

Forward and Inverse Kinematic Singularity analysis of Eight bar 3 – PPP Spatial Parallel Manipulator

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Abstract - An eight bar 3-PPP spatial parallel manipulator with forward and inverse kinematic singularities are analyzed. The determination of singularities in the workspace of 3- PPP spatial parallel manipulator with eight bar mechanism is an important role in machining process of several parallel manipulators for generating symmetric three dimensional coupler curves. The mathematical equations are derived for the proposed approach.

Keywords - Singularities; 3-Dof; Forward Kinematic Analysis; Inverse Kinematic Analysis; Eight Bar Spatial Parallel Manipulator

I. INTRODUCTION

A spatial kinematic eight bar manipulator with fixed and movable tetrahedrons, binary limbs, prismatic and spherical joints are selected. The selected spatial architecture has three degrees of freedom. The design of eight bar spatial parallel manipulator and its three dimensional symmetric coupler curves are more complex due to their kinematic structure. This paper presents forward and inverse kinematic analysis of eight bar spatial parallel manipulator to determine the singularities of a work space, because the singularities are more attractive in several researches. The mathematical equations are derived by using loop closer technique.

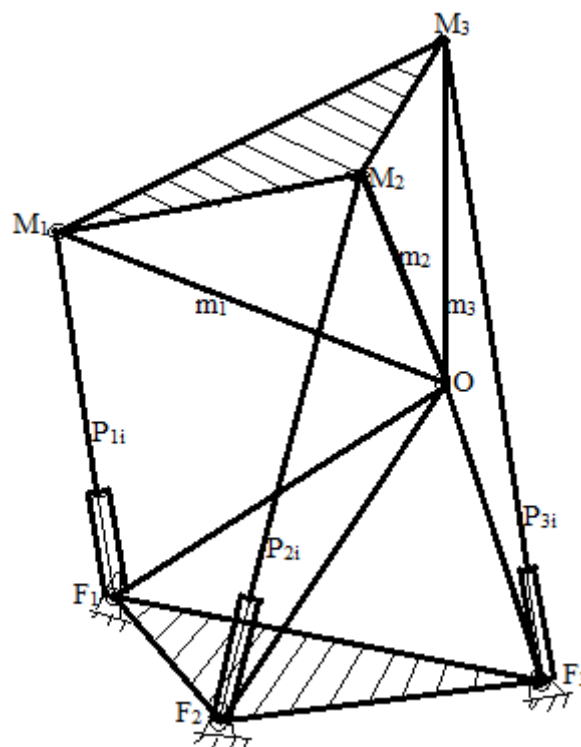


Fig (1) Eight bar 3-PPP spatial parallel manipulator

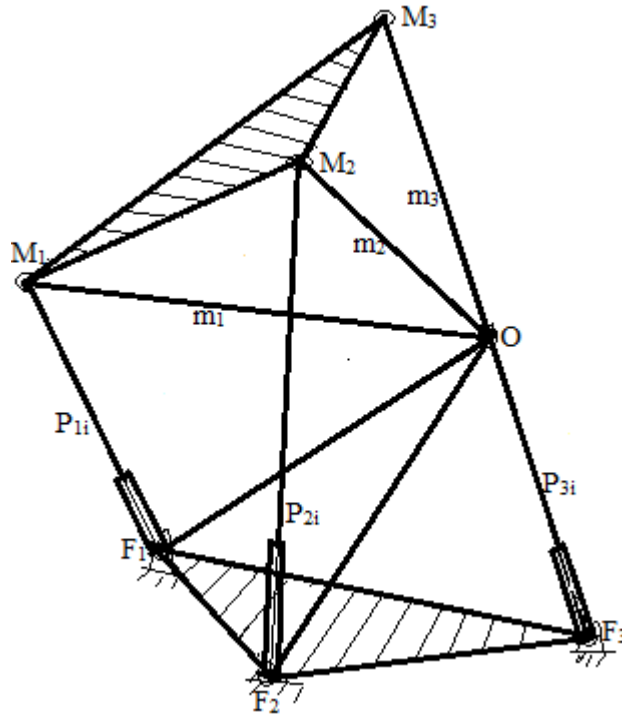


Fig (2) Forward Kinematic Singularity of Eight bar 3-PPP spatial parallel manipulator (one of the vectors $\overline{F_jOM_j}$ vanishes (lie on a straight line) then all actuators are locked)

