# Formulation Of Optimal Economic Order Quantity Under Different Inventory Models 

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#### Abstract

Decision making in production planning and inventory management is basically a problem of coping with large numbers and with a diversity of factors external and internal to the organization. The formulation and solution of an inventory model depend on the demand per unit time of an item which may be viewed as deterministic or probabilistic. The classical economic order quantity (EOQ) model assumes that items produced are of perfect, quality and that the unit cost production is independent of demand. However in realistic situation, product quality is never perfect, but it is directly affected by the reliability of the production process. Invertories are idel resources that possess economic value. There are vitally important to manufacturing firms because they store the value of the labour and processing activities used to make their products. Adequate inventories facilitate production and help to assure customers of good service, on the other hand carrying inventories ties up working capital on goods that sit ideal, and earning any return on investment. Hence the role of inventory management is to maintain adequate, but not excessive levels of inventories. The objective of basic inventory model is to determine the optimal order quantity that minimise the total incremental costs of holding


ABSTRACT
inventory and processing orders. In general the main objective of the inventory model is to minimize the cost of production and minimize the profit. In the present paper we have made an attempt of derive an economic order quantity formula in the different situations prevailing on the average cost of the item along with demand rate keeping finite production associated with the stock position. While analysing the problem we have formulated the EOQ for an optimal order level and the minimum average cost with uniform demand satisfying the shortage criterion.
Key word : Inventory model, EOQ model, Setup Cost, Shortage Cost.

## I. Introduction :

Inventory plays very important roles in decision science, economic, optimization design Science, Communication, Supply chain management, news vender problem, managements in style goods and perishable items and many other fields of science and technology. It can be considered as a necessary evil because the lack of synchronization in the production system makes holding inventory indispensable. Generally inventories deals with various functions such as: (i) Co-ordinating operations (ii) Smoothing production (iii)

Achieving economics of scale and (iv) Improving customer service. The problem which helps to determine the most optimal order quantity under stable conditions is generally known as classical economic order quantity (EOQ) inventory problem.

Though the theory of inventory have been studied more than a century, yet this field of study is drawing more attentions from the researchers of many other branches of mathematics. The main reason behind this attraction is to search a suitable replacement for the decision making for out requirement, which is essential to solve many real world optimization problems.

Over the years, a voluminous amount of research on this subject has been done and many interesting results have been published in various journals. Extensive survey of research work have been conducted by Bishop [1], Prasad [2], Zipkin [3], Spencer [4], Silver [5], urgreletti [6], Wanger [7] and Ziegler [8]. Rosenblatt and Lee [9] have discussued two sophisticated EOQ model with imperfect production which deteriorate over time under different scenarios.

The setup cost of the items produced is an important factor in EOQ model. First of all Kaplan [10] points out that there are three fundamental purposes for cost accounting system such as valuing inventory for financial statement, providing feedback to production managers and measuring the cost of individual stack keeping unit (SKU).

It is a natural curiosity to know, what decisions are requirement? And why one should replace it ! The objective of this presentation is not to describe these two questions vividly.

However on the cases of discussion it may be described briefly as "An inventory can be classified into" raw materials inventory, workin process inventory and finished goods inventroy. The raw materials inventory removes dependency between supplliers and plants. The work in process inventory removes dependency between various machines of a product line. The finished good inventory removes dependency between plants and its customers or market. The inventory model is primarily concerned with answering two fundamental questions :
(1) How much to order?
(2) When to order?


The answer to the first question determines the economic order quantity (EOQ) or Economic lot size problem or optimal order size by minimizing the following cost model.

An inventory problem exists if the amount of the goods in stock is subject to control and if there is at least one cost that decreases as inventory increases. The inventory model is based on the following facts.

### 1.1. Advantages :

(i) The economics of production with large run sizes.
(ii) The smooth and efficient running of the business.
(iii) The economics in transportation.
(iv) Faster and adequate service to customers.
(v) Profit from the market where prices are expected to rise

### 1.2 Disadvantages :

(i) House rent
(ii) Interest on invested capital
(iii) Physical handling
(iv) Accounting
(v) Depreciation and deterioration

In this paper we have generalized the work of Silver [11] and waters [12] where we have formulated economic lot size in general keeping demand rate is uniform with finite production along with shortage. Attempts also made to incorporate the economic lot size keeping different production cycle with finite production rate having no shortage.

