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Forecasting of Rice Stock using Winter's Exponential Smoothing and Autoregressive Moving Average Models

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Abstract—**This article discusses numerical computations of the Winter's exponential smoothing and autoregressive moving average models. Both of these models are used to predict the availability of rice stock at Indonesian National Logistics Agency or BULOG in City of Pekanbaru, Capital of Riau Province, Indonesia by considering the seasonal factors.**

Keywords*—*Time series, Winter's exponential smoothing, autoregressive moving average, mean square error

I. INTRODUCTION

Forecasting is very important in many types of organizations since predictions of future events must be incorporated into the decision-making process, such as total demand for products must be forecasted in order to plan total promotional effort, produce increasing numbers of defective items as the process operates over time or determine whether investment in new plants and equipment will be needed in future or plan production schedules and inventory maintenance [1, p. 2-3]. Making the right decisions in the future need to be supported by the existence of a good appropriate forecasting model.

Forecasting model has been widely examined by several researchers before, as done by Sahu and Kumar [5]. They examine a forecasting method for sales of milk product in Chattisgarh using single moving average method, double moving average method, single exponential smoothing method, semi average method and Naive method with weekly demand of data sets. They use four different measures of the accuracy of forecasting methods, that are mean square error, mean forecast error, mean absolute error, and root mean square error. Doganis *et al.* [4] apply radial basis function neural network architecture and a specially designed genetic algorithm methods for sales data of fresh milk. Osabouhien [3] examines and compares six basic time series forecasting models and aids of five different standard forecasting accuracy measures for forecasting the inflation data in Nigeria. Loganathan and Ibrahim [2] examine autoregressive moving average (ARMA) method using integration ways of seasonal factor model to predict tourism demand in Malaysia.

Application of forecasting models is also used for the availability of food stock especially basic food such as rice. Stock of rice is very important in maintaining the stability of the food in those countries where the majority of the population consumes rice, such as Indonesia. Majority of Indonesian people consume rice and make it as the main basic foodstuffs.

Indonesian National Logistics Agency or BULOG is an Indonesian government institution in charge of maintaining the stability of the price and the availability of basic food in Indonesia especially rice. In carrying out the current status, this body is assisted by several Regional Divisions, one of them is Regional Division of Riau and Riau Island Provinces. Monthly distribution of BULOG rice in Regional Division of Riau and Riau Island Provinces is influenced by the availability of a relatively early stock that depends on the amount of the rice needed by the people; the city of Pekanbaru residents are such a case.

According to the results of the census town of Pekanbaru in the year 2010 conducted by the Central Bureau of Statistics, the projected number of the population of city of Pekanbaru in 2015 is estimated around 1,093,416 people. Certainly the need of rice for the city of Pekanbaru must be directly proportional to the number of residents of Pekanbaru where its population continues to increase each year. Therefore, the agency requires the conditions for safe rice stock in the distribution of rice in the beginning of each month. Conditions of rice stocks should be supported by the right mathematical model that can predict the availability of rice stocks in the future.

There are several mathematical models that can be used to predict the availability of rice stocks in BULOG Pekanbaru. However, in this study, a mathematical model used and considered to be able to predict the magnitude of the availability of BULOG rice stocks are Winter's exponential smoothing and Autoregressive Moving Average models. Both of these models are well used to address the availability of data patterns that follow the trend of rice stocks and are influenced by seasonal factors. In their application, both models are compared to see which one is better by considering the value of their minimum mean square errors.

II. FORECASTING OF TIME SERIES MODEL

The time series is the set of sorted data in units of observation time [1, p. 18]. Models used in analyzing the patterns of relationships between variables that will be forecasted by the data are called time series models. In time series model, the right type of data pattern where the model can be tested is important step in choosing a right model. The data patterns can be differentiated into four types, namely the horizontal data patterns, trend data patterns, seasonal data patterns, and cyclical data patterns. After the pattern is identified, then the factors that affect the time series data patterns are analyzed.

