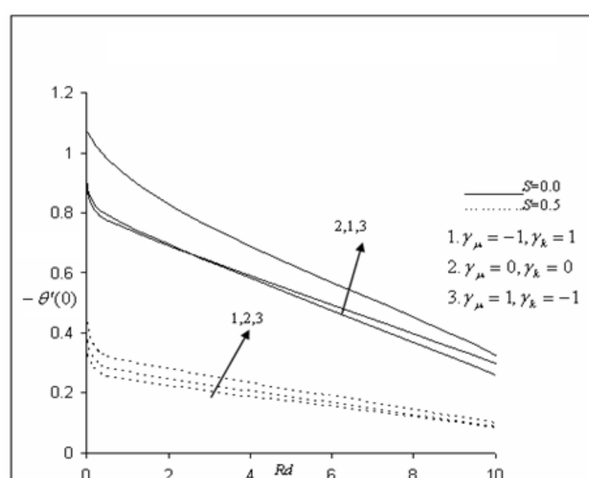

Fig 4 Variations of $-\theta'(0)$ with *Rd* for $\xi_0 = 0.25$

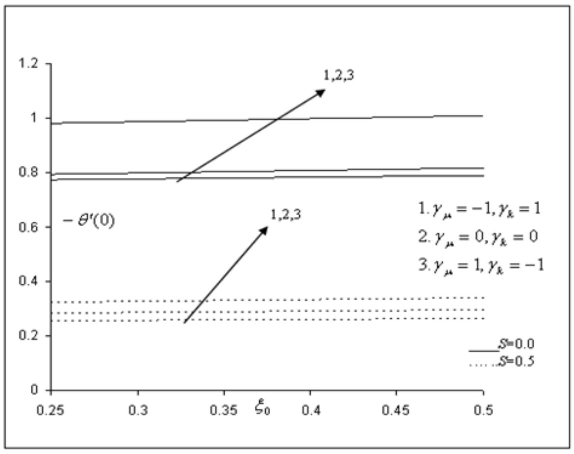

Fig.5 Variations of $-\theta'(0)$ with ξ_0 for $Rd = 0.5$

Variations in fluid velocity ($f'(\eta)$) are presented in figure.6. Reduced velocities can be observed in a stably stratified medium than in an unstratified medium. Velocities near the plate are relatively larger in an unstratified medium than in a stably stratified medium. Effect of variable fluid properties on fluid velocity is more significant in unstratified medium than in a stably stratified medium.

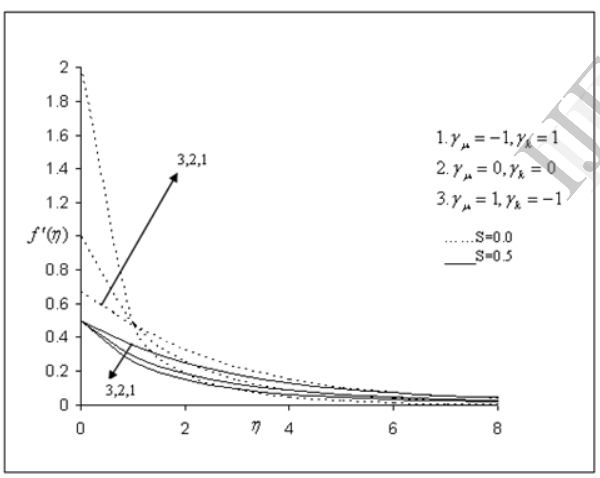

Fig .6 Variations of $f'(\eta)$ with for $Rd = 0.5$ & $\xi_0 = 0.25$

Variations in fluid temperature are presented in figures 7 and 8. Effect of increasing values of *Rd* and increasing values of ξ_0 on fluid temperature is to increase temperature in the boundary layer region near the cylinder and, effect increasing values of *S* is to diminish the fluid temperature. From figure 8, in a stably stratified medium, for fluids like Dichloro Difluro methane temperature in the VFP case is larger than the one in the CFP case. However for fluids like Methyl chloride temperature in the VFP case is smaller than the one in the CFP case. Temperature in the vicinity of the cylinder is smaller in a stably stratified medium than in an unstratified medium. Also Thermal Boundary layer thickness is larger in a stratified medium than in an unstratified medium.