The organisation of the article is follows : Following the introduction, section - (2) presents important notations and assumption to derive different theorems ralating EOQ. Numerical Examples and concluding remarks are presented in section 3 and 4 respectively.

## 2. Some Definitions and Notations

EOQ is that size of order which minimizes total annual (or any other time period as specified by individual firms) costs of carrying inventory and cost of ordering.

[Fig. 1 - Relationship between cost \& Quantity]

### 2.1. Assumptions of EOQ.

(a) Precise knowledge of demand of items and there uses rate are also assumed to be constant.
(b) Delivery of units ordered is virtually instantaneous.
(c) Price per unit, ordering and carrying costs are all assumed to be constant regardless of order quantity.
Purchasing Cost : Purchasing cost is based on the price per unit of the item. It may be constant or it may be offered at a discount that depends on the size of the order.

Setup Cost : Setup Cost represents the fixed charge incurred when an order is placed. This cost is independent of the size of the order.

Holding Cost : Holding cost represents the cost of maintaining the inventory in stock. It includes the interest on capital as well as the cost of storage, maintenance and handling.

Shortage cost : Shortage cost is the penalty incurred when we run out of stock. It includes potential loss of income as well as the more subjective cost of loss in customers good will.

### 2.2. Notations

$\mathrm{I}=$ The cost of carrying one rupee in Inventory for the unit time.
$h_{c}=$ Holding cost per unit per unit time.
$\mathrm{S}_{\mathrm{c}}=$ Shortage cost per unit per unit time
$=$ Setup cost per production run
$\mathrm{q}=$ Lot size per production run (i.e. the quantity produced in one run)
$\mathrm{t}_{\mathrm{d}}=$ Total demand
$\mathrm{d}_{\mathrm{r}}=$ Demand rate
K = Production rate
C = Average total cost per unit time
$\mathrm{t}_{\mathrm{i}}=$ time interval between run i and $\mathrm{i}+1$ for $\mathrm{i}=1,2, \ldots \mathrm{n}$.
$\mathrm{T}=$ total time.
$t=$ time interval between two consecutive ordering of inventory.
z = Stock level / order level
$\mathrm{L}=$ Lead time
$\mathrm{q}^{*}, \mathrm{t}^{*}, \mathrm{z}^{*}$ be the optional values of $\mathrm{q}, \mathrm{t}, \mathrm{z}$ respectively for which the cost C is minimum. Now we will prove the following theorems.

## Theorem-1 :

The minimum cost per unit time of the Economic lot size is proportional to the Square root of the demand rate keeping holding and setup cost constant.
Proof :
In this inventory model orders of equal size are placed at periodical intervals. The purchase price per unit is same irrespective of order size.

Let us suppose.
(i) Demand is uniform at a rate of $\mathrm{d}_{\mathrm{r}}$ units time.
(ii) Production rate is finite.
(iii) Lead time is zero
(iv) Shortages are not allowed.

Let ' $q$ ' be the units of quntity produced per production run at interval t .

Since the demand rate is $d_{r}$ units per unit time, then the total demand in one run of time interval $t$ is $d_{r} . t$

The quantity produced per production run is $\mathrm{q}=\mathrm{d}_{\mathrm{r}} \mathrm{t}$

The cost of holding inventory is $\frac{h}{2} h_{C} q \cdot t$
The total cost per production run of time $t$ is
given by $C=\frac{h}{2} h_{c} q t+s_{c}^{\prime}$
The average total cost per production run of time $t$ is
$C=\frac{1}{2} h_{c} q+\frac{s_{c}^{\prime}}{t}=\frac{1}{2} h_{c} q+\frac{s_{c}^{\prime} \cdot d_{r}}{q}$
For minimum value of C we have $\frac{d C}{d q}=0$
$\Rightarrow \frac{1}{2} h_{c}-\frac{s_{c}^{\prime}-d_{r}}{q^{2}}=0$
and the minimum order is

$$
\begin{equation*}
=q^{*}=\sqrt{\frac{2 s_{c}^{\prime} \cdot d_{r}}{h_{c}}} \tag{3}
\end{equation*}
$$