If the data pattern of time series is influenced by the trend and seasonal factors and unstable (not stationary), then it is good to use Autoregressive Moving Average or ARMA model in forecasting. The ARMA model is a mix between autoregressive (AR) model and the model of the moving average (MA). The ARMA model form as follows [1, p. 72]:

$$\left(1-\phi_1 B-\cdots-\phi_p B^p\right)X_t=\left(1-\theta_1 B-\cdots-\theta_q B^q\right)a_t \tag{1}$$

where $\left(1-\phi_1 B-\cdots-\phi_p B^p\right)$ is the coefficient of $AR(p)$ model and $\left(1-\theta_1 B-\cdots-\theta_q B^q\right)$ is the coefficient of $MA(q)$ model. This model is commonly known as the model of the $ARMA(p,q)$ where p and q are the order of the autoregressive and the moving average model respectively. The $ARMA(p,q)$ model in equation (1) requires a process of stabilization of time series data used in its application. This can be done by the process of transformation through stationary and differencing. When data are not stationary with respect to the mean, then the differencing process can be done using the following equation [8, h. 71]:

$$X_t^{'}=(1-B)^d X_t \tag{2}$$

or

$$X_t^{''}=(1-B^s)^D X_t \tag{3}$$

where d and D are the differencing order of non-seasonal and seasonal data pattern respectively. If the variance that causes the data stationary is disturbed, then the data can be transformed in the form of $\ln(X_t)$. When the data is stationary data series, then the $ARMA(p,q)$ model can be written as follows [1, p. 72]:

$$\Phi_P\left(B^s\right)\phi_p(B)(1-B)^d(1-B^s)^D X_t=\theta_q(B)\Theta_Q(B^s)a_t \tag{4}$$

where $(1-B)^d$ and $(1-B^s)^D$ follow the differencing process in equation (2) and (3). This model is called $ARIMA(p,d,q)(P,D,Q)^s$ model or $ARMA(p,q)$ model with seasonal factor and differencing process.

The steps in determinating the model $ARMA(p,q)$ with the process of differencing [1, p. 265-266] are the following:

A. *Model Identification*

This is a step of predicting an appropriate model of forecasting. The prediction of the model is done to the degrees of $AR(p)$ and $MA(q)$. In determination of the order of non-stationary $ARMA(p,q)$ model either seasonal or non-seasonal data at any given time series, can be done by identifying plot of the autocorrelation function (*ACF*) and the partial autocorrelation function (*PACF*) from stationary data. The *ACF* and *PACF* data can be obtained using the statistical software R. Then according to Montgomery [1, p. 256] and Suhartono [7, p. 217], the *ACF* and *PACF* theoretical patterns of seasonal and non-seasonal stationary can be seen from Table I.

TABLE I. The pattern of ACF and PACF of seasonal and non-seasonal *ARMA* model

Prediction Model	ACF	PACF
$MA(q)$	Cuts off after lag-q	Dies down exponentially or sinus at lag-q
$MA(Q)^s$	Cuts off after lag-QS	Dies down exponentially or sinus at lag-kS with k=1,2,...
$AR(p)$	Dies down exponentially or sinus at lag-p	Cuts off after lag-p
$AR(P)^s$	Dies down exponentially or sinus at lag-kS with k=1,2,...	Cuts off after lag-PS
$ARMA(p,q)$	Dies down exponentially or sinus at lag-q	Dies down exponentially or sinus at lag-p
$ARMA(P,Q)^s$	Cuts off after lag-QS	Cuts off after lag-PS

B. *Parameter Assessment*

This step is to determine the parameters for the selected model. The evaluation of these parameters is carried out by minimizing the mean square error values.

C. *Diagnoses Checking*

This step is the process of checking the properness of the selected model. The model is said to be proper if the model has a good significance and the model residual follows the normal distribution. The residual can be defined as the difference between the data and the forecasting values. Normality test for the model error is carried out using Box-Pierce statistical test with the following hypothesis:

H_0 : Residual having normal Gaussian models,

H_1 : Residual model is not a normal Gaussian.

Then after the model is declared eligible for the use of the process of checking the diagnoses, then the next is to do the process of forecasting.