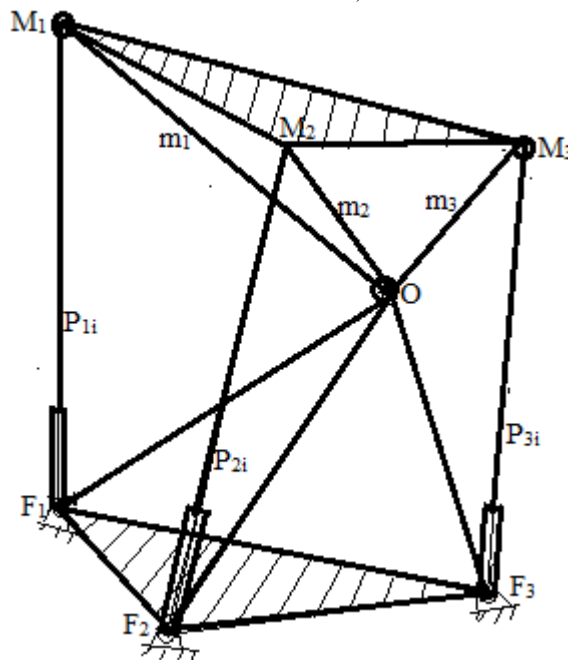


Fig (3) Forward Kinematic Singularity of Eight bar 3-PPP spatial parallel manipulator (two faces of the triangular vectors $\overline{F_jOM_j}$ lie on a same plane then all actuators are locked)

II. KINEMATIC ARCHITECTURE OF EIGHT BAR SPATIAL PARALLEL MANIPULATOR

Kinematic architecture of eight bar spatial parallel manipulator consists of fixed and movable tetrahedrons of its apex points are connected by spherical joint 'O', the three vertices of fixed tetrahedron $F_j|_{j=1,2,3}$ is connected by successive set of spherical joint, prismatic actuator, binary limb and spherical joint which is connected by the successive vertices $M_j|_{j=1,2,3}$ of movable tetrahedron. The binary limbs are actuated by linear actuators which are actuated by three linear

variable differential transducers. Therefore the distance between successive vertices of both fixed and movable tetrahedrons $\overline{F_j M_j} \Big|_{j=1,2,3}$ are variable, that means for each position the value of $\overline{P_{ji}} \Big|_{j=1,2,3 \text{ and } i=1,2,3,4 \text{ etc}}$ are variable.

III. JACOBIAN OF EIGHT BAR 3- PPP SPATIAL PARALLEL MANIPULATOR

The degree of freedom of the selected movable tetrahedron is three. The output rotational vector is described by angular velocities of the movable tetrahedron which is denoted as $[\omega_{mxi} \ \omega_{myi} \ \omega_{mzi}]^T$ and the input translational displacement vector of three prismatic actuators are denoted as $[P_{1i} \ P_{2i} \ P_{3i}]^T$. The loop closer equations for j^{th} limb and i^{th} position can be expressed as

$$[\overline{OM_{ji}}] \Big|_{j=1,2,3 \text{ and } i=1,2,3,4 \text{ etc}} = [\overline{OF_j} + \overline{F_j M_{ji}}] \Big|_{j=1,2,3 \text{ and } i=1,2,3,4 \text{ etc}} \quad (1)$$

The velocity vector loop equation can be obtained by making derivative of “(1)” with respect to time, then the velocity vector loop equations as

$$[\omega_{mi} * m_j] \Big|_{j=1,2,3 \text{ and } i=1,2,3 \text{ etc}} = [(P_{ji})(\omega_{ji}) * k_{ji}] \Big|_{j=1,2,3 \text{ and } i=1,2,3 \text{ etc}} + \left[\left(\frac{\dot{\overline{P_{ji}}}}{\overline{P_{ji}}} \right) \cdot (k_{ji}) \right] \Big|_{j=1,2,3 \text{ and } i=1,2,3 \text{ etc}} \quad (2)$$