From (1) and (2) the optimum value of $t$ and the minimum cost per unti time is given by

$$
\begin{equation*}
t^{*}=\sqrt{\left(\frac{2 s_{c}^{\prime}}{h_{c} \cdot d_{r}}\right)} \tag{4}
\end{equation*}
$$

$C_{\text {min }}=\frac{1}{2} h_{c} \cdot \sqrt{\frac{2 s_{c}^{\prime} \cdot d_{r}}{h_{c}}}+s_{c}^{\prime} \cdot d_{r} \sqrt{\frac{h_{c}}{2 s_{c}^{\prime} \cdot d_{r}}}=\sqrt{2 h_{c} \cdot s_{c}^{\prime} \cdot d_{c}}$
Considereing $\mathrm{h}_{\mathrm{c}}$ and are constants then the minimum cost per unit time is proportional to the square root of the demand rate.

## Theorem-2 :

The minimum cost per unit time of economic lot size is propertional to the square root of total demand value, the demand rate are different in different production cycles with finite production rate having no shortage.

## Proof :

Let $q$ be the units of quantity produced per production run. If $R$ is the total demand in the total period ' $t$ ', then the number of production cycles $=\frac{R}{q}=n$.
(since the shortages are not allowed)


Fig-2
As $\mathrm{t}_{1}, \mathrm{t}_{2}, \ldots \ldots . \mathrm{t}_{\mathrm{n}}$ be the times to the successive production cycles than $t=t_{1}+t_{2} \ldots+t_{n} \quad \ldots$ (5)

Thus ' $q$ ' the quantity produced at the beginning of each production run is supplied with different uniform demand rates in times $t_{1}$, $\mathrm{t}_{2}, \ldots . . \mathrm{t}_{\mathrm{n}}$ in the successive cycles.

The cost of holding inventoty for the period $t$ is
$h_{c}\left(\frac{1}{2} q t_{1}+\frac{1}{2} q t_{2}+\ldots . .+\frac{1}{2} q t_{n}\right)=\frac{1}{2} q h_{c} t$
and setup cost $=n s_{c}^{\prime}=\frac{R}{q} s_{c}^{\prime}$
The total cost for the period ' $t$ '

$$
\begin{equation*}
=\frac{1}{2} q h_{c} t+\frac{R}{q} s_{c}^{\prime} \tag{6}
\end{equation*}
$$

The average total cost per unit time

$$
\mathrm{C}=\frac{1}{2} q h_{c}+\frac{R s_{c}^{\prime}}{t q}
$$

For minimum value of C

$$
\frac{d C}{d q}=\frac{1}{2} h_{c}-\frac{R s_{c}^{\prime}}{t q^{2}}=0
$$

$$
\begin{equation*}
q^{*}=\sqrt{\frac{2 R s_{c}^{\prime}}{h_{c} t}} \tag{7}
\end{equation*}
$$

Hence (7) gives economic lot size formula. The minimum cost per unit time is

$$
\begin{align*}
& C_{\min }=\frac{1}{2} h_{c} \sqrt{\left(\frac{2 R s_{c}^{\prime}}{h_{c} t}\right)}+\frac{R s_{c}^{\prime}}{t} \sqrt{\frac{h_{c} t}{2 R s_{c}^{\prime}}} \\
& \text { or, } \quad C_{\min }=\sqrt{\frac{h_{c} s_{c}^{\prime} R}{2 t}}
\end{align*}
$$

It is clear that the relations (7) and (8) may be obtained from the corresponding relation of (3) and (4) of theorem (1) replacing $\mathrm{d}_{\mathrm{r}}$ (demand rate) by (average demand rate).

## Theorem 3 :

The minimum cost per unit time of economic lot size is $q=q^{*}=\left[\left\{\frac{2 s_{c}^{\prime}}{h_{c}}\left(\frac{d_{r} k}{k-d_{r}}\right)\right\}\right]$ is the economics lot size.
Proof: Let $\mathrm{K}>\mathrm{d}_{\mathrm{r}}$ the number of items produced per unit time.

