Forward and Inverse Kinematic Singularity analysis of Eight bar 3 – PPP Spatial Parallel Manipulator

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Abstract - An eight bar 3-PPP spatial parallel manipulator with forward and inverse kinematic singularities are analyzed. The determination of singularities in the workspace of 3-PPP spatial parallel manipulator with eight bar mechanism is an important role in machining process of several parallel manipulators for generating symmetric three dimensional coupler curves. The mathematical equations are derived for the proposed approach.

Keywords - Singularities; 3-Dof; Forward Kinematic Analysis; Inverse Kinematic Analysis; Eight Bar Spatial Parallel Manipulator

I. INTRODUCTION

A spatial kinematic eight bar manipulator with fixed and movable tetrahedrons, binary limbs, prismatic and spherical joints are selected. The selected spatial architecture has three degrees of freedom. The design of eight bar spatial parallel manipulator and its three dimensional symmetric coupler curves are more complex due to their kinematic structure. This paper presents forward and inverse kinematic analysis of eight bar spatial parallel manipulator to determine the singularities of a work space, because the singularities are more attractive in several researches. The mathematical equations are derived by using loop closer technique.

Fig (1) Eight bar 3-PPP spatial parallel manipulator
II. KINEMATIC ARCHITECTURE OF EIGHT BAR SPATIAL PARALLEL MANIPULATOR

Kinematic architecture of eight bar spatial parallel manipulator consists of fixed and movable tetrahedrons of its apex points are connected by spherical joint ‘O’, the three vertices of fixed tetrahedron $F_j$ for $j=1,2,3$ is connected by successive set of spherical joint, prismatic actuator, binary limb and spherical joint which is connected by the successive vertices $M_j$ for $j=1,2,3$ of movable tetrahedron. The binary limbs are actuated by linear actuators which are actuated by three linear
variable differential transducers. Therefore the distance between successive vertices of both fixed and movable tetrahedrons \( F_jM_j \) are variable, that means for each position the value of \( F_jM_j \) are variable.

### III. JACOBIAN OF EIGHT BAR 3- PPP SPATIAL PARALLEL MANIPULATOR

The degree of freedom of the selected movable tetrahedron is three. The output rotational vector is described by angular velocities of the movable tetrahedron which is denoted as \([\omega_{mx1} \quad \omega_{my1} \quad \omega_{mz1}]^T\) and the input translational displacement vector of three prismatic actuators are denoted as \([P_{1i} \quad P_{2i} \quad P_{3i}]^T\). The loop closer equations for \(j^{th}\) limb and \(i^{th}\) position can be expressed as

\[
[\overline{OM} _{j}]_{j=1,2,3 \text{ and } i=1,2,3,4 \text{ etc}} = (\overline{OF}_j + \overline{F}_jM_{j})_{j=1,2,3 \text{ and } i=1,2,3,4 \text{ etc}} \tag{1}
\]

The velocity vector loop equation can be obtained by making derivative of “(1)” with respect to time, then the velocity vector loop equations as

\[
[\omega_{mi} \cdot m_j]_{j=1,2,3 \text{ and } i=1,2,3 \text{ etc}} = [(P_{ji}) (\omega_{ji}) k_{ji}]_{j=1,2,3 \text{ and } i=1,2,3 \text{ etc}} + \left(\left(\overline{P}_{ji}\right), (k_{ji})\right)]_{j=1,2,3 \text{ and } i=1,2,3 \text{ etc}} \tag{2}
\]

Where \( \omega_{mi} = [\omega_{mx1} \quad \omega_{my1} \quad \omega_{mz1}]^T \) angular velocity vectors of the movable tetrahedron for \(i^{th}\) position; \(m_j = [m_1 \quad m_2 \quad m_3]\); \(P_{ji} = [P_{1i} \quad P_{2i} \quad P_{3i}]^T\) linear elongation or decrement along the direction of respective binary limbs; \(\overline{P}_{ji} = [\overline{P}_{1i} \quad \overline{P}_{2i} \quad \overline{P}_{3i}]^T\) linear elongation rate of \(j^{th}\) limb in \(i^{th}\) position; \(\omega_{ji} = [\omega_{x1i} \quad \omega_{y1i} \quad \omega_{z1i}]^T \) angular velocity of \(j^{th}\) limb in \(i^{th}\) position with respect to fixed frame of tetrahedron; \(k_{ji} = [k_{1i} \quad k_{2i} \quad k_{3i}]^T\) unit vector along the direction of \(F_jM_{ji} \) \(i=1,2,3 \text{ and } i=1,2,3,4 \text{ etc}\); \(\overline{OF}_j\) is fixed base therefore \(\omega_{ji}\) can be eliminated by making dot product on both sides of the above equation by \(k_{ji}\) then “(2)” can be written as