Where $\omega_{mi} = [\omega_{mxi} \ \omega_{myi} \ \omega_{mzi}]^T$ angular velocity vectors of the movable tetrahedron for i^{th} position; $m_j = [m_1 \ m_2 \ m_3]$; $P_{ji} = [P_{1i} \ P_{2i} \ P_{3i}]^T$ linear elongation or decrement along the direction of respective binary limbs; $\overline{P_{ji}} = [\overline{P_{1i}} \ \overline{P_{2i}} \ \overline{P_{3i}}]^T$ linear elongation rate of j^{th} limb in i^{th} position; $\omega_{ji} = [\omega_{1i} \ \omega_{2i} \ \omega_{3i}]^T$ angular velocity of j^{th} limb in i^{th} position with respect to fixed frame of tetrahedron; $k_{ji} = [k_{1i} \ k_{2i} \ k_{3i}]^T$ unit vector along the direction of $F_j M_{ji} \Big|_{j=1,2,3 \text{ and } i=1,2,3 \text{ etc}}$; $\overline{OF_j}$ is fixed base therefore ω_{ji} can be eliminated by making dot product on both sides of the above equation by k_{ji} then “(2)” can be written as

$$[(k_{ji} * m_j) \cdot \omega_{mi}] \Big|_{j=1,2,3 \text{ and } i=1,2,3 \text{ etc}} = \left[\left(\frac{\dot{\overline{P_{ji}}}}{\overline{P_{ji}}} \right) \right] \Big|_{j=1,2,3 \text{ and } i=1,2,3 \text{ etc}} \quad (3)$$

since $k_{ji} * k_{ji} = 0$ and $k_{ji} \cdot k_{ji} = 1$

The above equation can be written as

$$\left[[k_{ji} * m_j]^T \omega_{mi} \right] \Big|_{j=1,2,3 \text{ and } i=1,2,3 \text{ etc}} = J_p \left[\left(\frac{\dot{\overline{P_{ji}}}}{\overline{P_{ji}}} \right) \right] \Big|_{j=1,2,3 \text{ and } i=1,2,3 \text{ etc}} \quad \text{That means}$$

$$[J_{mji}(\omega_{mi})] \Big|_{j=1,2,3 \text{ and } i=1,2,3 \text{ etc}} = J_p \left[\left(\frac{\dot{\overline{P_{ji}}}}{\overline{P_{ji}}} \right) \right] \Big|_{j=1,2,3 \text{ and } i=1,2,3 \text{ etc}} \quad (4)$$

Where $J_{mji} = [k_{ji} * m_j]^T$ Jacobians of j^{th} limbs in i^{th} position in form of 3x3 matrices and J_p is 3x3 identity matrix.

IV. FORWARD KINEMATIC SINGULARITY ANALYSIS OF EIGHT BAR 3 - PPP SPATIAL PARALLEL MANIPULATOR

The forward kinematic singularities occur if determinant of $J_{mji} = 0$ that means $\left| [k_{ji} * m_j]^T \right| = 0$.

By the inspection of “Fig (2),” m_3 and P_{3i} lie on straight line, that means one of the three vectors vanishes because k_{ji} is the unit vector along the direction of P_{ji} that means it is in the style of ($i*i=0$). Under this condition the selected spatial manipulator gains one degree of freedom and the movable tetrahedron requires infinitesimal rotation along the line of action of m_3 and P_{3i} . Therefore all (3) actuators are locked. Similarly in “Fig (3),” two faces of the triangular vectors $\overline{F_2 OM_2}$ and $\overline{F_3 OM_3}$ lie on a same plane and the selected spatial manipulator gains one degree of freedom. Therefore the movable tetrahedron requires infinitesimal rotation about a line of intersection of plane $\overline{F_2 OM_2}$ and $\overline{F_3 OM_3}$, that means again all (3) actuators are locked.