If ' $q$ ' is the number of items produced per production run then the item produced per production is

$$
\begin{equation*}
t_{1}=\frac{q}{k} \tag{9}
\end{equation*}
$$

and the time of one complete production run i.e., $t=\frac{q}{d_{r}}$

If Q is the inventory level at the moment the production is completed then

$$
\begin{equation*}
Q=q-d_{r} t_{1}=q-d_{r} \frac{q}{k}=q\left(1-\frac{d_{r}}{k}\right) . \tag{11}
\end{equation*}
$$

The cost holding inventory for the period

$$
t=h_{c} \cdot\left(\frac{t Q}{2}\right)=\frac{1}{2} q h_{c}\left(1-\frac{d_{r}}{k}\right) t
$$

The total cost one run of period

$$
t=s_{c}^{\prime}+\frac{1}{2} q h_{c}\left(1-\frac{d_{r}}{k}\right) t
$$

The total cost per unit time

$$
C=\frac{s_{c}^{\prime}}{t}+\frac{1}{2} q h_{c}\left(1-\frac{d_{r}}{k}\right)
$$

The total cost per unit time

$$
\begin{equation*}
=\frac{s_{c}^{\prime} \cdot d_{r}}{q}+\frac{1}{2} q h_{c}\left(1-\frac{d_{r}}{k}\right) \tag{12}
\end{equation*}
$$

For minimum value of ' $c$ '

$$
\begin{align*}
& \frac{d c}{d q}=\frac{-s_{c}^{\prime} d_{r}}{q^{2}}+\frac{1}{2} h_{c}\left(1-\frac{d_{r}}{k}\right)=0 \\
& q=q^{*}\left[\left\{\frac{2 s_{c}^{\prime}}{h_{c}}\left(\frac{d_{r} k}{k-d_{r}}\right)\right\}\right] \tag{13}
\end{align*}
$$

Using (13) the minimum cost is

$$
\left.\left.\begin{array}{rl}
C_{\text {min }}=s_{c}^{\prime} & d_{r}
\end{array}\right] \frac{h_{c}}{2 s_{c}^{\prime}}\left(\frac{k-d}{d_{r} k}\right)\right] .
$$

$$
\sqrt{\left[2 h_{c} \cdot s_{c}^{\prime} d_{r}\left(1-\frac{d_{r}}{k}\right)\right]}
$$

and the time of one run is

$$
t^{*}=\frac{q^{*}}{d_{r}}=\sqrt{\left(\frac{2 s_{c}^{\prime} k}{h_{c} d_{r}\left(k-d_{r}\right)}\right)}
$$

## Theorem-4 :

The minimum cost of economic lot size in constant time ' $t$ ' with finite rate of production and having shortage is given by

$$
C_{\min }=\frac{h_{c} s_{c} Q}{2\left(h_{c}+s_{c}\right)}=\frac{h_{c} s_{c} d_{r} t}{2\left(h_{c}+s_{c}\right)}
$$

Proof : To proof this theorem we assumes the condition (i), (ii), (iii) of theorem (1) and allowing shortages with backlogg

Let z be the order to which the inventory is raised in the beginning of a run of time interval $t$ This inventory is reduced to zero in time $t_{1}=\frac{z}{d_{r}}$

Then the shortages arise and increase from 0 to $\mathrm{Q}-\mathrm{z}$ in the remaining time $\left(\mathrm{t}-\mathrm{t}_{1}\right)$ where Q is the total demand for one run.
$\mathrm{Q}=$ dr.t
The cost of holding inventory

$$
h_{c} \cdot \frac{1}{2} z \cdot t_{1}=\frac{h_{c} z^{2}}{2 d_{r}}
$$

The shortage cost for one run

$$
=s_{c} \cdot \frac{1}{2}(Q-z) \cdot\left(t-t_{1}\right)
$$

$$
\begin{aligned}
& =s_{c} \cdot \frac{1}{2}(Q-z) \cdot\left(t-t_{1}\right) \\
& =\frac{1}{2} s_{c}(Q-z) \cdot\left(\frac{R}{d_{r}}-\frac{z}{d_{r}}\right)=\frac{1}{2} \frac{s_{c}}{d_{r}}(Q-z)^{2}
\end{aligned}
$$