\[
\left[(k_{ji} \cdot m_j) \cdot \omega_{mi}\right]_{j=1,2,3 \text{ and } i=1,2,3 \text{ etc}} = \left[\overline{P}_{ji}\right]_{j=1,2,3 \text{ and } i=1,2,3 \text{ etc}} \tag{3}
\]

since \(k_{ji} \cdot k_{ji} = 0\) and \(k_{ji}, k_{ji} = 1\)

The above equation can be written as

\[
\left[(k_{ji} \cdot m_j)^T \cdot \omega_{mi}\right]_{j=1,2,3 \text{ and } i=1,2,3 \text{ etc}} = J_p \left[\overline{P}_{ji}\right]_{j=1,2,3 \text{ and } i=1,2,3 \text{ etc}} \tag{4}
\]

That means

\[
\left[J_{mji} \cdot \omega_{mi}\right]_{j=1,2,3 \text{ and } i=1,2,3 \text{ etc}} = J_p \left[\overline{P}_{ji}\right]_{j=1,2,3 \text{ and } i=1,2,3 \text{ etc}} \tag{4}
\]

Where \(J_{mji} = [k_{ji} \cdot m_j]^T\) Jacobians of \(j^{th}\) limbs in \(i^{th}\) position in form of 3x3 matrices and \(J_p\) is 3x3 identity matrix.

### IV. FORWARD KINEMATIC SINGULARITY ANALYSIS OF EIGHT BAR 3 - PPP SPATIAL PARALLEL MANIPULATOR

The forward kinematic singularities occur if determinant of \(J_{mji} = 0\) that means \(\left|k_{ji} \cdot m_j\right|^T = 0\).

By the inspection of “Fig (2),” \(m_3\) and \(P_{3i}\) lie on straight line, that means one of the three vectors vanishes because \(k_{ji}\) is the unit vector along the direction of \(P_{ji}\) that means it is in the style of \((i \neq 0)\). Under this condition the selected spatial manipulator gains one degree of freedom and the movable tetrahedron requires infinitesimal rotation along the line of action of \(m_3\) and \(P_{3i}\). Therefore all (3) actuators are locked. Similarly in “Fig (3),” two faces of the triangular vectors \(\overline{F}_2\overline{OM}_2\) and \(\overline{F}_3\overline{OM}_3\) lie on a same plane and the selected spatial manipulator gains one degree of freedom. Therefore the movable tetrahedron requires infinitesimal rotation about a line of intersection of plane \(\overline{F}_2\overline{OM}_2\) and \(\overline{F}_3\overline{OM}_3\) that means again all (3) actuators are locked.
V. INVERSE KINEMATIC SINGULARITY ANALYSIS OF EIGHT BAR 3 - PPP SPATIAL PARALLEL MANIPULATOR

Inverse kinematic singularities will occur, because $J_p$ is identity matrix and its determinant is zero. By the inspection of “(4)” $J_p$ is identity matrix and its determinant is zero. The inverse kinematic singularities will occur at its workspace boundary where one or more limbs are in fully stretched or retracted in positions. In this case the manipulator loses 1 or 2 or 3 degrees of freedom. Then the result is zero, there is no output motion of moving platform for an infinitesimal rotation of input limbs.

CONCLUSION

This paper presents singularities in the workspace of 3 – PPP spatial parallel manipulator with eight bar mechanism. The main contribution of this work is the possible singularity expressions one analyzed by forward and inverse kinematic motion equations which are derived by using closed loop technique. The forward kinematic singularities are possible if one of the vectors $\mathbf{F}_j^O \mathbf{M}_j$ vanishes that means lie on a straight line, then all actuators are locked or two faces of the triangular vectors $\mathbf{F}_j^O \mathbf{M}_j$ lie on a same plane, then also all actuators are locked. Similarly the inverse kinematic singularities are possible if one or more limbs are in fully stretched or retracted in positions. The future work can be extended by generating different cognate linkages to generate symmetric three dimensional coupler curves for eight bar 3- PPP spatial parallel manipulators.

REFERENCES


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