V. INVERSE KINEMATIC SINGULARITY ANALYSIS OF EIGHT BAR 3 - PPP SPATIAL PARALLEL MANIPULATOR

Inverse kinematic singularities will occur, because J_p is identity matrix and its determinant is zero. By the inspection of “(4)” J_p is identity matrix and its determinant is zero. The inverse kinematic singularities will occur at its workspace boundary where one or more limbs are in fully stretched or retracted in positions. In this case the manipulator losses 1 or 2 or 3 degrees of freedom. Then the result is zero, there is no output motion of moving platform for an infinitesimal rotation of input limbs.

CONCLUSION

This paper presents singularities in the workspace of 3 – PPP spatial parallel manipulator with eight bar mechanism. The main contribution of this work is the possible singularity expressions one analyzed by forward and inverse kinematic motion equations which are derived by using closed loop technique. The forward kinematic singularities are possible if one of the vectors $\overrightarrow{F_jOM_j}$ vanishes that means lie on a straight line, then all actuators are locked or two faces of the triangular vectors $\overrightarrow{F_jOM_j}$ lie on a same plane, then also all actuators are locked. Similarly the inverse kinematic singularities are possible if one or more limbs are in fully stretched or retracted in positions. The future work can be extended by generating different cognate linkages to generate symmetric three dimensional coupler curves for eight bar 3- PPP spatial parallel manipulators.

REFERENCES

- [1] Pundru Srinivasa Rao, “Forward and Inverse Kinematic Singularity analysis of Eight bar 3 – RRR Planar Parallel Manipulator,” IJERT, Vol.6, Issue 05, pp. 295-297, May 2017.
- [2] C. M. Gosselin, I.A. Bonev, D. Zlatanov, “Singularity Analysis of 3-DOF Planar Parallel Mechanisms via Screw Theory,” ASME J. Mech. Des. 125(3), pp. 573-581, 2003.
- [3] C.M. Gosselin, S. Lemieux and J. P.Merlet, “A New Architecture of Planar 3-DOF Parallel Manipulator,” Proc. IEEE Int. Conf. on Robotics and Automation, Minneapolis , MN, pp. 3738-3743, April 1996.
- [4] J.M. McCarthy, C.L. Collins, “The Singularity Loci of Two Triangular Paralle Manipulators,” Proc. IEEE Int. Conf. ICAR, Monterey, CA, pp. 473-478, July 1997.
- [5] R.P. Podhorodeski, M.A. Nahon, J.A. Carretero, “Workspace Analysis and Optimization of a Novel 3-DOF Parallel Manipulator,” IEEE J. Robotics and Automation, Vol. 15, No. 4, pp. 178-188, 2000.
- [6] J. Wang, Q. Zhang, W. Chou, “Inverse Kinematics and Dynamics of the 3-RRS Parallel Platform,” Proc. IEEE Int. Conf. Robotics and Research, Seoul, Korea, pp. 2506- 2511, 2001.
- [7] C.M. Gosselin and J. Angeles, “Singularity Analysis of Closed – Loop Kinematics Chains,” IEEE Transactions, Robotics and Automation, 6(3), pp. 281-290, 1990.
- [8] R. Boudreau, M. Arsenault, “The Synthesis of 3-DOF Planar Parallel Mechanisms with Revolute Joints for an Optimal Singularity Free Workspace,” J. Rob. Syst. 21(5), pp. 259-274, 2004.
- [9] R. Boudreau, M. Arsenault, “Synthesis of Planar Parallel Mechanisms While Considering Workspace, Dexterity, Stiffness, and Singularity Avoidance,” ASME J. Mech. Des., 128(1), pp. 69-78, 2006.
- [10] C. Gosselin, M.T. Masouleh, “Determination of Singularity Free Zones in the Workspace of Planar 3-PRR Parallel Mechanisms,” Vol. 129, pp. 649-652, June 2007.