The average total cost per unit time

$$
\begin{aligned}
C(z) & =\frac{1}{t}\left[\frac{h_{c}}{2 d_{r}} z^{2}+\frac{s_{c}}{2 d_{r}}(Q-z)^{2}\right] \\
& =\frac{d_{r}}{Q}=\left[\frac{h_{c}}{2 d_{r}} z^{2}+\frac{s_{c}}{2 d_{r}}(Q-z)^{2}\right]
\end{aligned}
$$

$$
c(z)=\frac{h_{c}}{2 Q} z^{2}+\frac{s_{c}}{2 Q}(Q-z)^{2}
$$

Since the time $t$ is fixed and known and $s_{c}^{\prime}$ is constant the average setup cost is also constant $\frac{s_{c}^{\prime}}{t}$ is not included on the average cost C(z)

For the minimum value of $\mathrm{C}(\mathrm{z})$

$$
\begin{align*}
& \frac{d C}{d z}=\frac{h_{c}}{Q} z-\frac{s_{c}}{Q}(Q-z)=0 \\
& \therefore \quad z=z^{*}=\frac{s_{c} Q}{h_{c}+s_{c}}=\frac{s_{c} d_{r} t}{h_{c}+s_{c}} \tag{17}
\end{align*}
$$

Which is the required optimal order level From (4) the minimum value of $C$ since

$$
\frac{d^{2} C}{d z^{2}}=\frac{h_{c}+s_{c}}{Q} \geq 0 \text { and from (3) minimum }
$$ cost per unit time is given by

$$
\begin{gathered}
C_{\min }=\frac{h_{c}}{2 Q}\left(\frac{s_{c} Q}{h_{c}+s_{c}}\right)+\frac{s_{c}}{2 Q}\left(Q-\frac{s_{c} Q}{h_{c}+s_{c}}\right)^{2}+\frac{d_{r} s_{c}^{\prime}}{Q} \\
C_{\min }=\frac{h_{c} s_{c} Q}{2\left(h_{c}+s_{c}\right)}=\frac{h_{c} s_{c} d_{r} t}{2\left(h_{c}+s_{c}\right)}
\end{gathered}
$$

If $h_{c} \neq 0$, then $\mathrm{h}_{\mathrm{c}}+\mathrm{s}_{\mathrm{c}}>\mathrm{s}_{\mathrm{c}} \quad \mathrm{Z}<\mathrm{Q}$
i.e. the order of level to which the inventory is raised is kept less than the total demand Q to create shortages.

## Theorem 5 :

The minimum cost of economic lot size under uniform rate of demand with finite rate of production and having shortages is given by

$$
2 \sqrt{\left[\frac{h_{c} s_{c} s_{c}^{\prime} d_{r}}{2\left(h_{c}+s_{c}\right)}\right]}=\sqrt{\left[\frac{2 h_{c} s_{c} s_{c}^{\prime} d_{r}}{h_{c}+s_{c}}\right]}
$$

Proof : In orer to prove the theorem we will assume all the assumption of theorem (4) only considering production is instances.

Let q be the order quantity per production run and $z$ be the order level to which the inventory is raised at the begining of a run of time interval t

$$
\begin{equation*}
\backslash \mathrm{Q}=\mathrm{d}_{\mathrm{r}} \mathrm{t} \tag{18}
\end{equation*}
$$

The cost of holding inventory $=\frac{h_{c} z^{2}}{d_{r}}$
Theshortage cost for one run

$$
\frac{s_{c}}{2 d_{r}}(q-z)^{2}=\frac{s_{c}}{2 d_{r}}\left(d_{r} t-z\right)^{2}
$$

The total cost per run

$$
=\frac{h_{c}}{2 d_{r}} z^{2}+\frac{s_{c}}{2 d_{r}}\left(d_{r} t-z\right)^{2}+s_{c}^{\prime}
$$

The average total cost per unit time

$$
\begin{equation*}
C(z, t)=\frac{h_{c}}{2 d_{r}} \frac{z^{2}}{t}+\frac{s_{c}}{2 d_{r}} \frac{\left(d_{r} t-z\right)^{2}}{t}+\frac{s_{c}^{\prime}}{t} . \tag{19}
\end{equation*}
$$