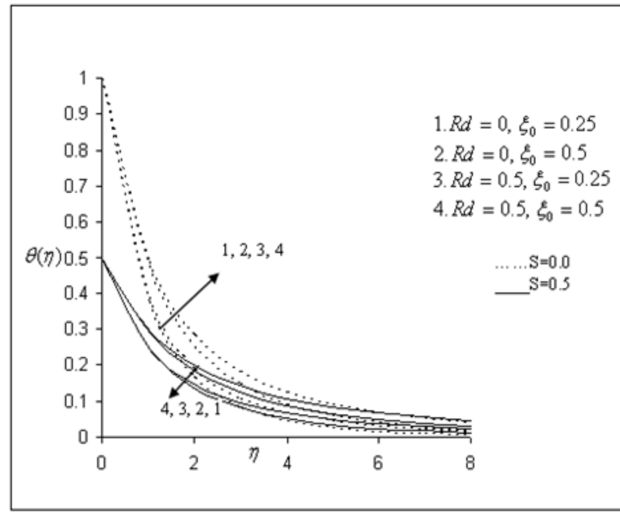

Fig .7 Variations of $\theta(\eta)$ with η for $\gamma_\mu = 0, \gamma_k = 0$

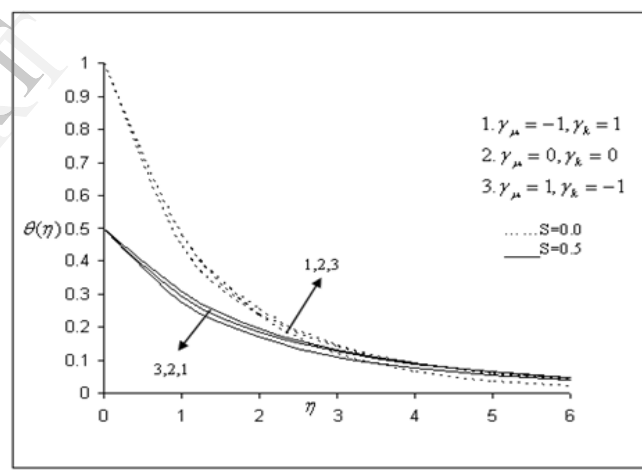

Fig. 8 Variations of $\theta(\eta)$ with η for $Rd = 0.5$ & $\xi_0 = 0.25$

Variations in stream function are presented in figures 9 and 10. As can be expected, stream function starts from zero value at the surface of the cylinder and asymptotically approaches a constant value far away from it.

From figure 9, both in an unstratified medium and in a stably stratified medium, with increasing intensity of radiation and with increasing values of ξ_0, stream function assumes larger numerical values. The values of stream function are smaller in a stably stratified medium than in an unstratified medium.

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From figure 10 it may be noted that, the variations in the values of the stream function between CFP and VFP cases are significant and they can be interpreted in an appropriate fashion when *S=0* and when *S=0.5*. When *S=0.5,* for fluids like Dichloro Difluoro methane stream function assumes larger values in the VFP case than in the CFP case while for fluids like Methyl chloride opposite is the behaviour between VFP and CFP case

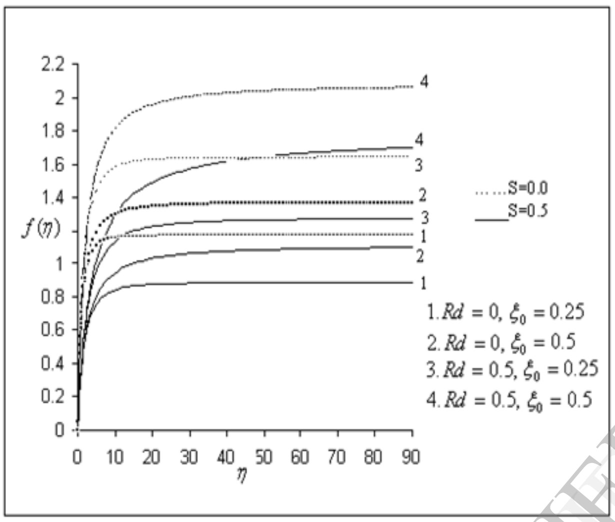

Fig. 9 Variations of $f(\eta)$ **with** η **for** $\gamma_\mu = 0,\ \gamma_k = 0$

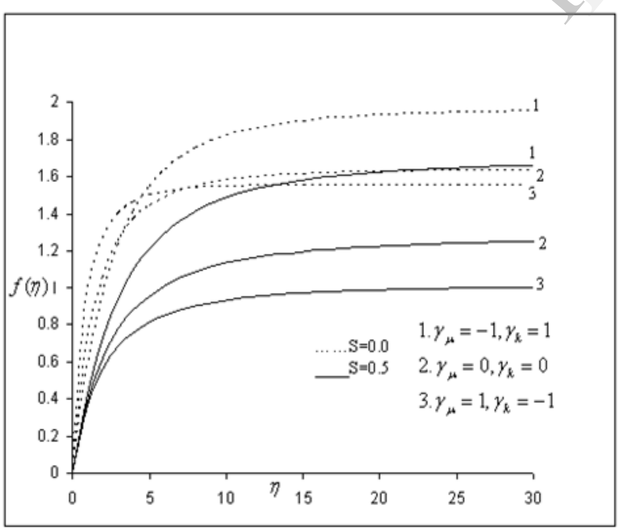

Fig.10 Variations of $f(\eta)$ **with** η **for** $C = 1,\ Rd = 0.5\ \&\ \xi_0 = 0.25$

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Conclusions:

1. Skin friction assumes negative values for all values of the parameters when $\xi_0 = 0.25$. As the radiation parameter takes increasing values, absolute values of the skin friction diminish. The diminishing trend is more pronounced in an unstratified medium rather than in a stratified medium.
2. For fluids like Methyl chloride absolute value of skin friction in the variable fluid property (VFP) case will be larger than that in the constant fluid property (CFP) case. However for fluids like Dichloro Difluro methane, opposite is the behaviour of skin friction.
3. Velocities near the cylinder are relatively larger in an unstratified medium than in a stably stratified medium. Effect of variable fluid properties on fluid velocity is more significant in unstratified medium than in a stably stratified medium.
4. Effect of increasing values of *Rd* and increasing values of ξ_0 on fluid temperature is to increase temperature in the boundary layer region near the cylinder and, effect increasing values of *S* is to diminish the fluid temperature.
5. When *S=0.5*, for fluids like Dichloro Difluoro methane, the stream function assumes larger values in the VFP case than in the CFP case while for fluids like Methyl chloride opposite is the behaviour between VFP and CFP cases.

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