If the forcasting conducted does not consider the stationary of the data for the data influenced by trend and seasonal factors, the Winter's exponential smoothing model is good for forecasting. This model is a model of exponential smoothing that uses three smoothing constants: constants for the overall, trend and seasonal. The Winter's exponential smoothing model uses two Winter Seasonal approaches [6, p. 15-21], namely:

1. *Multiplicative Seasonal Model*

   This model is applied for seasonal data from data variance of time series that has increased or decreased. The value of the forecast ($f_{t,k}$) for the period (t+k) reviewed at the end of the period t of this model is

$$f_{t,k} = (L_t + kT_t)S_{t+k-c} \tag{5}$$

   The smoothing values used are as follows:

   a. Base Smoothing

$$L_t = \alpha \frac{X_t}{S_{t-c}} + (1-\alpha)(L_{t-1} + T_{t-1}) \tag{6}$$

   b. Trend Smoothing

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1} \tag{7}$$

   c. Seasonal Smoothing

$$S_t = \gamma \frac{X_t}{L_t} + (1-\gamma)S_{t-c} \tag{8}$$

   where $0 \le \alpha, \beta, \gamma \le 1$, S_{t-c} are the estimation values of seasonal factors, c is the length of the seasonal, and k = 1, 2, ..., c.

2. *Additive Seasonal Models*

   For seasonal data with constant variance from data of time series, Winter's smoothing model with seasonal or additive seasonal model can be used. At the end of the period t, the value of forecast ($f_{t,k}$) for the period (t+k) is obtained from the equation:

$$f_{t,k} = L_t + kT_t + S_{t+k-c} \tag{9}$$

   The smoothing value used are as follows:

   a. Base Smoothing

$$L_t = \alpha(X_t - S_{t-c}) + (1-\alpha)(L_{t-1} + T_{t-1}) \tag{10}$$

   b. Trend Smoothing

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1} \tag{11}$$

   c. Seasonal Smoothing

$$S_t = \gamma(X_t - L_t) + (1-\gamma)S_{t-c} \tag{12}$$

The initial values are required in implementing a forecasting method. The initial values used in Winter's smoothing model are the following:

$$L_c = \frac{1}{c}(X_1 + X_2 + \cdots + X_c)$$

$$T_c = \frac{1}{c}\left(\frac{X_{c+1} - X_1}{c} + \frac{X_{c+2} - X_2}{c} + \cdots + \frac{X_{c+k} - X_k}{c}\right)$$

where c is the length of the seasonal data. The seasonal smoothing can use the initial values as follows:

a. Winter Multiplicative Seasonal

$$S_k = \frac{X_k}{L_c}$$

b. Winter Additive Seasonal

$$S_k = X_k - L_c$$

where k = 1, 2, 3,…, c. Furthermore the values of parameters α, β, and γ can be determined through a linear programming method for the purpose of minimizing mean square error. They are obtained with the help of solver in Microsoft Excel.

After a few forecasting models are obtained, then the next is to do a comparison to choose the best model. The comparison is done by looking at the results of the measurement of the degree of fault of the model. In this study, MSE is used to measure the error of the model, where the errors are expected to be very small and can represent the data.

III. FORECASTING OF RICE STOCKS

The first thing done in the method of forecasting is to analyze the pattern of the data. The data analyzed are the rice stocks at BULOG in Pekanbaru from January 2007 until December 2014. In this case it is not possible to use data from 1967 when the Agency was established, because the data is not available completely and not arranged neatly. Then to make it easier to analyze the pattern of the data, they can be plotted as presenting in Figure 1.

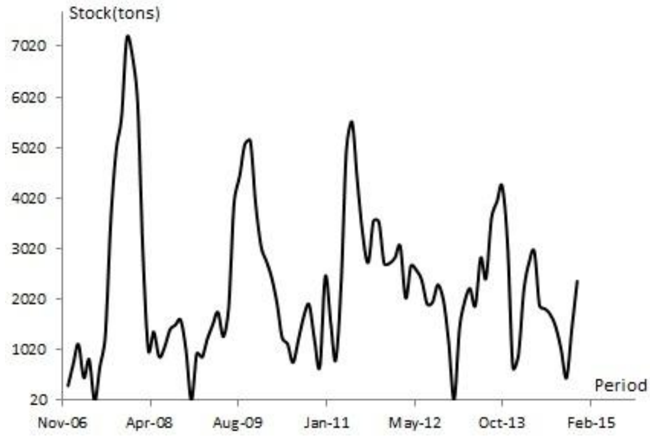

Fig. 1. Plot Data of Rice Stocks at BULOG of Pekanbaru

The data plot in Figure 1 indicates that the data is experiencing considerable fluctuation in between 500 to 7,000 tons of rice. In addition, there was a large increase in the month of December 2007, October 2009-October 2011 and June 2013. The biggest stock decline occurred in June 2007, December 2008, January to October 2013 and 2014. Then if it is done the analysis of the data of the trend pattern, then the trend of rice stocks at BULOG of

Pekanbaru shows that the availability of the rice stocks continue to decline gradually, in fact it can be considered fixed for each year.