Which is a function of two variables z and t for $\mathrm{C}(\mathrm{z}, \mathrm{t})$ to be minimum

$$
\begin{equation*}
\frac{\partial C}{\partial z}=\frac{h_{c}}{d_{r}} \frac{z}{t}-\frac{s_{c}}{t} \frac{\left(d_{c} t-z\right)}{t}=0 \tag{20}
\end{equation*}
$$

$$
\begin{align*}
\frac{\partial C}{\partial t} & =-\frac{h_{c}}{2 d_{r}} \frac{z^{2}}{t^{2}}+s_{c} \frac{\left(d_{r} t-z\right)}{t} \\
& -\frac{s_{c}}{2 d_{r}} \frac{\left(d_{r} t-z\right)^{2}}{t^{2}}-\frac{s_{c}^{\prime}}{t^{2}}=0 \tag{21}
\end{align*}
$$

From (20) we have

$$
\begin{align*}
& \frac{z}{d_{r} t}\left(h_{c}+s_{c}\right)=s_{c} \\
& z=\frac{s_{c} d_{r} t}{h_{c}+s_{c}} \tag{22}
\end{align*}
$$

Using in the value z in (21) we have

$$
\begin{array}{r}
-\frac{h_{c}}{2 d_{r}}\left(\frac{s_{c} d_{r} t}{h_{c}+s_{c}}\right)+\frac{s_{c}}{t}\left(d_{r} t-\frac{s_{c} d_{r} t}{h_{c}+s_{c}}\right) \\
-\frac{s_{c}}{2 d_{r} t^{2}}\left(d_{r} t-\frac{s_{c} d_{r} t}{h_{c}+s_{c}}\right)-\frac{s_{c}^{\prime}}{t^{2}}=0
\end{array}
$$

$$
\begin{aligned}
& \text { or } t^{2}=\frac{2 s_{c}^{\prime}\left(h_{c}+s_{c}\right)}{h_{c} s_{c} d_{r}} \\
& t=t^{*}=\sqrt{\left[\frac{2 s_{c}^{\prime}\left(h_{c}+s_{c}\right)}{h_{c} s_{c} d_{r}}\right]} \\
& \text { Now, } \frac{\partial^{2} C}{\partial z^{2}}=\frac{s_{c}+h_{c}}{d_{r} t} \\
& \begin{aligned}
& \frac{\partial^{2} C}{\partial t^{2}}=\frac{h_{c} \cdot z^{2}}{d_{r} t^{2}}+s_{c}\left(\frac{z}{t^{2}}\right) \\
&\left.=\frac{h_{c} \cdot z^{2}}{d_{r} t^{3}}+\left(\frac{s_{c} z^{2}}{d_{r} t^{3}}\right)+\frac{2 d_{r} z}{t^{2}}-\frac{2 z^{2}}{t^{3}}\right)+\frac{2 s_{c}^{\prime}}{t^{3}} \\
& \text { and } \\
& \frac{\partial^{2} C}{\partial t \partial z}=\frac{-h_{c} z}{d_{r} t^{2}}-\frac{s_{c} z}{d_{r} t^{2}} \\
& \frac{\partial^{2} C}{\partial t^{2}} \frac{\partial^{2} C}{\partial z^{2}}-\left(\frac{\partial^{2} C}{\partial t \partial z}\right)^{2} \\
&=\left(\frac{h_{c} z^{2}}{d_{r} t^{3}}+\frac{s_{c} z^{2}}{d_{r} t^{3}}+\frac{2 s_{c}^{\prime}}{t^{3}}\right)\left(\frac{h_{c}+s_{c}}{d_{r} t}\right)-\left(\frac{-h_{c} z}{d_{r} t^{2}}-\frac{s_{c} z}{d_{r} t^{2}}\right)^{2} \\
&=\frac{2 s_{c}^{\prime}\left(s_{c}+h_{c}\right)}{d_{r} t^{4}}
\end{aligned}
\end{aligned}
$$

Obviously for the value of t given by (23)

$$
\frac{\partial^{2} C}{\partial z^{2}}>0, \frac{\partial^{2} C}{\partial t^{2}}>0 \text { and } \frac{\partial^{2} C}{\partial z^{2}} \frac{\partial^{2} C}{\partial t^{2}}-\left(\frac{\partial^{2} C}{\partial t \partial z}\right)^{2}>0
$$

C is minimum for the value of t given by (23)

The optimum order quantity for minimum cost is given by

$$
\begin{aligned}
& q^{*}=d_{t} t^{*}=d_{r} \sqrt{\left[\frac{2 s_{c}^{\prime}\left(s_{c}+h_{c}\right)}{h_{c} s_{c} d_{r}}\right]} \\
& q^{*}=\sqrt{\left[\frac{2 d_{r} s_{c}\left(h_{c}+s_{c}\right)}{h_{c} s_{c}}\right]}
\end{aligned}
$$

which is the required economic lot size formula.
$\mathrm{C}_{\text {min }}$, which can be evaluated substantly the value of $z$ from (22) on (19) we have

$$
\begin{aligned}
& C(z, t)=\frac{h_{c}}{2 d_{r} t}\left(\frac{s_{c} d_{r} t}{h_{c}+s_{c}}\right)^{2}+ \\
& \frac{s_{c}}{2 d_{r} t}\left(d_{r} t-\frac{s_{c} d_{r} t}{h_{c}+s_{c}}\right)^{2}+\frac{s_{c}^{\prime}}{t} \\
& =\frac{h_{c} s_{c}^{2} d_{r} t}{2\left(h_{c}+s_{c}\right)^{2}}+\frac{s_{c} h_{c}^{1} d_{r} t}{2\left(h_{c}+s_{c}\right)^{2}}+\frac{s_{c}^{\prime}}{t} \\
& =\frac{h_{c} s_{c}^{2} d_{r} t\left(h_{c}+s_{c}\right)}{2\left(h_{c}+s_{c}\right)^{2}}+\frac{s_{c}^{\prime}}{t}=\frac{h_{c} s_{c} d_{r} t}{2\left(h_{c}+s_{c}\right)}+\frac{s_{c}^{\prime}}{t}
\end{aligned}
$$

substituting the value of $t$ from (23) we have

$$
\begin{gathered}
C_{\min }=\frac{h_{c} s_{c} d_{r}}{2\left(h_{c}+s_{c}\right)} \sqrt{\left[\frac{2 s_{c}^{\prime}\left(h_{c}+s_{c}\right)}{h_{c} s_{c} d_{r}}\right]} \\
+s_{c}^{1} \sqrt{\left[\frac{h_{c} s_{c} d_{r}}{2 s_{c}^{1}\left(h_{c}+s_{c}\right)}\right]} \\
=\sqrt{\left[\frac{h_{c} s_{c} s_{c}^{\prime} d_{r}}{2\left(h_{c}+s_{c}\right)}\right]}+\sqrt{\left[\frac{h_{c} s_{c} s_{c}^{\prime} d_{r}}{2\left(h_{c}+s_{c}\right)}\right]} \\
2 \sqrt{\left[\frac{h_{c} s_{c} s_{c}^{\prime} d_{r}}{2\left(h_{c}+s_{c}\right)}\right]}+\sqrt{\left[\frac{2 h_{c} s_{c} s_{c}^{\prime} d_{r}}{h_{c}+s_{c}}\right]}
\end{gathered}
$$

which is economic lot size formula.

## 3. Numerical Examples

## Example-1 :

A commodity is to supplied at a constant rate of 200 units perday supplies of any amount can be had at any required time but each ordering cost Rs. 50 cost of holding the commodity on inventory Rs. 2 per unit per day while the delay in the supply of the item induces a penality of Rs. 10 per unit delay of one day found the optimal policy ( $\mathrm{q}, \mathrm{t}$ ) where t is the reorder cycle period end of the convention level after recorder. What would be the best policy of the penality cost becomes.

## Solution:

$$
\begin{aligned}
& \mathrm{d}_{\mathrm{r}}=200 \text { cost per day } \\
& s_{c}^{1}=\text { Rs. } 50 \text { per order } \\
& \mathrm{h}_{\mathrm{c}}=\text { Rs. } 2 \text { per unit per days } \\
& \quad \mathrm{s}_{\mathrm{c}}=\text { Rs. } 10 \text { per unit per day } \\
& q^{*}=\sqrt{\frac{2 d_{r} s_{c}^{\prime}}{h_{c}}} \sqrt{\frac{h_{c}+s_{c}}{s_{c}}} \\
& =\sqrt{\frac{2 \times 50 \times 200}{2}} \sqrt{\frac{2+10}{10}} \\
& =109.5 \text { unit }=110 \text { units }
\end{aligned}
$$