In addition, the seasonal recurrence pattern also occurs in a few months and causes the data variance experiencing fluctuation. This indicates that the data pattern is influenced by the trend and seasonal factors. Therefore, it needs to be done the stationery data process (transformation and differencing) for the use of *ARMA*(p,q).

Then after the stationery process is done, identification of the model from stationary data is carried out using the *ACF* and *PACF* plot data. The *ACF* in Figure 2 from the data already stationary indicates that *ACF* does not significant on non-seasonal lags or cuts on lag-1, 2, 18 and *PACF* cut on lag-1, 2, 3, 4, 5, 18. It also occurs in seasonal lags that tend to be cut on lag-1,-2 and lag-12.

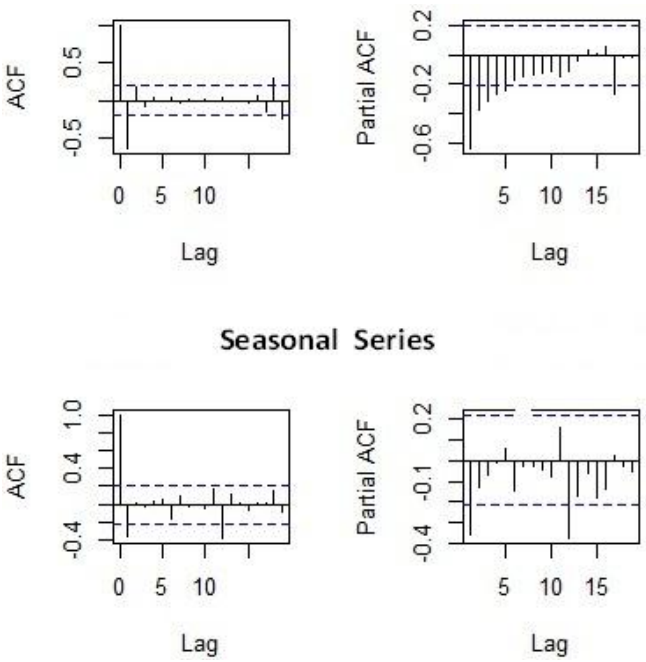

Fig. 2. Plot of *ACF* and *PACF* Data

By using the hint pattern of *ACF* and *PACF* on Table I, allegedly there are 3 pieces of the model that has the smallest error, namely *ARIMA*(2,1,2)(1,1,1)^{12}, *ARIMA*(2,1,1)(1,1,1)^{12}, *ARIMA*(2,2,2)(1,1,1)^{12}. Then suppose that the *ARIMA*(2,1,2)(1,1,1)^{12} the forecast value 1 (NR1) and so on. Then carried out estimation of parameter values to the model. Estimation of the value of this parameter is obtained with the help of statistical software R. The Results estimation of the values of the model parameters are represented in Table II. By using the value of α = 0.05 (degree of freedom), based on the p-value in the Table II, it appears that the model meets the average residual assumption model.

TABLE II. Estimation Parameters of the *ARMA*(p,q) Models

Models	AR(1)	AR(2)	SAR(1)	SAR(2)	MA(1)	SMA(1)	p-value
NR1	1,17	-0,44	-1,03	0,03	-0,23	-0,67	0,76
NR2	1,15	-0,42	-1,00	-	-0,24	-0,66	0,72
NR3	1,18	-0,42	-1,99	0,99	-0,22	0,67	0,99

So it can be said that the selected three models are worth for use in the forecasting process. As for the comparison of the results of the forecast model with ARIMA (2, 1, 2) (1, 1, 1) 12 NR 1, and so on can be seen in Table III.

Further forecasting is done by neglecting the stationary data model or by using stationery smoothing Winter. Due to the seasonal experience of fluctuation variance data, then a multiplicative seasonal Winter model can be used. By taking the initial values for the L_{t-1} = 2131,696612, T_{t-1} = -8,579, and S_t values obtained using estimated data in 2007 and 2008, it is obtained the forecasting values in Table III with a values of MSE = 88,36608423, α = 0,9985, β = 0, and γ = 0,7157.

TABLE III. BULOG Rice Stocks Forecast of Pekanbaru in 2015 (tones)

Months	NR1	NR2	NR3	Winter Multiplicative
January	2903,463	2885,508	2820,554	2383,02
February	3242,097	3213,174	3083,831	2266,48
March	2950,789	2922,527	2730,715	1392,98
April	2552,938	2537,522	2286,352	495,51
Mei	2553,830	2547,355	2208,217	728,28
June	2616.708	2630.208	2220.669	301,36
July	2248.334	2266.871	1804.382	565,04
August	2550,300	2569,260	2045,091	879,40
September	2719,739	2731,625	2156,044	1606,64
October	3086,776	3090,266	2465,697	2070,60
November	2717,398	2715,909	2095,726	2077,10
December	1673,699	1664,787	1068,366	2210,99
MSE of the Models	749041	750593	786941	88,36608423

At the end of the period t, the Winter's forecasting values can be used for equation (5) as following:

$$f_{t,k} = (L_t + kT_t)S_{t+k-c}$$

where values of smoothing for the base level, trend and seasonal can be upgraded by using the equations (6), (7) and (8) for the values of α, β, and γ obtained, namely:

$$L_t = 0{,}9984912 \frac{X_t}{S_{t-c}} + 0{,}0015088 \left(L_{t-1} + T_{t-1} \right)$$

and

$$S_t = 0{,}7157 \frac{X_t}{L_t} + 0{,}2843 S_{t-c}$$

Furthermore the value of the trend smoothing in the period ($t+k$) can use the value of the trend smoothing at the end of the period t.

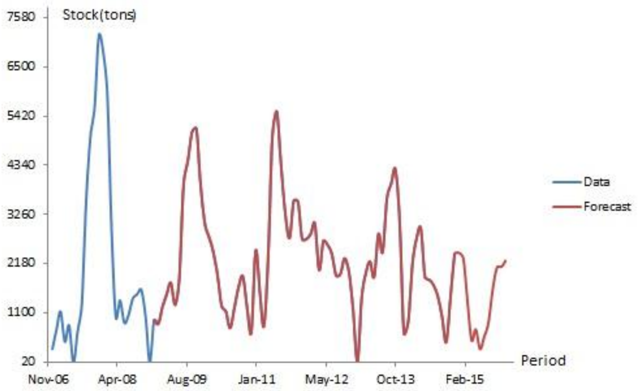

Fig. 3. Forecasting Plot of Winter Multiplicative Model

In the selection of the best model, the minimum of MSE value of the models become criteria. From Table III, Winter multiplicative model has smaller MSE value than *ARMA*(*p*,*q*) models. So the multiplicative model of Winter can be said better than model *ARMA*(*p*,*q*) in representing the data availability of rice stocks at BULOG of Pekanbaru in 2015 at the beginning of each month.

IV. CONCLUSION

Winter exponential smoothing model is a good method to predict data through the constant smoothing. The constants smoothing serve to overcome the factors that affect data such as base level, trend and seasonal. Unlike the case of *ARMA*(*p*,*q*) model, stationery data play an important role in this forecasting model, that serves to overcome the trend and seasonal. In application, the forecasting model of *ARIMA*(2,1,2) $(1,1,1)^{12}$, *ARIMA*(2,1,1) $(1,1,1)^{12}$, *ARIMA*(2,2,2) $(1,1,1)^{12}$ and Winter multiplicative with $\alpha = 0{,}9985$, $\beta = 0$ and $\gamma = 0{,}7157$ are the good models to use. But from the forecasting, the Winter multiplicative model has smaller value of MSE than the other models. So it can be said that the Winter's exponential smoothing with multiplicative seasonal model is a good model to use as a model for forecasting the availability stock rice at BULOG of Pekanbaru in the beginning of each month in 2015.

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