$$
t^{*}=\frac{Q^{*}}{d_{r}}=\frac{110}{200}=2 \text { day (approximate)this }
$$

means that the optimum order quantity of 110 units must be supplied after 2days if the penality cost becomes

$$
\begin{aligned}
& Q^{*}=\sqrt{\frac{2 d_{r} s_{c}^{\prime}}{h_{c}}}=\sqrt{\frac{2 \times 50 \times 200}{2}}=100 \text { units } \\
& t^{*}=\frac{Q^{*}}{D}=\frac{100}{200}=\frac{1}{2} \text { days }
\end{aligned}
$$

## Example - 2:

A company has to purchase of a T.V. tube where demand $d_{r}=2000$ tubes per annum, $=$ Rs. 150 per order and $h_{c}=$ Rs. 240 per item per annum, suppose now the supplier inform that if the order size at least 800 units he is prepared to supply the tubes at a discounted price of Rs. 9.80 per tube. Now we shall examine whether the offer of the supplier is attractive or not.

## Solution :

We have EOQ

$$
=Q^{*}=\sqrt{\frac{2 s_{c}^{\prime} d_{r}}{h_{c}}}=\sqrt{\frac{2 \times 2000 \times 150}{2.40}}=500 \text { units. }
$$

$$
\begin{aligned}
\text { Total cost } & =2000 \times 10+\frac{2000}{500} \times 150 \\
& +\frac{500}{2} \times 2.40=\text { Rs. } 21,200.00
\end{aligned}
$$

Now at a quantity $\mathrm{Q}=800$ to be purchase the price available is Rs. 9.80 .

Total cost for Q for 800

$$
\begin{aligned}
& =2000 \times 9.80+\frac{2000}{800} \times 150+\frac{800}{2} \times 2.40 \\
& =\text { Rs. } 20,935<\text { Rs. } 21,200
\end{aligned}
$$

Hence it is go for a price discount to order for 800 units or more.


Fig - 3
(Order size cost curve for price break model)

From the above figure we conclude that the price discount at a quantity of 800 units. At this stage the total cost is lower than the total cost corresponding to 500 units.

## 4. CONCLUSION :

In our work we have tried to specify some basic theorems and examples like the correct amount and quality of items on the hand at the time required with a minimum expenditure of investment constituent with the business expendiency. It implies minimizing the inventories while at the same time ensuring that stockouts donot occur and production does not suffer. In our work we have made an attent to include increased costs arise from the extra stock holding cost caused by the average stock level being higher due to the larger order quantity.

## References :

[1] Bishop, J.L., (1974) : Experience with a successful system for forecasting and inventory control. Operations Research, 22(6), 1224-1231.
[2] Prasad, S. (1994) : Classification of inventory models and system.

International Journals of production Economics, 34, 20922.
[3] Zipkin, P. (1995) : The second golden age of Inventory management. Paper presented at the INFORMS, Los Angeles.
[4] Spencer, M.S and J.F Cox (1995) : An analysis of the product - process Matrix and Repetitive manufacturing. International Journal of production Research, 33 (5), $1275=94$.
[5] Silver, E.A. (1981) : Operation research in inventory management. A review and cirtique. Opns. Res. 628-645.
[6] Urgeleitti, T.G. (1983) : Inventory control models and problems, Eur. J. Opl Res 14 : 1-12.
[7] Wagner, H.M. (1980) : Research portfolio for inventory management and production planning system. Opns Res. 28: 225-475.
[8] Ziegler, D (1996) : Selecting software for cycle counting APICS - The performance Advantage, November 44 47.
[9] Rosenblatt, M.J. and Lee, H.L. (1986) : Economic Production Cycles with imperfect production processes, IIE Trans. 14: 48-55.
[10] Kaplan, R.S. (1988); One cost system Isn't Enough, Havord Business Review (Jan.-Feb), 61-66.
[11] Silver, E. D. pyke and R. Peterson (1998), Inventory management and production planning and Scheduling, 3rd ed. Wiley, New York.
[12] Waters, C. (1992). Inventory Control and management, Wiley, New York.

